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Fixed-Points of Expansive Mappings in Supermetric Spaces

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Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

In the present research paper, we prove some fixed-point results for expansive mappings in the framework of super metric space.

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1 Introduction

A fascinating subfield of metric fixed-point theory is found in nonlinear functional analysis. Many discoveries and publications on the subject have been produced during the past century, all of which are relevant to Banach's

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fixed-point theorem. There are essentially two consensus theories for how to advance the metric fixed point: one involves lowering the constraints on the contraction mapping, and the other involves changing the abstract structure. Numerous generalizations and extensions have previously been made to metric spaces. Quasi-metric spaces, b-metric spaces, symmetric spaces, fuzzy metric spaces, dislocated metric spaces, partial metric spaces, 2-metric spaces, modular metric spaces, cone metric spaces, ultra-metric spaces, and numerous different combinations of these are examples of these. In fixed point theory, the study of expansive mappings is an extremely fascinating field of study. Expanding mappings were first introduced and various fixed-point theorems in complete metric spaces were demonstrated by Wang et al. in 1984 [1,2,3,4]. Daffer and Kaneko [5] established some common fixed-point theorems for two mappings in complete metric spaces and defined an expanding condition for a pair of mappings in 1992.

We prove some fixed-point theorems for expansive mapping in super metric space. Our findings extend the expansions of the metric space to a super metric space by Daffer and Kaneko [2] and [1].

2 Preliminaries

Erdal Karapinar and Andreea Fulga [6] introduced super-metric space. We were able to derive some fixed-point theorems in this structure, and we believe that this method could assist in alleviating the congestion and squeezing problems noted before.

Definition 2.1 (see [4]) Let $\mathfrak{D}$ is a non-empty set. We say that a function $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ is a super metric if it satisfies the following axioms:

(s1). $\forall\ \sigma, \varsigma \in \mathfrak{D}$, if $\eta(\sigma, \varsigma) = 0, then\ \sigma = \varsigma$.
(s2). $\forall\ \sigma, \varsigma \in \mathfrak{D}, \eta(\sigma, \varsigma) = \eta(\varsigma, \sigma)$.
(s3). *There exists* $s \geq 1$ *such that for every* $\varsigma \in \mathfrak{D}$, *there exist distinct sequences* $\{\sigma_n\}, \{\varsigma_n\} \subset \mathfrak{D}$, *with* $\eta(\sigma_n, \varsigma_n) \to 0$ *when* $n \to \infty$, *such that* $\eta(\varsigma_n, \varsigma) \leq s\eta(\sigma_n, \varsigma)$.

The tripled $(\mathfrak{D}, \eta, s)$ is called a super metric space.

Definition 2.2 (see [4]) On a super metric space $(\mathfrak{D}, \eta, s)$, a sequence $\{\sigma_n\}$:

(i). converges to σ in $\mathfrak{D}$ if and only *if* $\eta(\sigma_n, \sigma) = 0$.
(ii). is a Cauchy sequence in $\mathfrak{D}$ if and only if $\{\eta(\sigma_n, \sigma_m): m > n\} = 0$.

Proposition 2.3 (see [4]) On a super metric space, the limit of a convergent sequence is unique.

Definition 2.4 (see [4]) We say that a supermetric space $(\mathfrak{D}, \eta, s)$ is complete if and only if every Cauchy sequence is convergent in $\mathfrak{D}$.

Example 2.5 (see [4]) Let the set $\mathfrak{D} = \mathbb{R}, s = 2$, *and* $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ be an application defined as follows*:*

$$\eta(\sigma, \varsigma) = (\sigma - \varsigma)^2, \text{ for } \sigma, \varsigma \in \mathbb{R}\backslash\{1\}$$
$$\eta(1, \varsigma) = \eta(\varsigma, 1) = (1 - \varsigma^3)^2, \text{ for } \varsigma \in \mathbb{R}.$$

Then, the tripled $(\mathfrak{D}, \eta, s)$ forms a super metric space.

