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Pilot Placement for Time-Varying MIMO OFDM Channels with Virtual Subcarriers

Alieh Moradi, Hamidreza Bakhshi, Vahid Najafpoor
Electrical Engineering Department, Shahed University, Tehran, Iran
E-mail: *moradi@shahed.ac.ir*

Abstract

The rapid time-variation of a fading multipath environment can impair the performance of multiple-input multiple-output orthogonal frequency division multiplexing (MIMO OFDM). This paper proposes a pilot placement method for MIMO OFDM systems under time-varying channels with the guard band. The time-varying channel is described by complex exponential basis expansion model (BEM). We discuss the least square (LS) channel estimation to obtain the minimum mean square error (MSE) and derive the pilot allocation that can satisfy the minimum MSE with regard to guard band in time-varying channels. It is shown that optimal pilot clusters can distribute non-uniformly in frequency domain and minimize the MSE. We generalize our scheme over *G* OFDM symbols and compare it with comb pilots. It is demonstrated that the proposed approach is more effective than previous work. Simulation results validate our theoretical analysis.

1. Introduction

Orthogonal frequency division multiplexing (OFDM) gives an effective technique for high data rate transmission. A multiple-input multiple-output (MIMO) system increases system capacity and provides diversity in fading environment [1]. The combination of MIMO and OFDM offers high system throughput.

A pilot signal design is presented in [2] for MIMO OFDM system. The least square (LS) channel estimation is used for channel coefficients. To obtain the minimum MSE, an equal-powered, equal-spaced and phase shift orthogonal pilot sequence is suggested. In [3], the training sequences are designed for OFDM system where pilot tones satisfy the optimal conditions. The placement of pilot symbols is studied in [4] for single-carrier and OFDM systems. The problem of setting and design of pilot sequences is considered in [5] for the frequency-selective channel estimation. The Cramer-Rao Bound (CRB) in order to the minimum MSE of the channel estimation is computed for both single-input single-output and MIMO systems. The optimal pilot allocation is described in [6] for OFDM system. As supposed in it, the channel must be quasi static and have a finite impulse response. The energy exchange between the pilot and data is also derived optimally. In order to derive an op-

timum estimation, [7] also discussed the selection of pilot tones. As shown in it, the training sequences with equal space have the best performance. In [8], the training sequences with virtual subcarriers are presented. The pilot placement is non-uniform and gives the minimum MSE of the LS channel estimation. In practice systems, the presence of the virtual subcarriers can cause non-uniform pilot placement. To achieve optimal conditions, it applies the algorithm which is given in [9] for solving the linear inequalities. In [10], a pilot design scheme for the LS channel estimation is described in MIMO OFDM systems. A suboptimal pilot sequences is also suggested for reduction of the training overhead. General groups of pilot sequences are addressed in [11] for frequency-selective channel estimation in MIMO OFDM systems. Various pilot sequence allocations are arranged for each transmit antenna. These allocations are organized as follows: time-division multiplexing, frequency-division multiplexing, code-division multiplexing in the frequency domain and in the time domain, and different combinations of them.

For multiple antennas, a pilot design approach is proposed in [12], where the data sequences transmit over frequency-selective channels. This low complexity scheme is based on the maximum a lower bound of the average capacity. In [13], the estimation of the correlation

matrices for fading channels is considered. The pilot signals are used for obtaining the channel coefficients. The channel variations are modeled as multichannel autoregressive (AR). The peaks of the AR spectrum show the Doppler frequencies.

A pilot approach is considered in [14] for MIMO OFDM systems in time-varying channels. To increase the performance, the inter-carrier interference (ICI) due to time variations must be decreased in high delay environments. In [15], the channel estimation for MIMO OFDM systems is expressed in linearly time-varying channel. The channel coefficients of the linearly time-varying channel are based on complex exponential basis expansions model. In direct sequence code division multiple access (DS-CDMA) systems, [16] expresses an adaptive estimation of Doppler shifts for several Doppler sub-paths. The channel model is based on basis expansion model (BEM). For suppression of ICI in time-varying channel, [17] presents an analytical framework dependent on the known statistical properties with mobile radio channels. [18] uses a parallel removal technique for discarding ICI in frequency domain. It is supposed that the channel impulse response (CIR) changes linearly along a block period. For time-varying frequency-selective channels, a general framework is described in [19] to achieve the optimal pilot overhead and the optimal pilot boost. For single-input multiple-output time-varying channels, blind channel estimation is given in [20], where the time-varying channel is expressed as a complex exponential basis expansion model (CEBEM). For frequency-selective fading channel estimation, a rectangular-windowed least square estimation is studied in [21], relying on a polynomial model of the time-varying channel delays. For channel estimation, a new block differential codec is derived in [22]. The time-varying channels are described as a basis expansion model. By using the BEM, block differential design has a simple implementation, and derives the maximum Doppler diversity in the time-varying channels.

