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Theory and Properties of Atomic Spacetime

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Abstract

Following A. Einstein's aspirations for an atomic theory, a novel theory of spacetime quantization/atomization based on finite Atomic AString Functions evolving since the 1970s is offered. Atomization Theorems allow representing polynomials, analytic functions, and solutions of General Relativity via the superposition of solitonic atoms which can be associated with flexible spacetime quanta, metriants, or elementary distortions. With multiple interpretations discussed, discrete-continuous spacetime is conceptualized as a lattice network of flexible "solitonic atoms" adjusting locations to reproduce different metrics. The theory may offer some variants of unified field theory under research based on Atomic AString Function where, like in string theory, fields become interconnected having a common mathematical ancestor.

Keywords

Spacetime, Quantum, Atomic Function, AString, Soliton, Metriant, Unified Theory

1. Introduction and "Atomic Theory" of A. Einstein

In a 1933 lecture [1] cited below with some highlights, A. Einstein discussing some controversies of Quantum Mechanics mentioned the prospects of a novel "atomic theory" based on "mathematically simplest concepts and the link between them" to solve some "stumbling blocks" of continuous field theories to describe quantized fields.

"*The important point for us to observe is that all these constructions and the laws connecting them can be arrived at by the principle of looking for the mathematically simplest concepts and the link between them. In the limited number of the mathematically existent simple field types, and the simple equations possible between them, lies the theorist's hope of grasping the real in all its* *depth. Meanwhile the great stumbling-block for a field-theory of this kind lies in the conception of the atomic structure of matter and energy. For the theory is fundamentally non-atomic in so far as it operates exclusively with continuous functions of space, in contrast to classical mechanics, whose most important element, the material point, in itself does justice to the atomic structure of matter… I still believe in the possibility of a model of reality - that is to say, of a theory which represents things themselves and not merely the probability of their occurrence… But an atomic theory in the true sense of the word (not merely on the basis of an interpretation) without localization of particles in a mathematical model is perfectly thinkable. For instance, to account for the atomic character of electricity, the field equations need only lead to the following conclusions: A region of three-dimensional space at whose boundary electrical density vanishes everywhere always contains a total electrical charge whose size is represented by a whole number. In a continuum-theory atomic characteristics would be satisfactorily expressed by integral laws without localization of the entities which constitute the atomic structure. Not until the atomic structure has been successfully represented in such a manner would I consider the quantum-riddle solved.*"

Interestingly, some of Einstein's aspirations of a novel "atomic theory" with "simplest concepts and links between them" based on finite "regions of space" with "atomic structure" can be realized with the theory of Atomic Functions (AF) often called "mathematical atoms" pioneered in the 1970s by distinguished Academican of National Academy of Sceinces of Ukraine V.L. Rvachev and V.A. Rvachev [2]-[12] and developing by the author towards spacetime physics [13]-[20] since 2017 based on Atomic AString Functions and Atomic Solitons [2] [3] [4] [5] [21] [22] [23]. The purpose of this research is to demonstrate how two theories - General Relativity (GR) [13] [14] [15] [16] and Atomic AString Functions [2]-[12] [21] [22] [23]—can be combined as well as to offer an "atomic" mathematical interpretation of spacetime field as a superposition of flexible "solitonic atoms" (Atomic Solitons), with detailed discussion of the properties and novel intepretations of discrete-continuous atomic spacetime (§7 - 10). The combined Atomic Spacetime theory is based on formulated Atomization Theorems (§5) allowing representation of polynomials, analytic functions, and solutions of differential equations of mathematical physics including GR [13]-[33] via superposition of finite Atomic AString Functions resembling flexible quanta (§3, 7). It leads to spacetime atomization/quantization models on a lattice with elementary atomic metriants/quanta/distortions as "building blocks" of fields (§3, 7, 8, 9).

The main difficulty of integrating relatively new Atomic Functions theory [2]-[12] into GR [1] [13] [14] [15] [16] was to figure out how AFs known for their unique approximating properties of analytical functions and solutions of linear differential equations [6]-[12] can be applied for such complex GR equations including nonlinear Ricci tensors [13] [14] [15] [16]. The integration idea intuitively envisaged in [2] [3] and developed in [5] [23] is based on the combination of three properties—derivatives and integrals of finite Atomic Function expressed via AF itself [2]-[12], the ability of Atomic AString Functions via so-called Atomic Series to compose polynomials and analytic functions [6]-[12] [23], and "preservation of the analyticity" for Ricci and Einstein tensors proven in [5] [23]. It means if smooth spacetime geometry is represented as a superposition of finite AF splines, the deformations, metric, Ricci, and Einstein curvature tensors would also be some AF combinations because derivatives and complex multiplications of AFs are expressed via AF themselves. It offers a discrete-continuous interpretation of spacetime and other fields as a complex network/lattice of shifted and stretched "solitonic atoms" not only resonating with A. Einstein's [1] aspirations of a "perfectly thinkable" "atomic theory" with "simplest concepts and links between them" where finite "regions of space" can have "atomic structure" but also offering a quantization model of spacetime with a quantum described by a finite atomic spline. The background, challenges, and contributions to Atomic Functions theory are described in the historical review hereafter.

2. Brief History of Atomic and AString Functions

Theory of Atomic Functions (AF) [2]-[12] has been evolving since 1967-1971 when Academician of NAS of Ukraine V.L. Rvachev[1], had envisaged finite pulse function $up(x)$ for which derivatives (also pulses) would conveniently be similar to the original pulse shifted and stretched by the factor of 2:

$$up'(x) = 2up(2x+1) - 2up(2x-1) \text{ for } |x| \le 1, \; up(x) = 0 \text{ for } |x| > 1. \quad (1.1)$$

This and other similar functions possess unique properties of infinite differentiability, smoothness, nonlinearity, nonanalyticity, finiteness, and compact support like widely-used splines. What the most significant is that other functions like polynomials, trigonometric, exponential, and other analytic functions can be represented via a converging series of shifts and stretches of AFs. So, like from "mathematical atoms" [6]-[12], smooth functions can be composed of the AF superpositions, and because of that those "atoms" have been called Atomic Functions in the 1970s.

The foundation of AF theory has been developed in Ukraine since the 1970s by V.L. Rvachev and V.A. Rvachev [2] [3] [4] [5] [6] [25] [27] and enriched by many followers from different countries, notably by schools of V.F. Kravchenko [9] [10] [11] [12], B. Gotovac, H. Gotovac [26] [33], and the author [2] [3] [4] [5] [21] [22] [23], with the number of papers and books observed in [10] has grown to a few hundred. In 2017, the author noted [2] [3] [4] [5] that AF $up(x)$ (1.1) is a composite object consisting of two kink functions called AStrings [2] [3] [4] [5] making them more generic:

$$up(x) = AString(2x+1) - AString(2x-1) = AString'(x). \quad (1.2)$$

[1]Vladimir Logvinovich Rvachev (1926-2005), https://en.wikipedia.org/wiki/Vladimir_Rvachev, Academician of National Academy of Sciences of Ukraine, author of 600 papers, 18 books, mentor of 80 PhDs, 20 Doctors and Professors including the author.

Importantly, AString is not only a "composing branch" but also an integral of $up(x)$. Mutual relationships (1.1), (1.2) imply that theories and theorems involving AFs can be reformulated via AStrings. Composing AF pulse (1.1) via kink-antikink pair (1.2) of nonlinear AStrings resembles "solitonic atoms" (or bions) from the theory of soliton dislocations [5] [29] [30]. This led to the theory of Atomic Solitons [3] [5] where AString (1.2) becomes a solitonic kink while $up(x)$ is a "solitonic atom" made of AStrings. The ability of finite AFs to compose smooth polynomials, analytic functions and solutions of differential equations including GR leads to novel interpretations of spacetime and field composition from "solitonic atoms"/Atomic Solitons first described in [2] [3] [4] and later elaborated in [5] [23].

AString function (1.2) possesses another important property of composing/partitioning a line and smooth curves from a superposition of AStrings resembling the ideas of quantization of space lines, geodesics, and generally spacetime field published in 2018 [2] [3] as an "intuition theory". It assumes the representation of spacetime and gravity fields as a superposition of "solitonic atoms"/Atomic Solitons leading to ideas of "atomization of spacetime" or Atomic Spacetime [5] [23] hence supporting some A. Einstein's aspirations [1] of an "…atomic theory" with "mathematically simplest concepts and the link between them" to solve some "stumbling blocks" of continuous field theories to describe discrete quantized fields.

The mathematical foundation of the Atomic Spacetime theory is based on the sequence of *Atomization Theorems* [5] [23] described here (§5, 6) with full proof in [23]. Starting from the core theorems known since the 1970s and extended later [5] [23] for recently introduced AStrings with the so-called *Atomic Series* [5] [23], it was extended in [23] to new theorems for complex analytic functions, nonlinear theories, and finally nonlinear General Relativity (GR) equations [5] [23]. Interestingly, Atomic AString Functions can be not only introduced into GR but also deduced from GR (§6) [5] [23] so theoretically it could have been done by A. Einstein himself when in 1933 he mentioned finite "regions of space" with "discrete energies" (§1) [1] resembling finite atomic functions. The Atomization Theorems are not limited to spacetime but can also be applied to many physical theories including Quantum Mechanics, electromagnetism, elasticity, heat conductivity, soliton theories, and field theories [7]-[12] [22]-[50]. A unified representation of fields composed of superpositions of discrete finite Atomic AString Functions may offer some novel variants of unified theory under research now [2] [3] [4] [5] [19] [23] based on Atomic AString Functions where, like in string theory, fields become interconnected having a common mathematical ancestor.

Apart from mathematical formalism, this work describes multiple interpretations of spacetime within Atomic Spacetime theory, possible physical meanings of finite "solitonic atoms" and AStrings, and interrelations to other theories like string theory, loop quantum gravity, and field theories [19]-[51].