Example 2.6 (see [4]) *Let the set* $\mathfrak{D} = [0, +\infty]$ *and* $\eta: \mathfrak{D} \times \mathfrak{D} \to [0, +\infty)$ *be a function, defined as follows:*

$$\eta(\sigma, \varsigma) = \frac{|\sigma\varsigma - 1|}{\sigma + \varsigma + 1}, \textit{for } \sigma, \varsigma \in [0,1) \cup (1, +\infty], \sigma \neq \varsigma,$$

$$\eta(\sigma, \varsigma) = 0, \textit{for } \sigma, \varsigma \in [0, +\infty), \sigma = \varsigma,$$

$$\eta(\sigma, 1) = \eta(1, \sigma) = |\sigma - 1|, \textit{for } \sigma \in [0, +\infty].$$

We can easily see that η forms a super metric on $\mathfrak{D}$.

Proposition 2.7 (see [4]) *Let* $\Omega: \mathfrak{D} \rightarrow \mathfrak{D}$ be an asymptotically regular mapping on a complete super metric space $(\mathfrak{D}, \eta, s)$. Then, the Picard iteration $\{\Omega^n \sigma\}$ for the initial point $\sigma \in R$ is a convergent sequence on $\mathfrak{D}$.

Theorem 2.8 (see [4]) Let $(\mathfrak{D}, \eta, s)$ be a complete super-metric space and let $\Omega: \mathfrak{D} \rightarrow \mathfrak{D}$ be a mapping. Suppose that $0 < \alpha < 1$ such that $\eta(\Omega\sigma, \Omega\varsigma) \leq \eta(\sigma, \varsigma)$ for all $(\sigma, \varsigma) \in \mathfrak{D}$. Then Ω has a unique fixed point in $\mathfrak{D}$.

Theorem 2.9 (see [4]) Let $(\mathfrak{D}, \eta, s)$ be a complete super metric space and $\Omega: \mathfrak{D} \rightarrow \mathfrak{D}$ be a mapping, such that there exist $\alpha \in [0, 1)$ and that

$$\eta(\Gamma\sigma, \Gamma\varsigma) \leq k\left\{\eta(\sigma, \varsigma), \frac{\eta(\sigma, \Gamma\sigma)\eta(\varsigma, \Gamma\varsigma)}{\eta(\sigma, \varsigma) + 1}\right\}$$

Then, Ω has a unique fixed point.

3 Main Results

Our initial outcome is as follows.

Theorem 3.1 Let $\Omega: \mathfrak{D} \rightarrow \mathfrak{D}$ be a surjection and $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space. Assume that $a, b, c \geq 0$ with $a + b + c > 1$ such that

$$\eta(\Omega\sigma, \Omega\varsigma) \geq a\,\eta(\sigma, \varsigma) + b\,\eta(\sigma, \Omega\sigma) + c\,\eta(\varsigma, \Omega\varsigma) \tag{1}$$

$\forall\, \sigma, \varsigma \in \mathfrak{D}$ with $\sigma \neq \varsigma$. In $\mathfrak{D}$, Ω then possesses a fixed point.

Proof Considering the supposition. Ω is injective, as is evident. Let Θ represent Ω 's inverse mapping. After selecting $\sigma_0 \in \mathfrak{D}$, set $\sigma_1 = \Theta(\sigma_0)$, $\sigma_2 = \Theta(\sigma_1) = \Theta^2(\sigma_0), \ldots, \sigma_{n+1} = \Theta(\sigma_n) = \Theta^{n+1}(\sigma_0), \ldots$ We suppose that for any $n = 1,2, \ldots$, $\sigma_{n-1} \neq \sigma_n$ without losing generality. Alternatively, if $\sigma_{n_0-1} = \sigma_{n_0}$ exists for some n_0, then σ_{n_0} is a fixed point of Ω. As of (1), we possess