So as to discard the interference between neighboring channels in practical systems, a guard band is considered at each transmit data symbol. The guard band includes virtual subcarriers that are not used for transmission. In [23], a pilot placement is suggested regard to the guard band. It implies that comb pilots with the largest distance between pilot tones have the best effectiveness by exploiting the equivalence of the Toeplitz and circulant matrices. In this paper, we suggest an optimal pilot allocation that does not rest in the guard band based on the minimum MSE under time-varying channel. We also show that our new scheme has better performance than comb pilots.

The remainder of the paper is organized as follows:

Section 2 expresses the basis expansion model for time-varying channels and describes the MSE of the LS channel estimation. To obtain the minimum MSE, the optimal conditions are also derived in this section. In Section 3, we introduce a new pilot allocation that is optimal with respect to virtual subcarriers. We organize our method over G OFDM symbols in Section 4. Section 5 contrasts our scheme with comb pilots. Section 6 presents the simulation environment and outcomes, and shows improvement in our design. Eventually, the conclusions are expressed in Section 7.

Notations: $\text{tr}(.)$ is used to show the matrix trace. $(.)^*$, $(.)^T$ and $(.)^H$ express conjugate, transpose and conjugate transpose respectively. $\otimes$ is applied for the cyclic convolution. Expectation is presented by $E(.)$. The ceiling of a number is shown by $\lceil . \rceil$. Z denotes the set of integer numbers. Bold letters is presented for matrices (column vectors) and variables are displayed in italic.

2. System Model

The desired system is depicted in **Figure 1**. It is a MIMO OFDM system with N_t transmit antennas, N_r receive antennas and K subcarriers. The length of channel impulse response (CIR) is equal to L for each transmit-receive antenna pair. A cyclic prefix (CP) with length V is inserted to form a complete OFDM symbol. The CP is considered to be larger than the largest multipath delay. The channel estimation procedure is applied similarly at each receive antenna, so we assume to have N_t transmit antenna and one receive antenna.

Let $S_i(n)$ be the data symbol on nth time sample at ith transmit antenna. After removal the CP at the receiver, the output can be written as

$$Y(n) = \sum_{i=1}^{N_t} \sum_{l=0}^{L-1} h_{i,l}(n) S_i(n-l) + N(n) \qquad (1)$$

where $n = 0, 1, \cdots, K-1$ and $N(n)$ is additive Gaussian noise with zero mean and variance of σ_n^2. The channel coefficient can be approximated by a complex exponential model as

$$h_{i,l}(n) = \sum_{q=-Q/2}^{Q/2} h_{i,l,q} \, e^{-j\frac{2\pi qn}{N}} \qquad (2)$$

where $h_{i,l,q}$ is zero mean, complex Gaussian random variables with equal variance $\sigma^2(q,l)$. Since the BEM is periodic with period of N and the real channel is not periodic, N must be larger than the length of transmit data, *i.e.*, $N > K$. $Q = 2\lceil f_{\max} T_s K \rceil$, $f_{\max}$ is the maximum Doppler frequency, $T_s = T_1/K$ is the sample interval and T_1 is the total interval of data symbol [14].