3. Deriving Simple AString Metriant Function

Let's consider the problem of composing a straight x and curved $\tilde{x}(x)$ spaceline via superpositions over some finite metriant functions $m(x)$, $x \in [-1,1]$:

$$x = \sum_{k=-\infty}^{\infty} am((x-ka)/a); \ \tilde{x}(x) = \sum_{k=-\infty}^{\infty} c_k m((x-b_k)/a_k) \tag{3.1}$$

composing a spaceline from "elementary pieces" set at regular points ka resembling quanta of width $2a$ (**Figure 1**). We seek spaceline x to appear not only as a Lego-like translation (3.1) but also in "interaction zones" between quanta ($a = 1$) (**Figure 1(a)** and **Figure 1(b)**).

$$x \equiv \cdots m(x-1) + m(x) + m(x+1) + \cdots;$$
$$x \equiv m\left(x-\frac{1}{2}\right) + m\left(x+\frac{1}{2}\right), \ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]. \tag{3.2}$$

Reformulated for derivatives $p(x) = m'(x)$, the problem leads to a "partition of unity" [2]-[7] to represent a constant via a series of finite pulses:

$$1 \equiv \cdots p(x-1) + p(x) + p(x+1) + \cdots;$$
$$1 \equiv p\left(x-\frac{1}{2}\right) + p\left(x+\frac{1}{2}\right), \ x \in \left[-\frac{1}{2}, \frac{1}{2}\right]. \tag{3.3}$$

It can be achieved with widely used polynomial splines but it leads to a "polynomial trap" problem [24] imposing artificial polynomial order on spacetime models and not being able to compose a smooth curve $\tilde{x}(x)$ of arbitrary polynomial order. Instead, seeking a solution amongst finite functions for which derivatives are expressed via themselves

$$p'(x) = f(p(x)) = cp(ax+b) + dp(ax-b) \tag{3.4}$$

yields so-called atomic function (AF) $up(x)$ [2]-[12] discovered in the 1970s by V.L. Rvachev and V.A. Rvachev [6] (**Figure 1(b)**).

$$up'(x) = 2up(2x+1) - 2up(2x-1), \ p(x) = up(x). \tag{3.5}$$

The desired metriant function $m(x)$ from (3.2) would be the integral of $up(x)$ called *AString* in 2017 [2] [3] [4] [5]:

$$p(x) = up(x), m(x) = \int_0^x up(x)\mathrm{d}x = AString(x), \ x \equiv \sum_x AString(x-k). \tag{3.6}$$

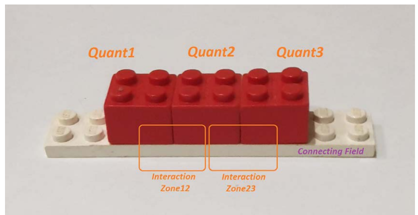
AString shaped as a kink (**Figure 1**) can compose both straight and curved lines from solitary pieces offering spacetime quantization models based on Atomic and AString Functions [2] [3] [4] [5] [23] [42] described hereafter.

4. Atomic and AString Functions

Let's describe Atomic [2]-[12] and AString [2] [3] [4] [5] Functions in more detail.

4.1. Atomic Function

Atomic Function (AF) (V.L. Rvachev, V.A. Rvachev, [6], 1971) $up(x)$ is a



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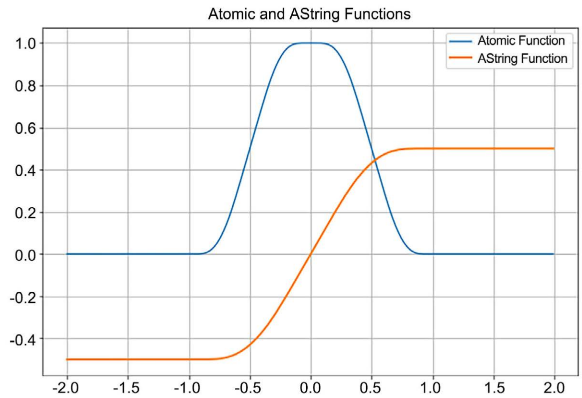

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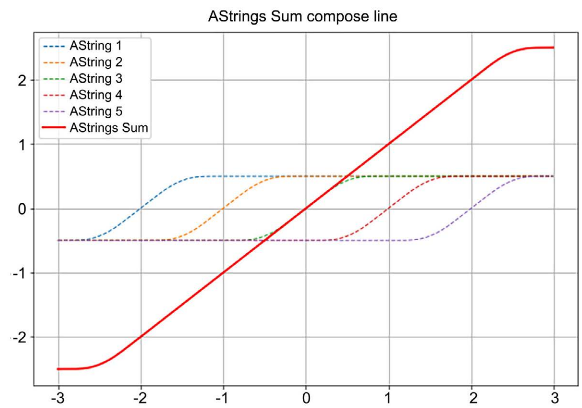

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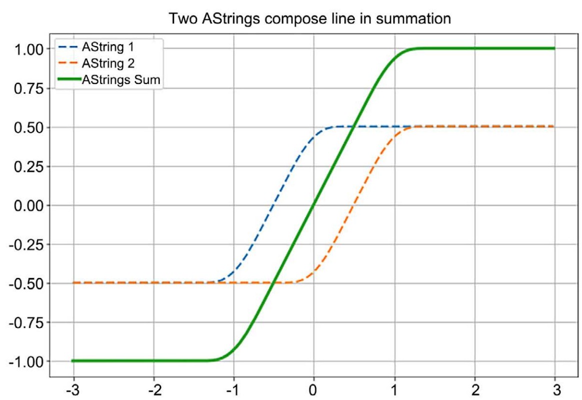

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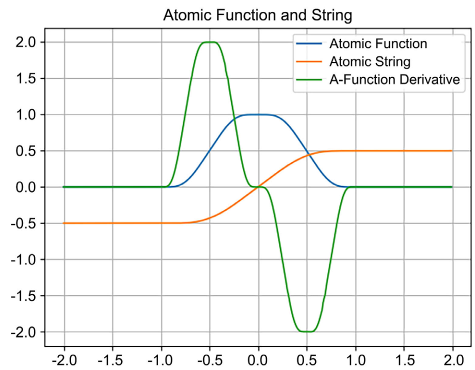
Figure 1. (a) Lego model with interaction zones; (b) Desired metriant function and its derivative; (c) Expansion of space by the sum of metriant functions; (d) Emergence of line $y = x$ by summing two metriant functions in "interaction zone".

finite compactly supported non-analytic infinitely differentiable function (**Figure 2**) with the first derivative expressible via the function itself shifted and stretched by the factor of 2.



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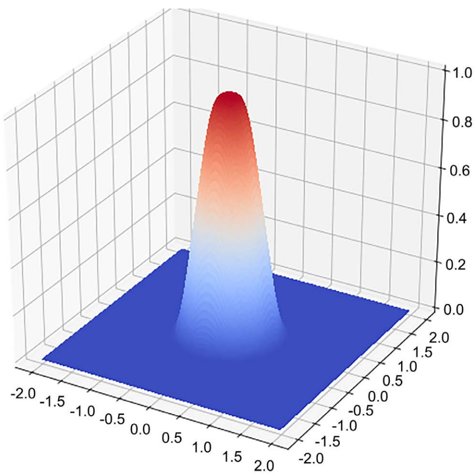

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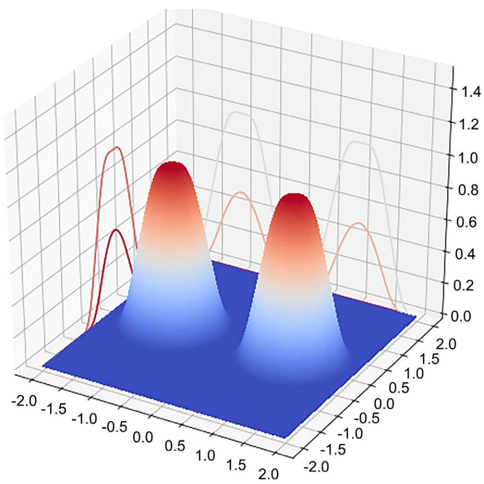

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Figure 2. (a) Atomic Function pulse with its derivative and integral (AString), (b) Atomic Function pulse ("solitonic atom") in 2D, (c) Two Atomic Function pulses ("solitonic atoms" or "atomic solitons").

$$up'(x) = 2up(2x+1) - 2up(2x-1) \text{ for } |x| \le 1, \; up(x) = 0 \text{ for } |x| > 1. \quad (4.1)$$

With exact Fourier series representation [2] [3] [4] [5] [7]-[12]

$$up(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{itx} \prod_{k=1}^{\infty} \frac{\sin(t2^{-k})}{t2^{-k}} dt, \; \int_{-1}^{1} up(x) dx = 1, \quad (4.2)$$

the values of $up(x)$ can be calculated with computer scripts [2] [4] [9] [10] [11] [12] [43].

Higher derivatives $up^{(n)}$ and integrals I_m can also be expressed via $up(x)$ [6]-[12] [25] [26]

$$up^{(n)}(x) = 2^{\frac{n(n+1)}{2}} \sum_{k=1}^{2^n} \delta_k up(2^n x + 2^n + 1 - 2k), \; \delta_{2k} = -\delta_k, \; \delta_{2k-1} = \delta_k, \; \delta_1 = 1;$$

$$I_m(x) = 2^{C_m^2} up(2^{-m} x - 1 + 2^{-m}), \; x \le 1;$$

$$I_m(x) = 2^{C_m^2} up(2^{-m+1} - 1) + \frac{(x-1)^{m-1}}{(m-1)!}, \; x > 1;$$

$$I_1(x) = up(2^{-1} x - 2^{-1}); \; I_1'(x) = up(x). \quad (4.3)$$

AF satisfies *partition of unity* [2]-[12] to exactly represent the number 1 by summing up individual overlapping pulses set at regular points… −2, −1, 0, 1, 2… (**Figure 3(a)**):

$$\cdots up(x-2) + up(x-1) + up(x) + up(x+1) + up(x+2) + \cdots \equiv 1. \quad (4.4)$$

This property is related to the following double symmetry [2]-[12]:

$$up(x) = up(-x), \; x \in [-1,1]; \; up(x) + up(1-x) = 1, \; x \in [0,1]. \quad (4.5)$$

Generic AF pulse of width $2a$, height c, and center positions b, d has the form

$$up(x,a,b,c,d=0) = d + c * up((x-b)/a) \cdot \int_{-a}^{a} cup(x/a) dx = ca . \quad (4.6)$$

Multi-dimensional atomic functions [2]-[8] [24] [27] (**Figure 2** and **Figure 3**) can be constructed as either multiplications or radial atomic functions:

$$up(x,y,z) = up(x) up(y) up(z),$$

$$up(r) = up\left(\sqrt{x^2+y^2+z^2}\right), \; \iiint cup\left(\frac{x}{a},\frac{y}{a},\frac{z}{a}\right) dxdydz = ca^3. \quad (4.7)$$

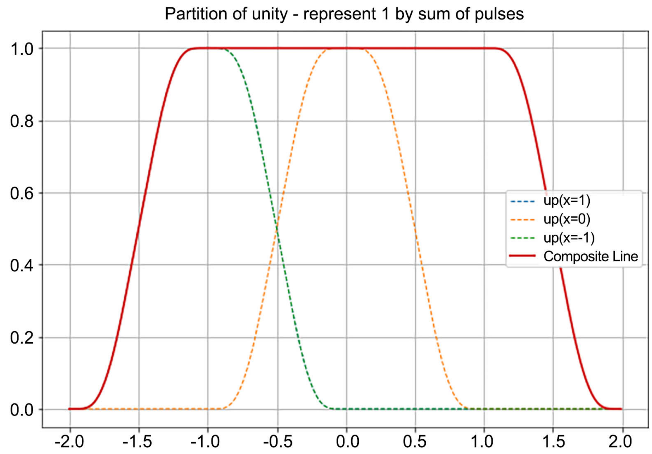
4.2. AString Function

AString function (**Figure 4**) first proposed in 2018 by the author [2] [3] [4] [5] is both an integral (4.3) and "composing branch" of $up(x)$:

$$AString'(x) = AString(2x+1) - AString(2x-1) = up(x). \quad (4.8)$$

AString has a form of a solitary kink (**Figure 4(a)**) which can compose a straight line $y = x$ both between and as a translation of AString kinks leading to spacetime "atomization"/quantization ideas (§3, 7):

$$x \equiv AString\left(x - \frac{1}{2}\right) + AString\left(x + \frac{1}{2}\right), \; x \in \left[-\frac{1}{2}, \frac{1}{2}\right];$$