$$\begin{aligned}
\eta(\sigma_{n-1}, \sigma_n) &= \eta(\Omega\Omega^{-1}\sigma_{n-1}, \Omega\Omega^{-1}\sigma_n) \\
&\geq a\,\eta(\Omega^{-1}\sigma_{n-1}, \Omega^{-1}\sigma_n) + b\,\eta(\Omega^{-1}\sigma_{n-1}, \Omega\Omega^{-1}\sigma_{n-1}) + c\,\eta(\Omega^{-1}\sigma_n, \Omega\Omega^{-1}\sigma_n) \\
&= a\,\eta(\Theta\sigma_{n-1}, \Theta\sigma_n) + b\,\eta(\Theta\sigma_{n-1}, \sigma_{n-1}) + c\,\eta(\Theta\sigma_n, \sigma_n) \\
&= a\,\eta(\sigma_n, \sigma_{n+1}) + b\,\eta(\sigma_n, \sigma_{n-1}) + c\,\eta(\sigma_{n+1}, \sigma_n)
\end{aligned}$$

Hence

$$(1 - b)\,\eta(\sigma_{n-1}, \sigma_n) \geq (a + c)\,\eta(\sigma_{n+1}, \sigma_n) \tag{2}$$

If $a + c = 0$, then $b > 1$. Inequality (2) implies that a negative number is greater than or equal to zero. This isn't feasible. Thus, $(1 - b) > 0$ and $a + c \neq 0$. Consequently,

$$\eta(\sigma_{n+1}, \sigma_n) \leq \lambda\,\eta(\sigma_{n-1}, \sigma_n) \tag{3}$$

where $\lambda = \frac{1-b}{a+c} < 1$ for all n. Hence

$$\eta(\sigma_{n+1}, \sigma_n) \leq \lambda^n\,\eta(\sigma_0, \sigma) \tag{4}$$

Taking $n \rightarrow \infty$ in inequality (5), we get

$$\eta(\sigma_n, \sigma_{n+1}) = 0. \tag{5}$$

We aim to demonstrate that the sequence $\{\sigma_n\}$ is a Cauchy sequence in the following. Let us now assume that $m, n \in N$, where $m > n$. If $\sigma_n = \sigma_m$, then $\Omega^m \sigma_0 = \Omega^n \sigma_0$. Thus, $\Omega^{m-n}(\Omega^n \sigma_0) = \Omega^n \sigma_0$ is implied. Thus, we have $\Omega^n \sigma_0$ is the fixed point of Ω^{m-n}. Furthermore,

$$\Omega\left(\Omega^{m-n}(\Omega^n \sigma_0)\right) = \Omega^{m-n}\left(\Omega(\Omega^n \sigma_0)\right) = \Omega(\Omega^n \sigma_0) \tag{6}$$

This indicates that the fixed point of Ω^{m-n} is $\Omega(\Omega^n \sigma_0)$. Consequently, $\Omega(\Omega^n \sigma_0) = \Omega^n \sigma_0$. Thus, the fixed point of Ω is $\Omega^n \sigma_0$. Assume for now that $\sigma_n \neq \sigma_m$. Next, employing (s3) and inequality (5), we obtain

$$\eta(\sigma_n, \sigma_{n+2}) \leq s\eta(\sigma_{n+1}, \sigma_{n+2}) \leq s\{\lambda^{n+1}\eta(\sigma_0, \sigma_1)\} = 0. \tag{7}$$

Hence, $\eta(\sigma_n, \sigma_{n+2}) = 0$. In a similar vein, we have

$$\eta(\sigma_n, \sigma_{n+3}) \leq s\eta(\sigma_{n+2}, \sigma_{n+3}) \leq s\{\lambda^{n+2}\eta(\sigma_0, \sigma_1)\} = 0. \tag{8}$$

It follows that $\{\eta(\sigma_n, \sigma_m): m > n\} = 0$ via induction. As a result, given a complete super metric space $(\mathfrak{D}, \eta, s)$, $\{\sigma_n\}$ is a Cauchy sequence that converges to $\sigma^* \in \mathfrak{D}$. We assert that the fixed point of Ω is σ^*. Since the map Ω is surjective. Hence, there is a point ς in Ω where $\sigma^\star = \Omega\varsigma$.