Define:
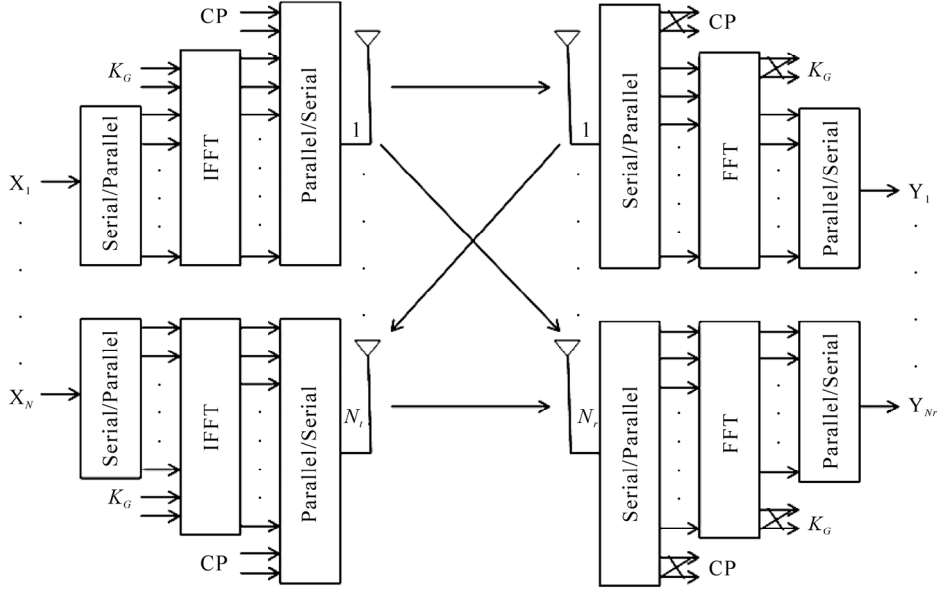
Figure 1. System model.

$$\left[\mathbf{F}_{K \times K}\right]_{k,n} = e^{-\frac{j2\pi kn}{K}}, 0 \leq k \leq K-1, 0 \leq n \leq K-1 \qquad (3)$$

$$H_i(k,q) = \sum_{l=0}^{L-1} h_{i,l,q}\, e^{-j\frac{2\pi ql}{N}} e^{-j\frac{2\pi kl}{K}} \qquad (4)$$

$$\mathbf{V}_q(0) = \mathbf{F}\left[e^{-j\frac{2\pi q \times 1}{N}}, \cdots, e^{-j\frac{2\pi qn}{N}}, \cdots, e^{-j\frac{2\pi q(K-1)}{N}} \right]^T \qquad (5)$$

then, with above definition, the fast Fourier transform (FFT) of $Y(n)$ is given by

$$Y(k) = \sum_{i=1}^{N_t} \sum_{q=-Q/2}^{Q/2} H_i(k,q)\, \mathbf{V}_q(k) \otimes \mathbf{X}_i + \overline{N}(k) \qquad (6)$$

where $k = 0, 1, \cdots, K-1$ and $\overline{N}(k)$ is the FFT of $N(n)$ with variance of σ^2. $\mathbf{X}_i$ is the transmit data vector in frequency domain, *i.e.*, $\mathbf{X}_i = \mathbf{F}\mathbf{S}_i$. $\mathbf{V}_q(k)$ denotes a cyclic shift vector of $\mathbf{V}_q(0)$ with a shifting length k.

As discussed in [15], the major power of the vector $\mathbf{V}_q(0)$ concentrates on two ends. So, $\mathbf{V}_q(0)$ can be approximated by

$$\mathbf{V}_q(0)$$
$$\approx \left[V_q(0), \cdots, V_q(T), 0, \cdots, 0, V_q(K-T), \cdots, V_q(K-1) \right]^T$$
$$V_q(n) = e^{-j\frac{2\pi qn}{N}}$$
$$\qquad (7)$$

where T is a positive number. $T = 2$ or 3 is sufficient for involving the principal lobe of $\mathbf{V}_q(0)$. Since the convolution between $\mathbf{V}_q(0)$ and $\mathbf{X}_i$ is only performed at

$2T + 1$ sequential tones around the tone k, we use pilot clusters with length $2T + 1$. We consider pilot clusters on a set of subcarriers $\Omega = \{k_1, k_2, \cdots, k_P\}$. P denotes the number of pilot clusters. So the pilot sequences at the pilot cluster k_p is

$\mathbf{X}_i(k_p) = X_i(k_p - T), \cdots, X_i(k_p + T)$ at ith transmit antenna.