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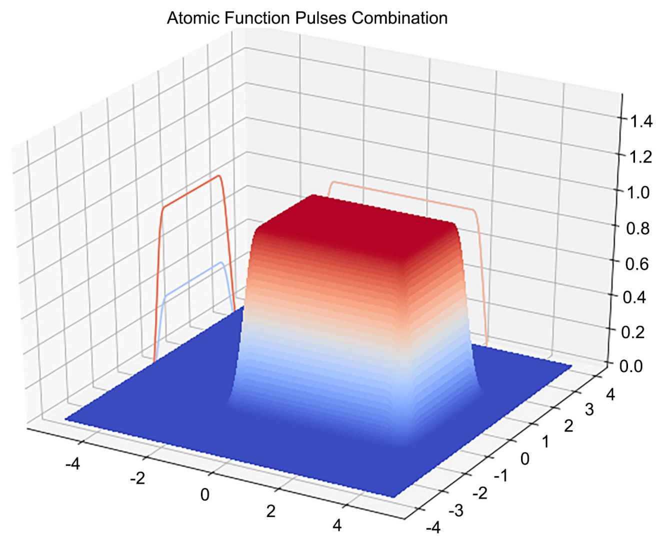

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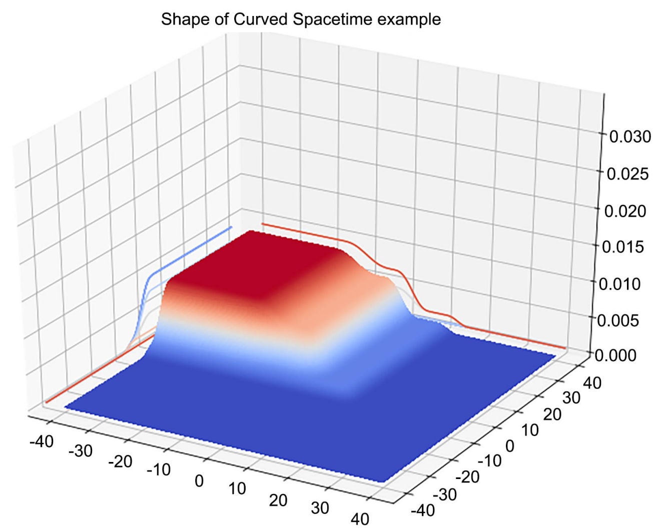

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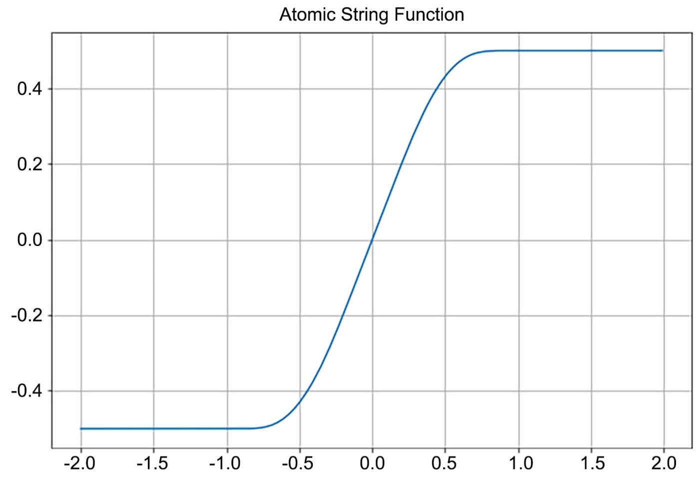
Figure 3. (a) Partition of unity with Atomic Functions; (b) Representation of flat surface via summation of Afs; (c) Curved surface as a superposition of "solitonic atoms".



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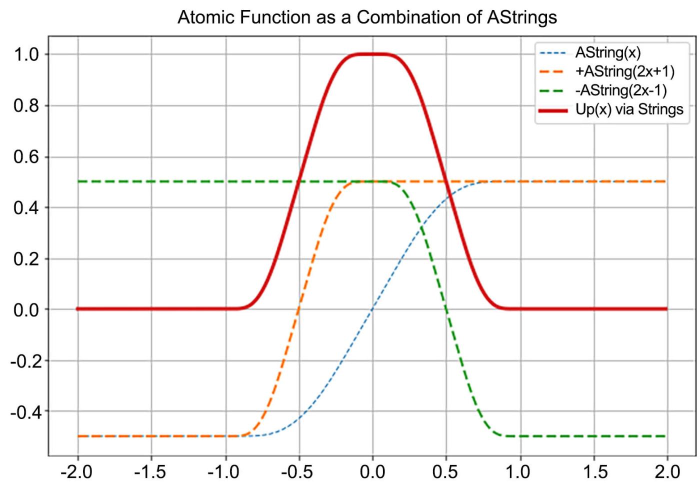

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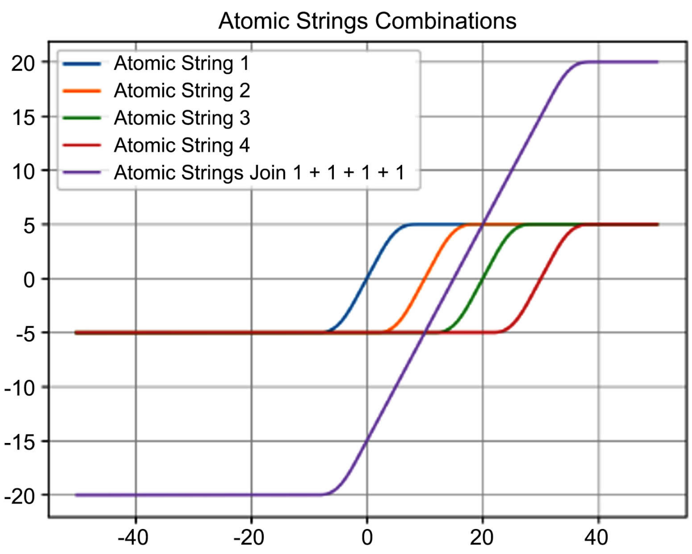


(c)

Figure 4. (a) Atomic String Function (AString), (b) Atomic function as a combination of two AStrings, (c) Representation of a straight line segment by summing of AStrings.

$$x \equiv \cdots AString(x-2) + AString(x-1) + AString(x) + AString(x+1) + AString(x+2)\cdots \quad (4.9)$$

The elementary AString kink function can be generalized in the form

$$AString(x,a,b,c,d=0) = d + c * AString((x-b)/a). \quad (4.10)$$

Importantly, Atomic Function pulse (4.6) can be presented as a sum of two opposite AString kinks (**Figure 4(b)**) making AStrings and AFs deeply related to each other:

$$up(x,a,b,c) = AString\left(x,\frac{a}{2},b-\frac{a}{2},c\right) + AString\left(x,\frac{a}{2},b+\frac{a}{2},-c\right). \quad (4.11)$$

4.3. Atomic Series, Atomic Splines and "Mathematical Atoms"

Atomic and AString Functions (or briefly *Atomics*) possess unique approximation properties described later in §5, 6. Like from "mathematical atoms" [6]-[12], as V.L. Rvachev called them, flat and curved smoothed surfaces/functions (**Figure 3**) can be composed of a superposition of Atomics via the so-called Generalized Taylor's Series [7] [8] [9] [24] [25] [26] [27] (or simply, *Atomic Series*) with an *exact* representation of polynomials of any order

$$\frac{1}{4}\sum_{k=-\infty}^{k=+\infty} k\,up\left(x-\frac{k}{2}\right) \equiv \sum_{k=-\infty}^{k=+\infty} AString(x-k) \equiv x;$$

$$\sum_{k=-\infty}^{k=+\infty}\left(\frac{k^2}{64}-\frac{1}{36}\right)up\left(x-\frac{k}{4}\right) \equiv x^2,$$

$$x^n \equiv \sum_{k=-\infty}^{k=+\infty} C_k\,up\left(x-k2^{-n}\right) = \sum_{k=-\infty}^{k=+\infty} C_k\left(AString\left(2\left(x-k2^{-n}\right)+1\right) - AString\left(2\left(x-k2^{-n}\right)-1\right)\right). \quad (4.12)$$

Notably, only a limited number of neighboring finite "atoms" are required to calculate a polynomial value at a given point.

It means Atomics can also represent/atomize any *analytic function* [28] (a function representable by converging Taylor's series) with known calculable coefficients:

$$y(x) = \sum_{m=0}^{\infty}\frac{y^{(m)}(0)}{m!}x^m = \sum_{m=0}^{\infty} B_m x^m = \sum_{m=0}^{\infty}\sum_{k=-\infty}^{k=+\infty} B_m C_k\,up\left(x-k2^{-m}\right) = \sum_{mk=-\infty}^{\infty} c_{mk}\,up\left(\frac{x-b_{mk}}{a_{mk}}\right) = \sum_{l=-\infty}^{l=+\infty} AString(x,a_l,b_l,c_l). \quad (4.13)$$

Analytic functions [28] represent a wide range of polynomial, trigonometric, exponential, hyperbolic, and other functions, their sums, derivatives, integrals, reciprocals, multiplications, and superpositions. Therefore, they all can be "atomized" via superpositions of Atomic and AString Functions with any degree of precision, which is the most important property.

Instead of sums (4.12), and (4.13), we will be using short notation with localized basis atomic functions $A_k(x)$ and function values y^k at node k assuming summation over repeated indices *k*:

$$y(x) = A_k(x) y^k; \quad f(x,y,z) = A_k(x,y,z) f^k \tag{4.14}$$

Composing functions from finite pieces can also be achieved with widely-used polynomial splines but with the limitation that n-order splines can exactly reproduce only n-order polynomials (eq cubic parabola cannot be exactly composed of quadratic splines). Atomic Splines [2]-[12] based on Atomic and AString Functions are more generic and also can provide smooth connection between splines, leading to spacetime and fields quantization ideas (§5 - 10) where Atomics pretend to be the "building blocks" of fields.

4.4. Atomic Solitons

Being *solutions* of special kinds of nonlinear differential equations with shifted arguments (4.1), (4.8), AStrings and Atomic Functions possess some mathematical properties of lattice solitons [29] [30] [31] [32] and have been called *Atomic Solitons* [2] [3] [4] [5]. AString is a solitonic kink whose particle-like properties exhibit themselves in the composition of a line (4.9) and kink-antikink "atoms" (4.8) (**Figure 4**). Being a composite object (4.8) made of two AStrings, AF $up(x)$ is not a true soliton but rather a *solitonic atom*, like "bions" or "dislocation atoms" [2] [4] [29] [30], as described in [2] [4].