$$\begin{aligned} \eta(\sigma_n, \sigma^\star) &= \eta(\Omega\sigma_{n+1}, \Omega\varsigma) \\ &\geq a\,\eta(\sigma_{n+1}, \varsigma) + b\ \eta(\sigma_{n+1}, \Omega\sigma_{n+1}) + c\,\eta(\varsigma, \Omega\varsigma) \\ &= a\,\eta(\sigma_{n+1}, \varsigma) + b\,\eta(\sigma_{n+1}, \sigma_n) + c\,\eta(\varsigma, \sigma^\star) \end{aligned} \tag{9}$$

which implies that as $n \to \infty$,

$$0 \geq (a + c)\,\eta(\varsigma, \sigma^\star)$$

Thus, $\varsigma = \sigma^\star$. As a result, $\sigma^\star$ is a fixed point for Ω. The proof is now complete.

Setting $b = c$ and $a = k$ in Theorem 3.1, We are able to get the following outcome.

Corollary 3.2 Let $\Omega: \mathfrak{D} \to \mathfrak{D}$ be a surjection and $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space. Assume that $k > 1$ such that

$$\eta(\Omega\sigma, \Omega\varsigma) \geq k\,\eta(\sigma, \varsigma) \tag{10}$$

$\forall\, \sigma, \varsigma \in \mathfrak{D}$ with $\sigma \neq \varsigma$. In $\mathfrak{D}$, Ω then possesses a fixed point.

Proof By putting $b = c = 0$ and $a = k$ in condition (1), we may deduce from Theorem 3.1 that Ω has a fixed point $\sigma^\star$ in $\mathfrak{D}$.

Corollary 3.3 Let $\Omega: \mathfrak{D} \to \mathfrak{D}$ be a surjection and $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space. Assume that n is a positive integer and that k is a real number greater than 1 such that

$$\eta(\Omega^n\sigma, \Omega^n\varsigma) \geq k\,\eta(\sigma, \varsigma) \tag{11}$$

$\forall\, \sigma, \varsigma \in \mathfrak{D}$ with $\sigma \neq \varsigma$. In $\mathfrak{D}$, Ω then possesses a fixed point.

Proof Ω^n has a fixed point $\sigma^\star$, according to Corollary 3.2. However, since $\Omega^n(\Omega\sigma^\star) = \Omega(\Omega^n\sigma^\star) = \Omega\sigma^\star$, also Ω^n has $\Omega\sigma^\star$ as a fixed point. As a result, $\Omega\sigma^\star = \sigma^\star$, where $\sigma^\star$ is Ω 's fixed point. The fixed point of Ω is unique since it is also the fixed point of Ω^n.

Theorem 3.4 Let $\Omega: \mathfrak{D} \to \mathfrak{D}$ be a continuous surjection and $(\mathfrak{D}, \eta, s)$ be a complete *super* metric space. If there exist a constant $k > 1$ such that for any $\sigma, \varsigma \in \mathfrak{D}$, there is

$$M(\sigma,\varsigma) \in \{\eta(\sigma,\varsigma), \eta(\sigma,\Omega\sigma), \eta(\varsigma,\Omega\varsigma)\} \tag{12}$$

satisfying

$$\eta(\Omega\sigma,\Omega\varsigma) \geq k\ M(\sigma,\varsigma) \tag{13}$$

$\forall\ \sigma,\varsigma \in \mathfrak{D}$. Then Ω has a unique fixed point in $\mathfrak{D}$.