Define:

$$w_{i,q}(k_p) = \mathbf{V}_q(k) \otimes \mathbf{X}_i(k_p) \qquad (8)$$

$$H_{i,l,q} = h_{i,l,q}\, e^{-j\frac{2\pi ql}{N}} \qquad (9)$$

$$R_{i,l,q}(k_p) = w_{i,q}(k_p)\, e^{-j\frac{2\pi k_p l}{N}} \qquad (10)$$

Then at all pilot clusters, the Equation (6) can be written as

$$\mathbf{Y} = \mathbf{R}\mathbf{H} + \overline{\mathbf{N}} \qquad (11)$$

where $\mathbf{R}$ is a $P \times N_t L(Q+1)$ matrix. The least square channel estimation, supposing $\mathbf{R}^H \mathbf{R}$ has full rank and $P \geq N_t L(Q+1)$, is given by [24]

$$\hat{\mathbf{H}} = \left(\mathbf{R}^H \mathbf{R}\right)^{-1} \mathbf{R}^H \mathbf{Y} \qquad (12)$$

From (12), the MSE of the LS channel estimation can be obtained as

$$\mathrm{MSE} = \frac{1}{(Q+1)LN_t} E\left\{ \left\| \hat{\mathbf{H}} - \mathbf{H} \right\|_2^2 \right\}$$
$$= \frac{\sigma^2}{(Q+1)LN_t} \mathrm{tr}\left(\mathbf{R}^H \mathbf{R}\right) \qquad (13)$$

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In order to obtain the minimum MSE of the LS channel estimation with fixed energy E_i allocated for pilot sequences, we must have

$$\mathbf{R}^H \mathbf{R} = E_i \, \mathbf{I}_{LN_t(Q+1)} \tag{14}$$

where $\mathbf{I}$ shows an identical matrix. For achieving (14) and minimizing the MSE in (13), we must satisfy the following condition as

$$r(m) = \sum_{k \in \Omega} w_{i,q}^*(k) w_{i,q}(k) e^{j\frac{2\pi km}{K}} = 0 \tag{15}$$
$$m \in \{-L+1, \cdots, L-1\} \text{ and } m \neq 0$$

where $r(0) = E_i$. For $q \neq q'$, we have

$$s(m) = \sum_{k \in \Omega} w_{i,q}^*(k) w_{i,q'}(k) e^{j\frac{2\pi km}{K}} = 0 \tag{16}$$
$$m \in \{-L+1, \cdots, L-1\} \text{ and } q,q' \in \{-Q/2, \cdots, Q/2\}$$

finally, for two different transmit antennas and $q \neq q'$, we require

$$d(m) = \sum_{k \in \Omega} w_{i,q}^*(k) w_{j,q'}(k) e^{j\frac{2\pi km}{K}} = 0 \tag{17}$$
$$m \in \{-L+1, \cdots, L-1\} \text{ and } i \neq j$$

3. Pilot Placement with Guard Band

In practical system, some of subcarriers are not use for transmission that called virtual subcarriers. The virtual subcarriers set in the guard band. For avoiding the interference between communication channels, the guard band is considered at each transmit data sequence. The transmit subcarriers are assumed to be belonged to

$$\Pi = \left\{ 0, \pm 1, \cdots, \pm \left(\frac{K}{2} - \frac{K_G}{2} \right) \right\}$$

where K_G denotes the number of the virtual subcarriers. The purpose is to estimate the channel impulse response under time-varying channel with guard band. Therefore, we suggest the pilot placement that minimizes the MSE in the presence of the guard band under the BEM.

As discussed in previous section, for minimizing the MSE, the conditions (15), (16) and (17) must equal to zero. In this paper, for simplification, it is assumed K is even and K_G is odd. Let $E\Delta K = K$. Define M as the number of pilot cluster groups at each transmit symbol, where E is the number of pilot clusters and ΔK is the distance between pilot clusters at each groups. So, we have M pilot cluster groups at each transmit antenna. Since pilot tones are outside the guard band, the distance between pilot clusters at each group must be larger than the length of the guard band, *i.e.*, $\Delta K > K_G$. The condition of full rank $\mathbf{R}^H \mathbf{R}$ shows that we should have at

least $N_t L(Q+1)$ pilot clusters, *i.e.* $E \geq N_t L(Q+1)$. Consider $a^{(h)}$ is the power allocated for each pilot in hth group, where $\sum_h a^{(h)} = E_i/E$. The following pilot allocation is proposed as

$$\left| w_{i,q}(k_p) \right|^2 = \begin{cases} a^{(h)}, & k_p = \mu + e\Delta K + h(2T+1); \\ 0, & \text{elsewhere;} \end{cases} \tag{18}$$

where $\mu = -\left(K/2 - (K_G+1)/2\right) + T$, $e = 0,1,\cdots,E-1$ and $h = 0,1,\cdots,M-1$. The problem is to find M and imply that the pilot allocation (18) satisfies (15), (16) and (17) with regard to the guard band.