5. Atomization Theorems

Unique properties of Atomic and AString Functions (Atomics) allow formulating *Atomization Theorems* stating how scalar, vector, tensor functions, and solutions of linear and nonlinear differential equations can be represented via a series of Atomics/Atomic Splines/Atomic Solitons leading to spacetime quantization and field unification ideas. To shorten the content, the proof would be provided only to a few theorems referring to [23] for more detailed descriptions with proof.

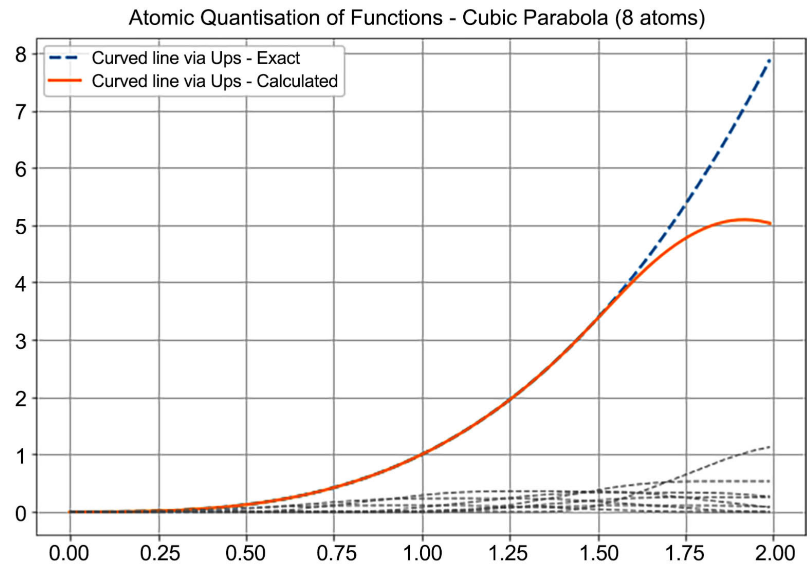
Theorem 1 (*Polynomial atomization theorem*). Polynomials of any order can be exactly represented/atomized via the Atomic Series of Atomic and AString Functions:

$$\left(x^n\right)^{(n)} = c\sum_{k=-\infty}^{k=+\infty} up(x-k) \equiv c,$$
$$x^n = I_n\left(c\sum_{k=-\infty}^{k=+\infty} up(x-k)\right) = \sum_{k=-\infty}^{k=+\infty} C_k up\left(x - k2^{-n}\right). \tag{5.1}$$

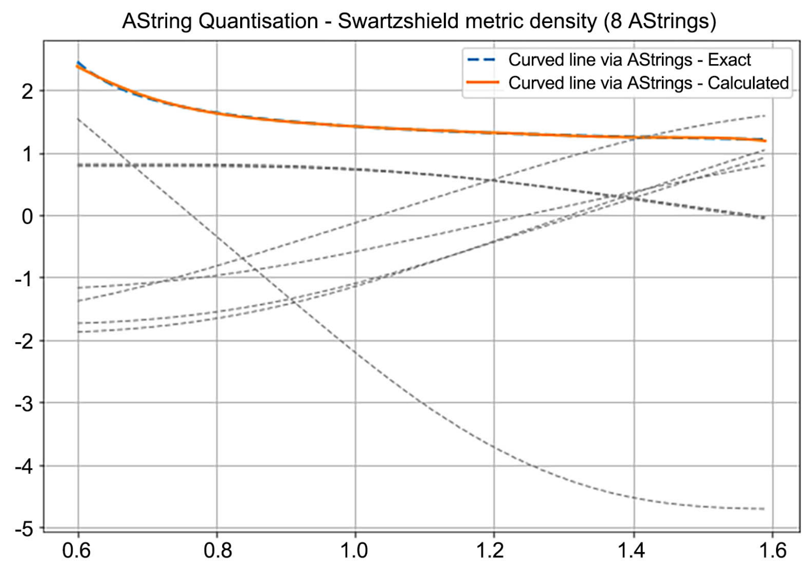
$$P_n(x) = x^n + a_1 x^{n-1} + \cdots + a_n \equiv \sum_k C_k up\left(\frac{x-ka}{a}\right)$$
$$= \sum_k AString\left(x, a_k, b_k, c_k\right) = A_k(x) P_n^k. \tag{5.2}$$

Based on (4.12) and finiteness, it states that only a few neighboring Atomics are required to calculate a polynomial value at a given point (**Figure 5(a)**).

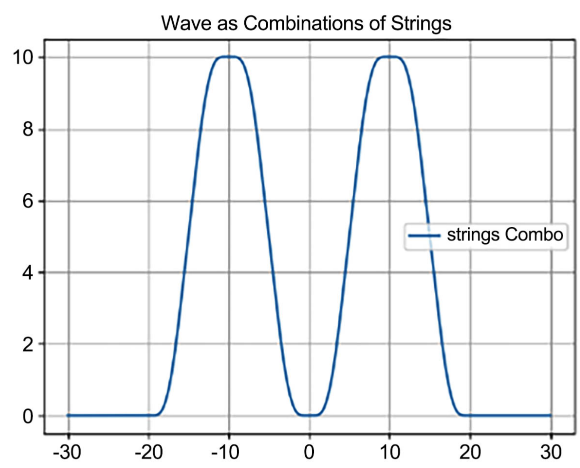
Theorem 2 (*Analytic atomization theorem*). Analytic functions representable by converging Taylor's series via polynomials can be represented/atomized via converging Atomic Series of localized Atomic and AString Functions:


Atomic Quantisation of Functions - Cubic Parabola (8 atoms)
Curved line via Ups - Exact
Curved line via Ups - Calculated
0 1 2 3 4 5 6 7 8
0.00 0.25 0.50 0.75 1.00 1.25 1.50 1.75 2.00

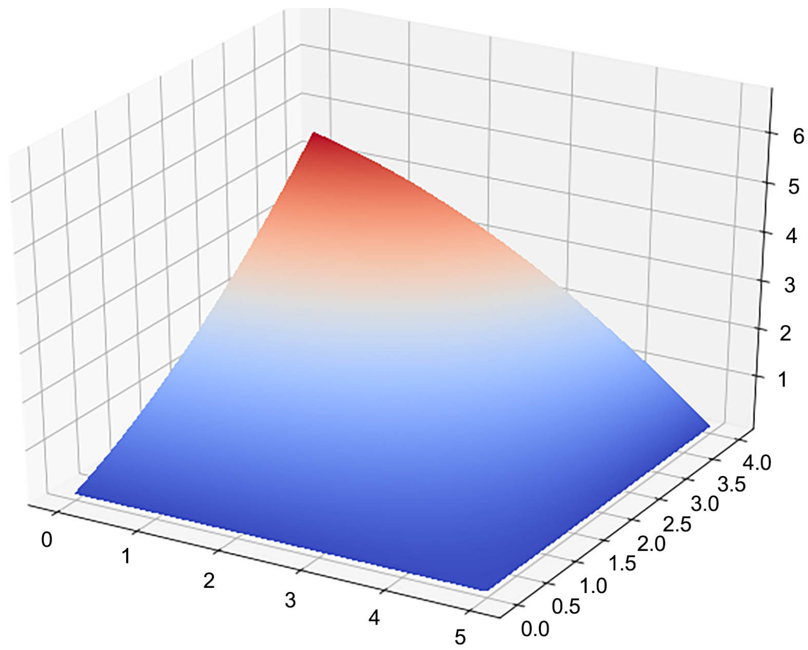
(a)


AString Quantisation - Swartzshield metric density (8 AStrings)
Curved line via AStrings - Exact
Curved line via AStrings - Calculated
2 1 0 -1 -2 -3 -4 -5
0.6 0.8 1.0 1.2 1.4 1.6

(b)


Wave as Combinations of Strings
strings Combo
0 2 4 6 8 10
-30 -20 -10 0 10 20 30

(c)



(d)

Figure 5. Representing sections of polynomials and analytic functions with AStrings and Atomic Functions (a) Cubic parabola via 8 Atomic Functions; (b) Schwarzschild metric function; (c) Wave-like formation; (d) 3d surface.

$$\begin{aligned} y(x) &= \sum_{m=0}^{\infty} \frac{y^{(m)}(0)}{m!} x^m = \sum_{m=0}^{\infty} B_m x^m = \sum_{m=0}^{\infty} \sum_{k=-\infty}^{k=+\infty} B_m C_k up\left(x - k2^{-m}\right) \\ &= \sum_{mk=-\infty}^{\infty} c_{mk} up\left(\frac{x - b_{mk}}{a_{mk}}\right) = \sum_{l=-\infty}^{l=+\infty} AString\left(x, a_l, b_l, c_l\right) = A_k(x) y^k. \end{aligned} \tag{5.3}$$

By definition, an analytic function [28] is representable via conversing Taylor's series by polynomials which in turn can be expressed via Atomics (5.1). It means physical fields described by exponential, trigonometric, hyperbolic, and other analytic functions are also atomizable. This theorem proven in [23] can be generalized to various combinations of analytic functions [23] [28] [45] with the following.

Theorem 3 (*Complex analytic atomization theorem*). Complex functions $y(x)$ that are sums $y = y_1 + y_2$, products $y = y_1 y_2$, reciprocals $y = 1/y_1$ ($y_1 \neq 0$), inverse $y(y_1) = x$, derivatives $y = y_1'$, integrals $I(y)$, and superposition $y = y_1(y_2)$ of analytic functions $y_1(x), y_2(x)$ can be represented/atomized by Atomic Series over Atomic and AString Functions.

In general, polynomic, trigonometric, exponential, and other analytic functions are the solutions of some linear differential equations (LDE) implying that Atomization Theorems can be extended to differential equations [7]-[12] [23] [27].

Theorem 4 (*LDE atomization theorem*). Solutions of linear differential equations (LDE) with constant coefficients can be represented/atomized via series over Atomic and AString Functions:

$$L(y) = y^{(n)}(x) + a_1 y^{(n-1)}(x) + \cdots + a_{n-1} y'(x) + a_n y(x) = 0;$$
$$y(x) = A_k(x) y^k. \tag{5.4}$$

This theorem can be generalized [23] to equations with variable analytic coefficients frequently appearing in mathematical physics.

Theorem 5 (*Variable LDE atomization theorem*). Solutions of linear differential equation with variable coefficients $a_k(x)$ representable by analytic functions can be represented/atomized via Atomic Series over Atomic and AString Functions:

$$L(y, a_k(x)) = y^{(n)}(x) + a_1(x) y^{(n-1)}(x) + \cdots + a_n(x) y(x) = 0;$$
$$y(x) = A_k(x) y^k. \tag{5.5}$$

Atomization of composite functions like $\arctan(\exp(x)), \operatorname{sech}(x)$ satisfying nonlinear sine-Gordon and Schrodinger differential equations [31] [32] imply that the Atomization procedure is also applicable to some nonlinear differential equations.