Proof A sequence $\{\sigma_n\}_{n=1}^{\infty}$ can be obtained in a manner similar to the Theorem 3.2 proof, such that $\sigma_{n-1} = \Omega\sigma_n$. We suppose that for any $n = 1,2,\ldots$, $\sigma_{n-1} \neq \sigma_n$ without losing generality. Alternatively, if $\sigma_{n_0-1} \neq \sigma_{n_0}$ for some n_0 then σ_{n_0} is a fixed point of Ω. Therefore, based on inequality (13),

$$\eta(\sigma_{n-1},\sigma_n) = \eta(\Omega\sigma_n,\Omega\sigma_{n+1}) \geq kM(\sigma_n,\sigma_{n+1})$$

where

$$M(\sigma_n,\sigma_{n+1}) = \{\eta(\sigma_n,\sigma_{n+1}), \eta(\sigma_n,\sigma_{n-1})\}$$

The next two scenarios need to be taken into consideration:

- If $M(\sigma_n,\sigma_{n+1}) = \eta(\sigma_n,\sigma_{n-1})$ then from (13), we have

$$\eta(\sigma_{n-1},\sigma_n) \geq k\eta(\sigma_n,\sigma_{n-1})$$

which implies $\eta(\sigma_{n-1},\sigma_n) = 0$ that is $\sigma_{n-1} = \sigma_n$. This is a contradiction.

- If $M(\sigma_n,\sigma_{n+1}) = \eta(\sigma_n,\sigma_{n+1})$ then from (13), we have

$$\eta(\sigma_{n-1},\sigma_n) \geq k\ \eta(\sigma_n,\sigma_{n+1})$$

We derive that $\{\sigma_n\}_{n=1}^{\infty}$ is a Cauchy sequence in a complete super metric space $(\mathfrak{D},\eta,s)$ by proceeding as in Theorem 3.1. Thus, $\{\sigma_n\}_{n=1}^{\infty}$ is a sequence that converges to a point $\sigma^\star \in \mathfrak{D}$. As Ω is continuous, $\sigma^\star$ is obviously a fixed point of Ω. The proof is now complete [7-9].

We now prove that, in the context of super metric spaces, common fixed points for a pair of two weakly compatible self-mappings that fulfill expansive conditions exist. Jungck first proposed the idea of commuting maps in Jungck [10]. A generalization of the idea of commuting maps, compatible mappings were first proposed by Jungck in Jungck [11]. In Muraliraj and Jahir Hussain [12], Jungck expanded on the idea of suitable maps in the following ways [13-16].

Definition 3.5 Given a nonempty set $\mathfrak{D}$, let Υ and Ω be two self-mappings on it. If Υ and Ω commute at all of their coincidence points, meaning that for every σ in $\mathfrak{D}$, $\Upsilon\sigma = \Omega\sigma$ and $\Upsilon\Omega\sigma = \Omega\Upsilon\sigma$, then Υ and Ω are considered weakly compatible.

Let's now demonstrate our result.

Theorem 3.6 Assume that the super metric space $(\mathfrak{D},\eta,s)$ is complete. Let Υ and Ω be self-mappings of $\mathfrak{D}$ and $\Omega(\mathfrak{D}) \subseteq \Upsilon(\mathfrak{D})$ that are weakly compatible. Assume that $k > 1$ exists such that

$$\eta(\Upsilon\sigma,\Upsilon\varsigma) \geq k\ \eta(\Omega\sigma,\Omega\varsigma),\ \ \forall\ \sigma,\varsigma \in \mathfrak{D}\,. \tag{14}$$

If either $\Omega(\mathfrak{D})$ or $\Upsilon(\mathfrak{D})$ are complete subspaces. Then there exists a unique common fixed point of Υ and Ω in $\mathfrak{D}$.