1) for $e = 0$ and $h = 0$, the relation (18) can be expressed as

$$\left| w_{i,q}(k_{p\text{first}}) \right|^2 = a^{(0)}, \quad k_{p\text{first}} = -\left(\frac{K}{2} - \frac{K_G+1}{2} \right) + T \tag{19}$$

as seen, k_p is the middle tone at each pilot cluster. So, the first tone places at $k_p - T$ and the last ton sets at $k_p + T$. $k_{p\text{first}} - T = -\left(K/2 - (K_G+1)/2\right)$ is the first edge of the subcarriers that is outside the guard band.

2) for $e = E-1$ and $h = M-1$, we have

$$\left| w_{i,q}(k_{p\text{last}}) \right|^2 = a^{(M-1)}, \tag{20}$$
$$k_{p\text{last}} = \mu + (E-1)\Delta K + (M-1)(2T+1)$$

where $k_{p\text{last}} + T$ is the last tone of the last cluster that should be outside the guard band. It means

$$k_{p\text{last}} + T \leq \frac{K}{2} - \frac{K_G+1}{2} \tag{21}$$

$$\mu + (E-1)\Delta K + (M-1)(2T+1) + T \leq -\mu + T \tag{22}$$

then, M can be written as

$$M \leq \frac{\Delta K - K_G}{2T+1} \tag{23}$$

for above M, our proposed pilot placement do not include the virtual subcarriers. **Figure 2** illustrates subcarriers with guard band.

Now, we should demonstrate this pilot allocation zeros (15), (16) and (17).

3) with substituting condition (18) in (15), it can be given by

$$\begin{aligned} r(m) &= \sum_{e,h} a^{(h)} e^{j2\pi(\mu+e\Delta K+h(2T+1))m/K} \\ &= a^{(0)} e^{j2\pi(\mu)m/K} \sum_e e^{j2\pi m(e\Delta K)/K} \\ &+ a^{(1)} e^{j2\pi(\mu+2T+1)m/K} \sum_e e^{j2\pi m(e\Delta K)/K} \\ &\vdots \\ &+ a^{(M-1)} e^{j2\pi(\mu+(M-1)(2T+1))m/K} \sum_e e^{j2\pi m(e\Delta K)/K} \end{aligned} \tag{24}$$
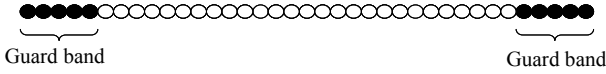
Guard band Guard band

Figure 2. Subcarriers with guard band.

Table 1. Placement of pilot clusters for $N_t = 2$, $K = 516$, $K_G = 23$, $L = 3$ **and** $E = 12$.

Group I	Group II	Group III	Group IV
−244	−239	−234	−229
−201	−196	−191	−186
−158	−153	−148	−143
−115	−110	−105	−100
−72	−67	−62	−57
−29	−24	−19	−14
14	19	24	29
57	62	67	72
100	105	110	115
143	148	153	158
186	191	196	201
229	234	239	244

it is implied

$$\sum_{e=0}^{E-1} e^{j2\pi m(e\Delta K)/K} = 1 + e^{j2\pi m\Delta K/K} + \cdots + e^{j2\pi m\Delta K(E-1)/K}$$

$$= 1 + e^{j\frac{\phi}{E}} + e^{j2\frac{\phi}{E}} + \cdots + e^{j\frac{(E-1)\varphi}{E}} \quad (25)$$

$$= 0$$

where $\varphi = 2\pi m$. We corroborate that $r(m)$ is zero for our pilot allocation. For minimizing the MSE, $s(m)$ and $d(m)$ should also become zero.