Theorem 6 (*NDE atomization theorem*). Solutions of nonlinear differential equations (NDE) with linear differential operator $L(y)$ and nonlinear analytic function $f(y)$ can be represented/atomized via Atomic Series over Atomic AString Functions:

$$L(y, a_k(x)) = y^{(n)}(x) + a_1(x) y^{(n-1)}(x) + \cdots + a_n(x) y(x) = f(y);$$
$$y(x) = A_k(x) y^k. \tag{5.6}$$

The preservation of analyticity and Theorem 3 imply that compositions $y = y_1(y_2)$ of analytic functions are also analytic [23] [45], so the theorems 1 - 6 can be Generalized to the following theorem.

Theorem 7 (*Complex NDE atomization theorem*). Solutions of nonlinear differential equations with functional-differential operator F preserving analyticity of function $y(x)$ can be represented/atomized via Atomic Series over Atomic and AString Functions.

$$F\left(y(x), \frac{\partial^m y}{\partial x^n} \right) = 0;$$
$$y(x) \equiv \sum\nolimits_l up(x, a_l, b_l, c_l) = \sum\nolimits_k AString(x, a_k, b_k, c_k) = A_k(x) y^k. \tag{5.7}$$

Applying Theorem 2 to the widely used Fourier Series leads to the following theorem.

Theorem 8 (*Waves atomization theorem*). Any smooth function with a *finite spectrum* [7] [8] [9] [23] can be represented/atomized via Atomic Series over Atomic and AString Functions:

$$\begin{aligned} y(x) &\equiv \sum\nolimits_{i=1}^n d_i \sin(e_i x - f_i) = \sum\nolimits_l up(x, a_l, b_l, c_l) \\ &= \sum\nolimits_k AString(x, a_k, b_k, c_k) = A_k(x) y^k. \end{aligned} \tag{5.8}$$

Theorems 1 - 8 can be extended to many dimensions noting that multidimensional *n*-order polynomials in *m*-dimensions $P_{mn} = P_n(x_1, \cdots, x_m)$ which are some multiplications of 1D polynomials exactly representable by Atomics (Theorem 1) are also exactly representable by multiplications of Atomic Functions (multidimensional atomic functions (4.9) $UP(a_k, b_k, c_k)$ which in turn are AStrings combinations (4.8).

Theorem 9 (*3D atomization theorem*). Representable by converging Taylor's series, multidimensional analytic functions with their sums, multiplications, reciprocals, derivatives, integrals, and superpositions can be represented/atomized via Atomic Series over localized multidimensional Atomic and AString Functions:

$$\begin{aligned} P_{mn} = P_n(x_1, \cdots, x_m) &= \prod_{i=1}^{m} P_n(x_i) = \sum_k \prod_{i=1}^{m} up_i(x_i, a_{ik}, b_{ik}, c_{ik}) \\ &= \sum_k UP(a_k, b_k, c_k) = \sum_l AString(a_l, b_l, c_l) = A_k(x_1, \cdots, x_m) P_{mn}^k. \end{aligned} \tag{5.9}$$

Similar to the 1D case with theorems 4, 5, the atomization procedure can be extended to multi-dimensional differential equations containing linear differential operators like Laplacian and Poisson operators widely used in mathematical physics:

$$L(y_1, \cdots, y_m)(x_1, \cdots, x_m) = a_{ijmn} \frac{\partial^m y_i}{\partial x_j^n} = 0; \nabla_i = \frac{\partial}{\partial x_i}; \Delta = \sum \frac{\partial^2}{\partial x_i^2}; \Delta + k; \Delta\Delta.$$

$$y_i(x_j) \equiv \sum_{ijjkl} up_i(x_j, a_{ijl}, b_{ijl}, c_{ijl}) = \sum_{jjkl} AString_i(x_j, a_{ijk}, b_{ijk}, c_{ijk}). \tag{5.10}$$

In summary, formulated Atomization Theorems proven in [23], demonstrate how polynomials, complex multi-dimensional analytical functions, and solutions of linear and nonlinear differential equations can be represented/atomized via superpositions of localized Atomic and AString Functions.

6. Atomization Theorems in General Relativity

Theorems 1 - 9 lead to the following theorems provided with proof and targeting Einstein's General Relativity (GR) theory [13] [14] [15].

6.1. Atomization Theorems for Metric, Curvature, and Ricci Tensors

Considering together multidimensional Atomic Series (5.9), (5.10) and Atomization Theorems 1-6 leads to the following theorems important for General Relativity.

Theorem 10 (*Tensor's atomization theorem*). First $\partial_i = \frac{\partial}{\partial x_i}$ and second derivatives $\partial_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}$ as well as the metric tensor g_{ij} defining interval on a curved surface $\mathrm{d}s^2 = g_{ij}(x_n)\mathrm{d}x_i\mathrm{d}x_j$ preserve analyticity and being applied to analytic functions $y_k(x_l)$ lead to analytic functions representable/atomizable by Atomic Series over Atomic and AString Functions.

Proof. Being linear differential operators, both first and second derivative operators preserve analyticity because derivatives of multidimensional polynomials $B_{lm}x_l^m$ would also be polynomials exactly representable via multidimensional Atomic Functions and AStrings (5.9) using Atomic Series (5.2). For curved spacetime surfaces/geometries described by some analytic functions $\tilde{x}_i = \tilde{x}_i(x_j)$; $\mathrm{d}\tilde{x}_i = \frac{\partial \tilde{x}_i}{\partial x_j}\mathrm{d}x_j$, the derivatives and their multiplications would also be analytic, hence representable by Atomics (Theorems 2, 3, 9). This theorem can be proved in another way by noting that all derivatives and integrals of Atomics are expressed via themselves (4.3), (4.8), and if space geometry analytic functions $\tilde{x}_i(x_j)$ are the sum of Atomics, then all derivatives and metric tensors would also be some Atomics combinations:

$$g_{ij}(x_n) = \sum_{ijnk} up(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} AString(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}). \quad (6.1)$$

Proof obtained. This theorem means that for analytic spacetime geometries/configurations, their deformations, curvatures, metrics, and geodesics would also be some Atomics superpositions, with a range of analytical surfaces and spacetime metrics known in GR [13] [14] [15] described later. Furthermore, due to the properties of analytic function superpositions to preserve analyticity (Theorem 3), the last theorem can be extended to nonlinear Ricci tensors important in GR [13] [14] [15].

Theorem 11 (*Ricci tensor atomization theorem*). Nonlinear Ricci tensor R_{jk} and Christoffel operators Γ_{ij}^k preserve analyticity and applied to analytic functions would yield analytic functions representable/atomizable by Atomic Series via Atomic and AString Functions.

Proof. Christoffel operators [13] [14] [15], which include multiplications of functions to their spatial derivatives, transform analytic metric tensor functions (6.1) representable by polynomials into more complex polynomials representable by Atomics via Atomic Series (5.3). Similarly, Ricci tensors are also a combination of derivatives and multiplications of Christoffel symbols [13] [14] [15] which preserve analyticity, hence representable via Atomics:

$$\Gamma_{ij}^k = \frac{1}{2}g^{kl}(\partial_i g_{jl} + \partial_j g_{il} - \partial_l g_{ij}); \; R_{jk} = \partial_i \Gamma_{jk}^i - \partial_j \Gamma_{ik}^i + \Gamma_{ip}^i \Gamma_{jk}^p - \Gamma_{jp}^i \Gamma_{ik}^p \quad (6.2)$$

$$R_{ij}(x_n) = \sum_{ijnk} up(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}) = \sum_{ijnl} AString(x_n, a_{ijnl}, b_{ijnl}, c_{ijnl}). \quad (6.3)$$

Proof obtained. This theorem can be intuitively understood in the sense that polynomials are "hard to destroy" because their multiplications, derivatives, integrals, and superpositions would also be polynomials representable by Atomics. It also means that not only spacetime metrics but also curvature tensors can be "atomized" using shifts and stretches of finite AString and Atomic Functions which, as described later, may be associated with flexible spacetime quanta.

6.2. Atomization Theorem for General Relativity

The sequence of Theorems 1-10 converges into the following theorem for Einstein's General Relativity [13] [14] [15].

Theorem 12 (*Atomic Spacetime Theorem*). For analytic manifolds, Einstein's curvature tensor preserves analyticity and yields spacetime shapes, deformations, curvatures, and matter/energy tensors representable via multi-dimensional Atomic and AString Functions superpositions. Solutions of General Relativity equations can be represented/atomized by converging Atomic Series over finite Atomic and AString Functions:

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = \sum\nolimits_{\mu\nu i} \boldsymbol{UP}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right) \\ = \sum\nolimits_{\mu\nu i} \boldsymbol{AString}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right). \tag{6.4}$$

Proof. For analytic manifolds—spacetime geometries described by analytic functions $\tilde{x}_i = \tilde{x}_i\left(x_j\right)$ representable by converging Taylor's series - the metric tensors $g_{\mu\nu}$ composed of derivatives and their multiplications would also be some analytic functions (Theorem 10). Being injected into Christoffel operators (6.2) and then Ricci tensors $R_{\mu\nu}$, they would yield another set of analytic functions (Theorem 11) representable by Taylor's series because the derivatives, multiplications, and superposition of analytic functions would also be analytic (Theorem 3). The curvature scalar $R = g^{\mu\nu} R_{\mu\nu}$ in (6.4) also preserves analyticity because of the cross-multiplication of polynomials and their derivatives would also be polynomials. Injected into (6.4), those tensors produce Einstein's tensor $G_{\mu\nu}$ and energy-momentum tensor $T_{\mu\nu}$ ($\frac{8\pi G}{c^4}$ is a constant) supposedly representable by polynomials via multi-dimensional Taylor's series. Because a polynomial of any order is exactly representable/atomizable via Atomic and AString Functions (Theorem 1), the spacetime curvature, metric, and energy/momentum tensors would be the superpositions of multi-dimensional Atomic $\boldsymbol{UP}$ and $\boldsymbol{AString}$ functions, derivatives of which are expressed via themselves. Due to fundamental relation (4.8) $up(x) = AString'(x) = AString(2x+1) - AString(2x-1)$, the Atomic Function $up(x)$ is a sum of two AStrings which can be associated with a finite quantum/metriant being able, within one model, to compose straight $x = \sum\nolimits_k AString(x, a, ka, a)$ and curved $\tilde{x} = \sum\nolimits_k AString(x, a_k, b_k, c_k)$ lines from elementary AString pieces resembling quanta (§3, 6, 7, 8).