Proof Let $\sigma_0 \in \mathfrak{D}$. Since $\Omega(\mathfrak{D}) \subseteq \Upsilon(\mathfrak{D})$, choose σ_1 such that $\varsigma_1 = \Upsilon\sigma_1 = \Omega\sigma_0$. In general, choose σ_{n+1} such that $\varsigma_{n+1} = \Upsilon\sigma_{n+1} = \Omega\sigma_n$, then from condition (14),

$$\begin{aligned}\eta(\varsigma_{n+1},\varsigma_{n+2}) &= \eta(\Omega\sigma_n,\Omega\sigma_{n+1}) \leq \frac{1}{k}\,\eta(\Upsilon\sigma_n,S\sigma_{n+1})\\ &= \frac{1}{k}\,\eta(\Omega\sigma_{n-1},\Omega\sigma_n)\\ &= \frac{1}{k}\,\eta(\varsigma_n,\varsigma_{n+1})\\ &= \lambda\,\eta(\varsigma_n,\varsigma_{n+1}) \end{aligned} \tag{15}$$

where $\lambda = \frac{1}{k} < 1$. Repeating (15) $(n+1)$-times, we obtain

$$\eta(\varsigma_{n+1},\varsigma_{n+2}) \leq \lambda^{n+1}\,\eta(\varsigma_0,\varsigma_1) \tag{16}$$

Taking the limit n tends to infinity in inequality (16), we get

$$\eta(\varsigma_{n+1},\varsigma_{n+2}) = 0. \tag{17}$$

Proceeding like Theorem 3.1, we obtain that $\{\varsigma_n\}_{n=1}^{\infty}$ is a Cauchy sequence in a complete *super* metric space $(\mathfrak{D},\eta,s)$. So, the sequence $\{\varsigma_n\}_{n=1}^{\infty}$ converges to a point $\sigma^{\star} \in \mathfrak{D}$. Hence, $\varsigma_n \to \sigma^{\star}$ as $n \to \infty$ and then

$$\varsigma_n = \Omega\sigma_n = \Upsilon\sigma_n = \sigma^{\star}. \tag{18}$$

Since $\Omega(\mathfrak{D})$ or $\Upsilon(\mathfrak{D})$ is complete and $\Omega(\mathfrak{D}) \subseteq \Upsilon(\mathfrak{D})$, there exists a point $\theta \in \mathfrak{D}$ such that $\Upsilon\theta = \sigma^{\star}$. Now from (14), we have

$$\eta(\Omega\theta,\Omega\sigma_n) \leq \frac{1}{k}\eta(\Upsilon\theta,\Upsilon\sigma_n) \tag{19}$$

Proceeding to the limit as $n \to +\infty$ in (19), we have

$$\eta(\Omega\theta,\sigma^{\star}) \leq \frac{1}{k}\eta(\Upsilon\theta,\sigma^{\star}) \tag{20}$$

which implies that $\Omega\theta = \sigma^{\star}$. Therefore $\Omega\theta = \Upsilon\theta = \sigma^{\star}$. Since Ω and Υ are weakly compatible, therefore $= \Omega\Upsilon\theta$, that is $\Upsilon\sigma^{\star} = \Omega\sigma^{\star}$.

Now we show that $\sigma^{\star}$ is a fixed point of Υ and Ω. From (14), we have

$$\eta(\Upsilon\sigma^{\star},\Upsilon\sigma_n) \geq k\,\eta(\Omega\sigma^{\star},\Omega\sigma_n) \tag{21}$$

Proceeding to the limit as $n \to \infty$ in (21), we have

$$\eta(\Upsilon\sigma^{\star},\sigma^{\star}) \geq k\,\eta(\Omega\sigma^{\star},\sigma^{\star}) \tag{23}$$

which implies that $\Upsilon\sigma^{\star} = \sigma^{\star}$. Hence $\Upsilon\sigma^{\star} = \Omega\sigma^{\star} = \sigma^{\star}$.

Uniqueness. Suppose that $\sigma^{\star} \neq \varsigma^{\star}$ is also another common fixed point of Υ and Ω. Then

$$\eta(\Upsilon\sigma^{\star},\Upsilon\varsigma^{\star}) \geq k\,\eta(\Omega\sigma^{\star},\Omega\varsigma^{\star}) \tag{24}$$

implies that $\sigma^{\star} = \varsigma^{\star}$. This completes the proof.

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Competing Interests

Authors have declared that no competing interests exist.

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