4) for two different transmit antennas and $q \neq q'$, the training sequence is designed as

$$w_{i,q}(k_p) = \sqrt{a^h} e^{-j2\pi(i-1+q+Q/2)Le/E} \quad (26)$$

After, setting (26) in (17), it can be obtained

$$d(m) = \sum_h a^{(h)} e^{j\frac{2\pi(\mu+h(2T+1))m}{K}} \sum_{e=0}^{E-1} e^{j\frac{2\pi\lambda e}{E}} = 0 \quad (27)$$

$$m \in \{-L+1,\cdots,L-1\} \text{ \& } i \neq j \text{ or } q \neq q'$$

where $\lambda = (i-j+q-q')L+m$. Since $E \geq N_t L(Q+1)$, λ is always smaller than E and $\sum_{e=0}^{E-1} e^{j2\pi\lambda e/E}$ becomes zero for all m. Therefore, we introduce a new pilot placement under time-varying channel with regard to the guard band that minimizes the MSE and achieves the optimal conditions. We can generalize our scheme over G OFDM symbols.

Table 1 shows an example of our pilot allocation in four groups. Consider two transmit antennas. Let $K = 516$, $L = 3$, $K_G = 23$, $Q = 1$ and $T = 2$. As known $E \geq N_t L(Q+1)$, so we choose $E = 12$ and $\Delta K = 43$. Relation (23) provides $M = 4$. Each group has 12 pilot clusters and each cluster requires 5 pilot tones.

4. Pilot Placement over *G* OFDM Symbols

In this section, we suppose G OFDM symbols at each transmit antenna. Let $w_{i,q}^{(g)}(k)$ be $w_{i,q}(k)$ at gth OFDM symbol. With this consideration, condition (14) changes to

$$\sum_{g=0}^{G-1} \mathbf{R}^{(g)H} \mathbf{R}^{(g)} = E_i \mathbf{I}_{LN_t(Q+1)} \quad (28)$$

where $\mathbf{R}^{(g)}$ is $\mathbf{R}$ at gth OFDM symbol.

From (28), conditions (15), (16) and (17) can be written as

$$\sum_{g=0}^{G-1} \sum_{k \in \Omega} w_{i,q}^*(k) w_{i,q}(k) e^{j\frac{2\pi km}{K}} = 0 \quad (29)$$

$$m \in \{-L+1,\cdots,L-1\} \text{ \& } m \neq 0$$

For $q \neq q'$, it is given by

$$\sum_{g=0}^{G-1} \sum_{k \in \Omega} w_{i,q}^*(k) w_{i,q'}(k) e^{j\frac{2\pi km}{K}} = 0 \quad (30)$$

$$m \in \{-L+1,\cdots,L-1\} \text{ \& } q,q' \in \{-Q/2,\cdots,Q/2\}$$

For two different transmit antennas and $q \neq q'$, it can be obtained as

$$\sum_{g=0}^{G-1} \sum_{k \in \Omega} w_{i,q}^*(k) w_{j,q'}(k) e^{j\frac{2\pi km}{K}} = 0 \quad (31)$$

$$m \in \{-L+1,\cdots,L-1\} \text{ \& } i \neq j$$

To achieve (29), we can write (18) as

$$\sum_{g=0}^{G-1} \left| w_{i,q}(k_p) \right|^2 = \begin{cases} a^{(h)}, & k_p = \mu + e\Delta K + h(2T+1); \\ 0, & \text{elsewhere}; \end{cases} \quad (32)$$

where $\sum_{h=0}^{M-1} a^{(h)} \neq 0$.

From (30) and (31), we can express

$$\sum_{g=0}^{G-1} w_{i,q}^*(k_p) w_{j,q'}(k_p)$$

$$= \begin{cases} a^{(h)}, & k_p = \mu + e\Delta K + h(2T+1); \\ 0, & \text{elsewhere}; \end{cases} \quad (33)$$