Proof obtained. In a nutshell, this theorem tells that the smooth spacetime field is representable/atomizable via superpositios of finite AStrings and Atomic Functions, the derivatives of which are expressed via themselves meaning the spacetime shape, deformations, curvatures, and energy/momentum tensors can also be represented as some superposition of Atomics. Now, this idea first hypothesized in 2017 [3] is based on a set of theorems. It offers an "atomic model" of spacetime [2] [3] [4] [5] resonating with A. Einstein's 1933 [1] "*perfectly thinkable*" "*atomic theory*" dealing with "*simplest concepts and links between them*" with finite regions of space (§1). Let's note that Atomization is not a simple discretization of space – separation of a volume into adjacent finite elements [22] [24] [38]. Here, the "finite elements" (AStrings) are smoothly overlapping

(§3, **Figure 1**) and capable to describe both expansions of space (4.9) and localized solitonic atoms $up(x)$ (4.8).

Reversing of Atomization Theorems (1-12) allows deriving Atomic and AString Functions from General Relativity equations (6.4), as described in [23].

6.3. Deriving Atomics from General Relativity Equations

Previous Atomization Theorems in GR were based on the historical assumption that we know the mathematical properties of Atomic Functions [2]-[12] and try to introduce them to spacetime physics as it was done in [2] [3] [4] [5]. The intriguing question is whether it is possible to do the opposite—to derive Atomic Functions from GR, so in theory, A. Einstein could have done it himself, especially in 1933 when in [1] he envisaged an "*atomic theory*" with "*…region of three-dimensional space at whose boundary electrical density vanishes everywhere*" resembling finite functions like Atomics (§1). The following theorem shows how it can be achieved by applying backward the Atomization Theorems 1-12.

Theorem 13 (*Atomic Spacetime quantum theorem*). It is possible to find an infinitely differentiable finite pulse spline function that can represent analytic solutions of General Relativity and polynomials of any order via superpositions, and such a function should have a form of Atomic Function $up(x)$ with derivative

$$up'(x) = 2up(2x+1) - 2up(2x-1) \tag{6.5}$$

and integral

$$AString'(x) = up(x) = AString(2x+1) - AString(2x-1). \tag{6.6}$$

Being localized solitary functions capable to compose flat and curved spacetime fields in overlapping superposition, those spline functions may be interpreted as flexible quanta of spacetime.

Proof. Preservation of analyticity in Einstein's curvature tensor, Ricci tensor, and Christoffel operators (Theorems 11 and 12) implies that analytic metric tensor functions $g_{ij}(x_n)$ being injected in those operators would produce other analytic functions representable by polynomials – because the multiplication of derivatives of polynomials to other polynomials would also be polynomials. Analyticity of metric tensor $g_{ij}(x_n)$ representable by Taylor's series via polynomials (for which derivatives and integrals would also be polynomials) implies that spacetime geometry $\tilde{x}_i(x_n)$ and geodesics should also be analytic functions representable by polynomials (Theorem 10). This theorem would be proven if we find some basis spline pulse-like function $p(x) \in [-1,1]$ which in translation would exactly compose polynomials of any order (Theorem 1) $x^n = \sum_k c_k p\left(\frac{x-b_k}{a_k}\right)$. Firstly, following §3, we have to eliminate the polynomial spline candidates (like B-splines or cubic Hermitian splines [21] [22] [23]) as they are unable to exactly compose a polynomial of *any* order. The desired spline function should be a polynomial of infinite order, or simply belong to class $C(\infty)$ of absolutely smooth functions. Secondly, we have to eliminate trigonometric and other exponential-based spline functions like Gaussians or Sigmoids because by summing only a few pulses they cannot exactly reproduce even the simplest polynomials (a line, or a constant). To satisfy Theorem 1, the choice has narrowed to infinitely differentiable spline functions $p(x)$ capable to compose any polynomial $x^n = \sum_k c_k p\left(\frac{x-b_k}{a_k}\right)$ and also satisfy the "partition of unity" $c = \sum_k cp(x-k)$. Desired infinite differentiability implies that the spline function's derivative should be expressed via the function itself $p'(x) = F(p(x))$, or in simplest linear form $p'(x) = F(p(x)) = kp(ax+b) - kp(ax-b)$ which with symmetry condition $p(x) = p(-x)$, normalization $p(0) = 1$ and finiteness $p(x) = 0$, $|x| > 1$ lead to Atomic Function $p(x) = up(x)$, $up'(x) = 2up(2x+1) - 2up(2x-1)$ as described in §3. Using this spline function to compose the 3D polynomials, the geometry of spacetime $\tilde{x}_i(x_n)$ along with metric tensor $g_{ij}(x_n)$, Ricci tensor $R_{ij}(x_n)$ and Einsteinian tensor $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$ lead to Theorems 11 and 12 and the ability to express GR solutions via a series of Atomic Function pulses. Due to symmetry (2.5), the Atomic Function $up(x)$ can be represented via the sum of two simpler *AString* kink functions (3.8) $up(x) = AString(2x+1) - AString(2x-1) = AString'(x)$, so GR solutions can also be expressed via AStrings. AStrings can describe continuous spacetime line expansion via superpositions of finite functions $x \equiv \sum_k AString(x-k)$ hence may be associated with some finite quanta of space/quanta of length.

Proof obtained. This conceptually important theorem allows deducing finite Atomic and AString Functions from GR noticing a crucial property of GR operators to preserve analytic functions and polynomials and the unique ability of Atomic AString Functions to exactly represent them. There is not much of a surprise that spacetime and other smooth fields can be represented by some splines or "mathematical atoms", as founders called them in the 1970s [6] [7] [8] [9] [10]. The challenge was to formally work out [5] [23] how complex nonlinear Einstein's GR equations can yield simply looking Atomic Splines, and preservation of analyticity and atomization of polynomials were the key hints to achieve this.

7. Atomic Spacetime

7.1. Spacetime Atomization Model

Theorems 10-13 provide a theoretical foundation for atomization/quantization of spacetime field based on Atomic and AString Functions when GR equations and solutions, along with Ricci, curvature, and metric tensors, can be represented via Atomic Series over multidimensional Atomic and AString Functions (4.7):

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu} = \sum\nolimits_{\mu\nu i} \boldsymbol{UP}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right)$$
$$= \sum\nolimits_{\mu\nu i} \boldsymbol{AString}\left(x_i, a_{\mu\nu i}, b_{\mu\nu i}, c_{\mu\nu i}\right), \tag{7.1}$$

$$R_{ij}\left(x_n\right) = \sum\nolimits_{ijnk} \boldsymbol{UP}\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right)$$
$$= \sum\nolimits_{ijnl} \boldsymbol{AString}\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right), \tag{7.2}$$

$$g_{ij}\left(x_n\right) = \sum\nolimits_{ijnk} \boldsymbol{UP}\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right)$$
$$= \sum\nolimits_{ijnl} \boldsymbol{AString}\left(x_n, a_{ijnk}, b_{ijnk}, c_{ijnk}\right), \tag{7.3}$$

$$\tilde{x}_i\left(x_l\right) = \sum\nolimits_k \boldsymbol{AString}\left(x_l, a_{lk}, b_{lk}, c_{lk}\right). \tag{7.4}$$

These formulae express the mathematical fact that it is possible to compose analytical manifolds (**Figure 6**) by adjusting parameters of overlapped localized Atomic and AString Functions splines, or like in Lego-game, compose a smooth shape from elementary pieces resembling quanta. Also, if finite Atomics, for which derivatives are expressed via themselves, represent spacetime shape $\tilde{x}_i\left(x_l\right)$ (7.4), the series over Atomics would also describe spacetime deformations, curvatures, metrics, Ricci's, Einstein's, and energy-momentum tensors. It becomes quite similar to widely-used Fourier series based on representation of smooth fields by superposition of harmonic functions for which derivatives, integrals and multiplications would also be some harmonics. But unlike Fourier series dealing with continuous infinite functions (which A. Einstein [1] also complained about, §1), the Atomic Series allows to come up with the concept of an atomic quantum–finite regions of space overlapping to produce complex geometries and fields.

Because AString can compose a line and a curve from "elementary pieces" resembling quanta (§3, 6, 8) one can envisage a spacetime field as a complex network of flexible spacetime quanta (**Figure 4** and **Figure 6**). The notion of "quantum" here is not directly related to Quantum Mechanics and Quantum Gravity [18] [19] [34]-[39] but rather to the finiteness of "solitonic atoms" capable to compose shapes and fields from small pieces.

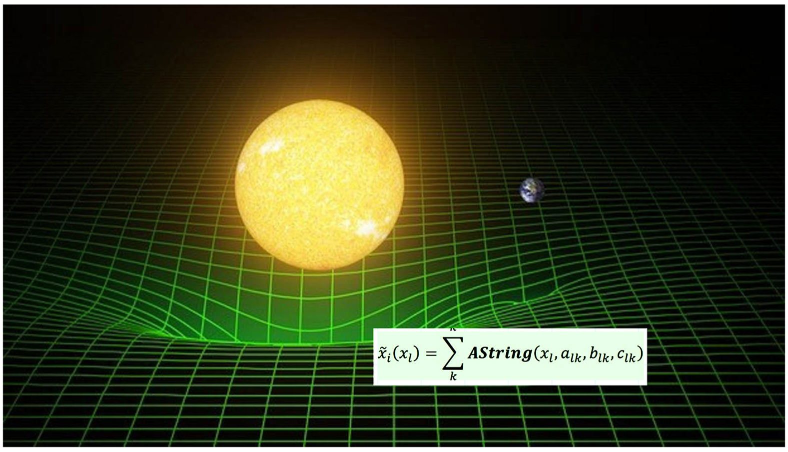
7.2. Atomization of Known General Relativity Solutions

The idea of Atomic Spacetime atomization/quantization can be demonstrated for known GR solutions [13] [14] [15] [16].

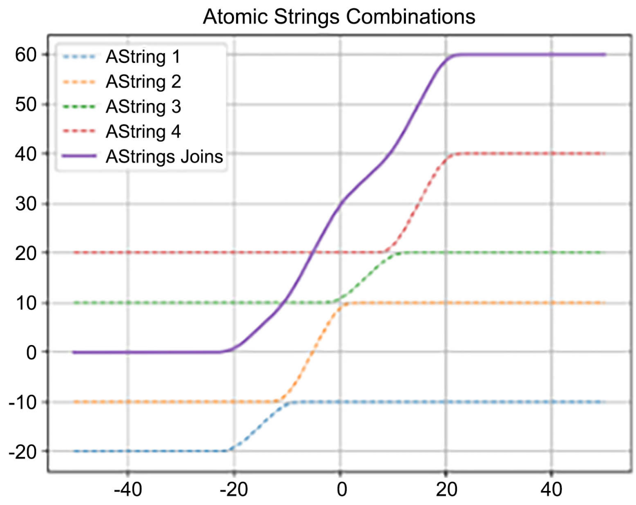
Einstein-Minkowski solution $T_{\mu\nu} = 0$, $g_{ij} = 1$ for homogeneous uniform spacetime/universe [13] [14] [15] [16] [20] is simply atomizable/quantizable via translations of identical overlapping AString quanta (§3, **Figure 3**) [2] [3] [4] [5] in vector notation:

$$\boldsymbol{AQuantum}\left(x_1, x_2, x_3, t, a, \rho, c_l\right)$$
$$= AString\left(x_1, a, a, \rho a\right)\boldsymbol{e}_1 + AString\left(x_2, a, a, \rho a\right)\boldsymbol{e}_2$$
$$+ AString\left(x_3, a, a, \rho a\right)\boldsymbol{e}_3 + AString\left(t, a/c_l, a/c_l, \rho a/c_l\right)\boldsymbol{e}_t, \tag{7.5}$$

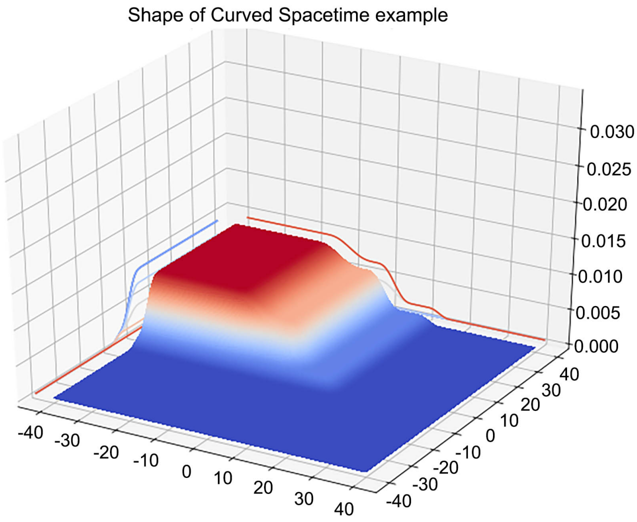
or schematically (**Figure 3**, **Figure 4**, **Figure 6** and **Figure 7(c)**)



(a)



(b)



(c)

Figure 6. (a) Curved spacetime composed of AStrings, (b) Joining AStrings of different heights simulates spacetime curving, (c) Curved spacetime geodesics represented via joints of 3D solitonic atoms.

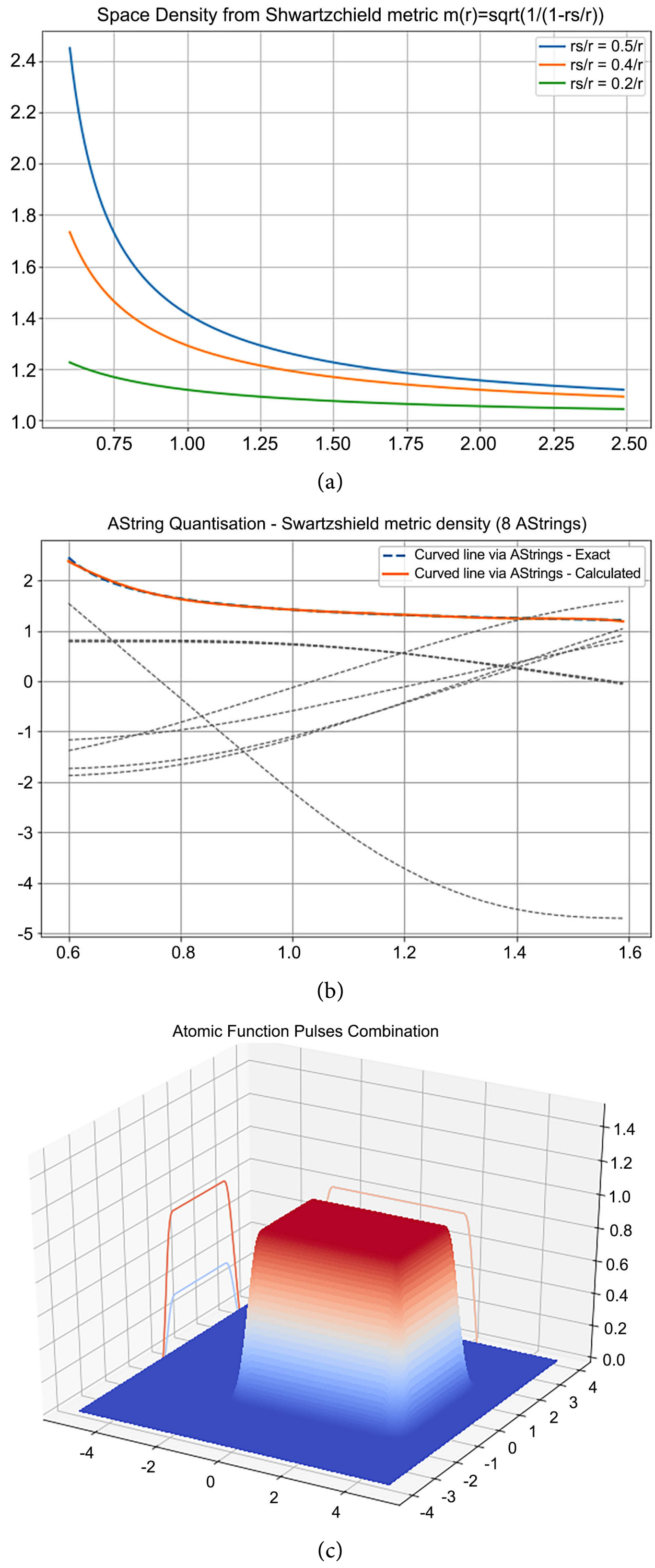


Figure 7. (a) Space density function from Schwarzschild GR solution, (b) Representing Schwarzschild metric via AStrings, (c) Uniform spacetime field as a superposition of solitonic atoms.

$$\boldsymbol{UniformSpace}\left(x_1,x_2,x_3,t\right)=\sum_k \boldsymbol{AQuantum}\left(x_1,x_2,x_3,t,a,\rho,c_l\right). \quad (7.6)$$

Friedmann solution for expanding spatially homogeneous universe with metric [13] [14] [15] [16]

$$\mathrm{d}s^2=a\left(t\right)^2\mathrm{d}\overline{s}^2-c^2\mathrm{d}t^2;\ \mathrm{d}\overline{s}^2=\mathrm{d}r^2+S_k\left(r\right)^2\mathrm{d}\Omega^2 \quad (7.7)$$

includes analytic function $S_k\left(r\right)$ representable via Atomic Series (5.4), (5.5) as per Theorems 3,4:

$$\begin{aligned} S_k\left(r\right)&=r\,\mathrm{sinc}\left(r\sqrt{k}\right)=r-\frac{kr^3}{6}+\frac{kr^5}{120}-\cdots \\ &=\sum_k c_k up\left(\frac{r-b_k}{a}\right)=\sum_k AString\left(r,a_k,b_k,c_k\right). \end{aligned} \quad (7.8)$$

Scale factor $a\left(t\right)$ [13] [14] [15] [16] being an analytic power function [28] is also representable via Atomics:

$$a\left(t\right)=a_0 t^{\frac{2}{3(w+1)}};\ a\left(t\right)\sim t^{2/3},\ w=0;\ a\left(t\right)\sim t^{1/2},\ w=1/3, \quad (7.9)$$

$$a\left(t\right)=\sum_k up\left(t,a_k,b_k,c_k\right)=\sum_l AString\left(t,a_l,b_l,c_l\right). \quad (7.10)$$

Schwarzschild solution (**Figure 7**) for radial bodies and black holes has spacetime metric [13] [14] [15] [16] [20]

$$\mathrm{d}s^2=-A\left(r\right)c^2\mathrm{d}t^2+B\left(r\right)\mathrm{d}r^2+r^2\mathrm{d}\Omega;$$

$$A\left(r\right)=\left(1-\frac{r_s}{r}\right);\ B\left(r\right)=\left(1-\frac{r_s}{r}\right)^{-1}. \quad (7.11)$$

Analytic (outside of singularity) function $A\left(r\right)$ and it's reciprocal $B\left(r\right)$ (also analytic as per Theorem 3) representable via converging Taylor's series is also representable via Atomics (5.4):

$$A\left(r\right)=\sum_k c_k up\left(\frac{r-b_k}{a}\right)=\sum_k AString\left(r,a_k,b_k,c_k\right),\ r\neq 0. \quad (7.12)$$

In summary, the atomization of known GR solutions confirms the main idea that analytic spacetime fields are representable via the superposition of finite AStrings and Atomic Functions.

8. Atomic Spacetime Properties and Interpretation

Representing spacetime field within the Atomic Spacetime model (§6, 7) via the superposition of overlapping "solitonic atoms" described by Atomic AString Functions leads to the following interpretations of spacetime first proposed in [2] [3] [4] and elaborated in [5] [23].

8.1. No Point in Space

In the Atomic Spacetime model, there is no "point in space", and space is rather a superposition of *finite atoms* resembling quanta (**Figures 2-7**) with some width a and "intensity" c described for hypothetical 1D case by Atomic Function (4.6) centered at points b and d of space:

$$up(x,a,b,c,d=0)=d+c*up((x-b)/a)\cdot\int_{-a}^{a}cup(x/a)\mathrm{d}x=ca. \quad (8.1)$$

The energy (integral) of this 1D atom/quantum is *ca*, or ca^3 in 3D case (4.7).

8.2. Quanta Interacting in Overlapping Zones

In the Atomic Spacetime model, neighboring spacetime quanta $cAString((x-b)/a)$ (3.6), (4.9), (7.1) *overlap* (**Figure 1**, **Figure 3** and **Figure 4**) to produce smooth expansion, for example, in 1D case:

$$\tilde{x}(x)=\sum_k c_k AString((x-b_k)/a_k)=\sum_k AString(x,a_k,b_k,c_k) \quad (8.2)$$

Here, metric and density $\rho(x)$ of spacetime are described by *overlapping* Atomic Functions

$$\rho(x)=\tilde{x}'(x)=\sum_k up(x,a_k,b_k,c_k);\ \mathrm{d}\tilde{x}=\rho(x)\mathrm{d}x; \quad (8.3)$$
$$\rho(x)=\sum_k up(x,a_k,b_k,c_k).$$

Important to note that, like Lego blocks or kitchen tiles, the individual blocks/tiles are not overlapping, but a field keeping them together does overlap, as shown in **Figure 1**. So, Atomic Quantization is not a simple discretization of space (3.1)—separation of a volume into independent "finite elements" [21] [22]. Here, the collocated finite elements [2] [3] [4] [5] [26] overlap/interact with each other to ensure smooth connections between them.

8.3. Spacetime Is "Emerging" between Quanta

In the Atomic Spacetime model, spacetime is "emerging" in interaction zones between quanta as described in §3 and shown in **Figure 1** with Formulaes (3.1)-(3.7), (4.9), (7.4)

$$x\equiv AString\left(x-\frac{1}{2}\right)+AString\left(x+\frac{1}{2}\right),\ x\in\left[-\frac{1}{2},\frac{1}{2}\right]. \quad (8.4)$$

Spacetime expansion then is a translation of overlapping interacting AString quanta

$$x\equiv\sum_k AString(x-k) \quad (8.5)$$

This may support the physical idea of the "emergence of space" expressed by some Loop Quantum Gravity theorists [18] notably L. Smolin [37].