5. Comparing the Proposed Method with Comb Pilot

We described pilot sequences that can minimize the MSE for time-varying channel with virtual subcarriers. Now, we interest to contrast our method with comb pilots which have equal space and equal power without crossing the guard band. In [23], comb pilots are used for single input single output OFDM systems in the presence of the guard band. Without considering the virtual subcarriers, pilots with equal space and equal power can optimize (13) with uniform eigenvalues. As known, comb pilots do not place in the guard band and can not optimize the MSE of the LS channel estimation. So, [23] presents a semi-optimal scheme. The equivalence of $r(m)$, with assumption of [23], is given by

$$r(n) = \sum_{k=0}^{K} R(k) e^{j\frac{2\pi kn}{K}} \tag{34}$$
$$n \in \{-L+1, -L+2, \cdots, L-2, L-1\}$$

for $k \in \Omega$, $R(k)$ is equal to 1, *i.e.*, $R(k) = 1$. Let α_P be the number of pilot clusters which distribute symmetric about the origin. Then, we have

$$\Omega = \left\{ \pm \frac{d}{2}, \pm \frac{3d}{2}, \cdots, \pm \frac{d+(\alpha_P - 2)d}{2} \right\}.$$ The inverse discrete Fourier transform of $R(k)$ is written as

$$r(n) = \frac{\sin\left(\frac{\pi d}{K}\alpha_P n\right)}{\sin\left(\frac{\pi d}{K}n\right)} \tag{35}$$

the energy of $r(n)$ diminishes rapidly with raising n for comb pilots. It is shown in [23] that comb pilots with larger distance between pilot tones have better signal to noise ratio because the largest space has the narrowest main lobe.

If we choose $E_i = 1$, $r(m)$ of condition (15) will be equal to one for $m=0$, *i.e.*, $r(0) = E_i$ and will become zero for $m \neq 0$. Thus, $r(m)$ illustrates an impulse function. With this consideration, $r(m)$ is equivalence to $r(n)$ given in [23]. Our proposed method achieves conditions (15), (16) and (17). In comparison with comb pilots, $r(m)$ has the narrowest main lobe and becomes zero suddenly in $m \neq 0$. Therefore, our proposed pilot setting is optimum while comb pilots are sub-optimal.

6. Simulation and Results

We consider two transmit and two receive antennas for MIMO OFDM system. The modulation mode is chosen as BPSK. The channel is supposed to have $L = 3$ and $V = $

5. The OFDM symbol size is selected as $K=516$ and the length of the guard band is $K_G = 23$. We assume that the channel coefficients are distributed identically and independently by complex Gaussian random process. The symbol rate is $f_s = 1 \, \text{MHz}$ or $T_s = 1 \, \mu\text{s}$ and the maximum Doppler frequency is $f_{\max} = 100 \, \text{Hz}$. $N = 5120$ is chosen, where $N > K$. We select $Q = 1$ and $T = 2$, so the length of the pilot cluster is $2T + 1 = 5$. As known, E must be equal or larger than $N_t L(Q+1)$. Consider $E = 12$ and $\Delta K = 43$ because of $E\Delta K = K$. The relation (23) shows that M can be at most 4. We only suppose two groups in our simulation in **Figure 3** and **Figure 4**, *i.e.*, $M = 2$. The number of total pilot clusters is $ME = 24$ and the total pilot tones are equal to $(2T + 1)\,ME = 120$. The multi-path density profile is considered as $\varphi_c(\tau) = \exp(-0.1\tau/T_s)$ and the Doppler power spectrum is $S_c(f) = \left(\pi\sqrt{f_{\max}^2 - f^2}\right)^{-1}$, where $f_{\max} \geq f$.

Figure 3 shows the bit error rate (BER) versus signal to noise ratio (SNR) for the time-varying BEM. The distance between comb pilot cluster 1 is $d = 20$ and the pilot clusters are belonged to the set $\Omega = \{\pm 10, \pm 30, \cdots, \pm 230\}$. The set of comb pilot cluster 2 is $\Omega = \{\pm 9, \pm 27, \cdots, \pm 207\}$, where the distance between them is chosen as $d = 18$. The amplitude of pilot tones is equal to each other. The pilot clusters of proposed approach are placed at $\Omega = \{\pm 244, \pm 201, \cdots, \pm 229\}$, where these tones are selected from group I and group IV in Table I. The amplitude of pilot tones is followed by relation (26). Since we only have two groups, so $a^{(1)}$ and $a^{(2)}$ are zero. As seen, the comb pilot cluster 1 has better effectiveness than comb pilot cluster 2 because of the larger distance between them. The proposed approach yields performance improvements greatly compared to comb pilot clusters.

Figure 4 illustrates the MSE versus SNR for comb pilot cluster 1, comb pilot cluster 2 and proposed approach. By comparison between curves, it is concluded that the proposed scheme is the best.

Figure 5 compares the MSE of proposed approach for different Doppler frequencies as $f_{\max} = 100 \, \text{Hz}$, 200 Hz and 500 Hz. We consider four groups and the total pilot tones are 240, therefore $a^{(1)}$ and $a^{(2)}$ are non-zero. The simulation results show that the effectiveness of the MSE is decreased with increasing Doppler frequency.

7. Conclusion

In this paper, the problem of the pilot placement was studied for time-varying channel estimation with virtual subcarriers. First, we described the BEM for the LS channel estimation under time-varying channels and then, we expressed the optimal conditions based on the mini-
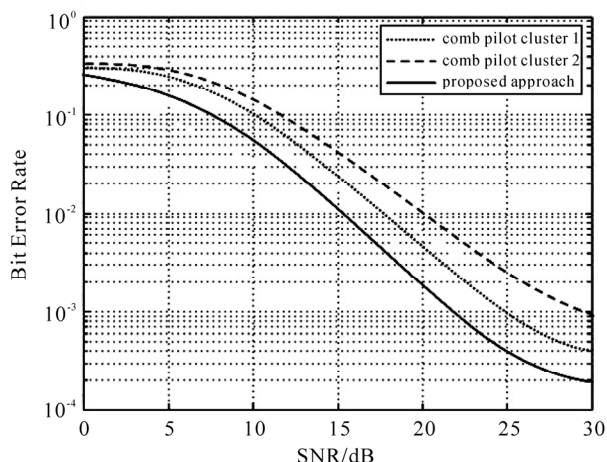
Figure 3. BER performance for pilot placement, $K = 516$, $K_G = 23$, $E = 12$ **and** $L = 3$.
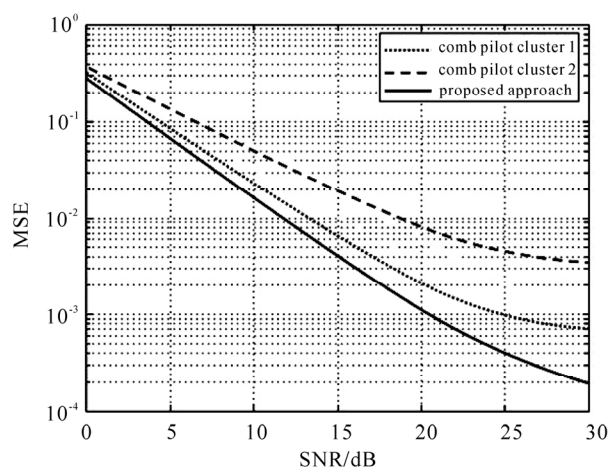

Figure 4. MSE performance for pilot placement, $K = 516$, $K_G = 23$, $E = 12$ **and** $L = 3$.
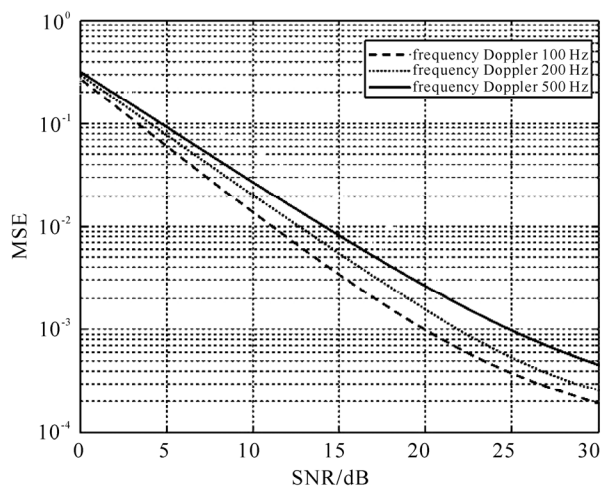

Figure 5. MSE performance for different Doppler frequency, $K = 516$, $K_G = 23$, $E = 12$ **and** $L = 3$.

mum MSE. We suggested a new pilot allocation that is distributed non-uniformly with non-equally powered pilot tones. We arranged the proposed pilot cluster over *G* OFDM symbols and gave an example of the pilot setting in four pilot groups. After contrasting our design with comb pilots, it was shown that the proposed scheme improves SNR. Simulation results also verify our idea.

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