8.4. Spacetime Atomization Is Not the Simple Spacetime Discretization

Atomic Spacetime Quantization is not a simple discretization of space (3.1) – separation of a volume into independent finite elements [21] [22]. For example, some authors [38] proposing a "quantum of length" use the typical discretization of space and *approximation* of derivatives in GR and Quantum Mechanics equations with finite differences. It introduces unphysical "unsmooth connections" between finite elements regardless of the order of approximating polynomial splines. Here, the neighboring finite elements [2] [3] [4] [5] [23] [26] interact with each other (§3) in overlapping zones to compose a polynomial of any order in superposition (Theorem 1) (**Figure 1** and **Figure 5**).

8.5. Atomic Quantization vs. Harmonic Oscillator Model

The abovementioned discretization of spacetime and fields is related to Harmonic Oscillator (HO) model which, as L. Susskind mentioned [39], is a generic mathematical model for many models in physics. In the HO model, a field is represented by a chain of *rigid* balls oscillating on elastic *springs*. However, due to the rigidity of nodes, the deformation of one elastic spring is not fully passed to the neighboring spring creating the field un-smoothness problem. The Atomic Spacetime model is more advanced as it keeps the notion of "nodes" but enforces a field smoothness by using smooth Atomic Functions (§3, 4). Here, a field is composed not of rigid balls but flexible "solitonic atoms" (**Figure 1** and **Figure 2** and **Figure 5** and **Figure 7**) resembling strings in string theory [17].

8.6. Spacetime Is a Field Composed of Solitonic Atoms

Being solitary solutions of special nonlinear differential equations (4.1), (4.8), Atomic and AString Functions possess mathematical properties of solitons [29] [30] [31] [32] called Atomic Solitons [2] [3] [4] [5]. So, in the Atomic Spacetime model, the spacetime field is a combination of solitonic atoms. Expansion of space is a superposition of $AString(x)$ kinks while spacetime density, metric, and curvature are the superposition of pulse-like "solitonic atoms" $cup((x-b)/a)$ of different intensity (§6, 7). Bringing spacetime into the realm of a solitonic theory offers a solitonic interpretation of spacetime and gravity. In some physical theories, particle-wave solitons (Frenkel-Kontorova model [4] [30], Skyrmions [29], lattice solitons [29] [31] [35] [36], or sine-Gordon solitons [4] [32]) are considered as "fundamental building blocks" of fields. Atomic Solitons expand this idea not only to spacetime but also to other fields [2] [3] [4] [5] [23] supporting the idea like in string theory [17] [35] [36] that nature may be made of soliton wave-particles.

8.7. Spacetime Is a Flexible "Fabric" of Solitonic Atoms

In the Atomic Spacetime model (§6, 7), spacetime is a *flexible* continuum/field which can be modeled by varying the widths and intensities/heights of individual AStrings (**Figures 2-7**). In atomic physics and solitonic dislocation models [2] [4] [29] [30], uneven fields can be created either by "absence" or by "grouping" of some atoms in otherwise regular structures. To uphold GR [13] [14] [15] [16] where matter curves the spacetime, one can envisage the simple model [2] [3] [4] [5] when the presence of "matter" alters the distribution of solitonic atoms (6.4) ultimately creating flexible textures shown in **Figures 5-7**. How it physically happens is a question to physicists. However, based on conventional knowledge, seemingly "empty" space consists of quantum vacuum fields [40] [41], the configuration of which can be mathematically described as a superposition of "solitonic atoms" which can grow, shrink, group, and evolve ultimately creating flexible "fabric" [2] [3] [5] [20] of spacetime (§7). This fabric is made of not "rigid" Lego-like blocks but rather flexible "solitonic atoms" $cup(x/a)$ which can vary in intensity c and width a. In summary, if spacetime is a "fabric", Atomics can be conceptualized as "pieces", or "solitonic atoms" of this fabric.

8.8. Expansion of Spacetime Means Bigger Quanta

In the modern inflation cosmological model [14] [15] [16] [20] [33], spacetime is expanding since Big Bang. In the Atomic Spacetime model, it implies that spacetime is getting bigger not by increasing the total number of quanta (which would imply the existence of an external "source of quanta") but by increasing the size/width a and location b of every "atom" $cup((x-b)/a)$ which is especially visible in atomized Friedmann solution (7.7) - (7.10). It supports the abovementioned idea that spacetime atoms are flexible rather than rigid. The enlargement of every atom may mean that Big Bang energy is pumping pressure into every quantum without having a distinct center of expansion.

8.9. Solitonic Wave-Particle Duality of Spacetime Quanta

In nature and mathematics, solitons possess the property of a wave-particle duality [2] [4] [29] [30] [31] [32] when a soliton behaves like a particle (in interactions) and as a wave (in propagation). Atomic Solitons $AString(x,a,b,c)$ and $up(x,a,b,c)$ also uphold this concept. Being a pulse, $up(x)$ can be associated with a wave while the summation of pulses $cup((x-b)/a)$ composes a continuum resembling particles/atoms. In this sense, spacetime can be conceptualized as a conglomerate of solitonic wave particles on a lattice.

8.10. Spacetime Field Is Composed of AStrings

Mathematically, Atomic Function $up(x)$ appearing in all spacetime atomization equations (§3 - 8) is a simple combination of two opposite AString kinks (4.8), **Figure 4(b)**:

$$up(x) = AString(2x+1) + AString(1-2x). \tag{8.6}$$

It implies that the spacetime as a combination of shifts and stretches of $up(x)$ (§3, 6, 7) is ultimately "made of" AStrings. When two AString kinks are pointing in one direction, the sum of them describes the expansion of space (3.2) - (3.4), (4.9), while kink-antikink pair of opposite AStrings produces "solitonic atoms" $up(x)$ (**Figure 4(b)**) capable to compose enclosed continua. It invites the hypothesis of the existence of a physical process in nature that forms either enclosed atoms or extended structures from the same blocks. The idea is similar to "soliton dislocations" in Frenkel-Kontorova solitonic model [2] [4] [29] [30] when two "opposite" dislocations form stable "atoms" while two collocated dislocations just extend the structure. Because AString derivatives are also expressed via AStrings (4.8), the spacetime shape, deformations, and curvatures are also some complex AString combinations (**Figure 7**).

This Atomic Spacetime mathematical composition idea is quite similar to the widely-used Fourier series for representing fields via harmonics to which, like in string theory [17] [35] [36], one can attribute a meaning of a "fundamental block" of fields (fields are made of vibrating of strings). But here this "fundamental block" is AString which, unlike a harmonic, is a finite function resembling the finiteness of a quantum.

8.11. AString Metriant as a Quantum of Space

The concept of a *metriant* has been proposed in the 1960s-1990s by A. Veinik [34], one of the pioneers of Generalized Thermodynamics (GT), who treated spacetime as a GT phenomenon. Like others, the metric phenomenon of space should have its unique conservable extensor (metrior), related intensial (metrial), and elementary quantum – metriant [34]. AString, as a simple solitonic kink function capable to compose flat and curved spacetime fields in superposition (§3 - 7) is a natural candidate for the metriant model, as was first proposed in [2] [3] [4] and later elaborated in [5] [23].

8.12. AString as a Flexible Quantum of Length

Extending the abovementioned concept of AString metriant, it is interesting to observe that the AString function appears in calculating a length along a curved spacetime geodesic. In a hypothetical 1D model of a curved spacetime $\tilde{x}(x)$, a "density" function $\rho(x)=\tilde{x}'(x)$, for example from Schwarzschild metric (7.11) of black holes $\rho(x)=\left(1-\frac{r_s}{x}\right)^{-1/2}$, is representable via translations of Atomic Functions (7.12):

$$\rho(x)=\tilde{x}'(x)=\sum\nolimits_k up(x,a_k,b_k,c_k). \tag{8.7}$$

Then, a so-called "proper distance" in curved space [13] [14] [15] [16] $L(x)$ can be calculated via path integral

$$\begin{aligned} L(x) &= \int_0^x \mathrm{d}\tilde{x}(x)=\int_0^x \rho(x)\mathrm{d}x=\int_0^x \sum\nolimits_k up(x,a_k,b_k,c_k)\mathrm{d}x \\ &= \sum\nolimits_k AString(x,a_k,b_k,c_k). \end{aligned} \tag{8.8}$$

Because AString is the integral (4.8) of Atomic Function $up(x)$, the length function $L(x)$ can be interpreted as a sum of some AStrings. For uniform space $\rho(x)=\rho$, a length L becomes a sum of the same numbers $L=L_0+\cdots+L_0$ while in curved space, it is not true, and "pieces of length" can be different: $L=L_1+\cdots+L_n$. So, the AString function appears in the calculation of distances and can be interpreted as a flexible "quantum of length". Unlike other quantization theories postulating *one* "rigid" minimal "quantum of length" [38], in Atomic Spacetime theory, where AString quanta/metriants $AString(x,a,b,c)$ are flexible, quanta can "stretch" adjusting to variable spacetime metric. It also correlates with Special Relativity [13] [14] [15] where spacetime can shrink, and consistent spacetime quantization models should account for that.

8.13. Atomic Spacetime Energy

Atomization of space—representing spacetime field (7.1) - (7.5) as a superposition of solitonic atoms—upholds the ideas of "zero-point energy" and quantum vacuum fields theories [40] [41] where spacetime can hold energy which can be calculated/quantized as a sum of energies of individual atoms. Indeed, an integral of 3D Atomic Function (4.7) equal ca^3 where *a* is the "width"/size of a quantum while c is the "intensity" of a spacetime field at a given point. If space is uniform (7.6), the energies of "atoms" will be the same $E = E_0 + \cdots + E_0$ while for deformed space, it is not true, and individual amounts of energy can be different: $E = E_1 + \cdots + E_n$. Like in elasticity theory, the more "stressed" areas would hold more energy.

8.14. Spacetime Is both Discrete and Continuous

In the Atomic Spacetime model, spacetime is assumed as both discrete and continuous; discrete, being composed of finite "solitonic atoms", and continuous, being united with smooth connections between "atoms" blended into the continuous medium. Here, finite smooth Atomic Functions serve as a "bridge" between discrete/finite/quantum world and continuous macro-world where, like in the Lego game, smooth continua can be assembled from finite pieces with preservation of smoothness between them. In this sense, Einsteinian curved space can be conceptualized as continuous as a liquid or elastic medium composed of smooth connections of finite "atoms" $cup\left(\left(x-b\right)/a\right)$ similar to "continuous" oceans made of "finite" water molecules.

8.15. Fractalic Spacetime

Being infinitely divisible functions, Atomic and AString Functions possess well-known fractalic properties relevant to a fractal curve of *infinite length containing a finite area* and *Koch snowflake fractals* [2] [4] [8] [9] [10] [11] [12]. A line can be partitioned into sections with larger AStrings (3.2) - (3.4), (4.9) but then each section can be subdivided into smaller AStrings creating a repeated fractalic pattern. It means quantized/atomized spacetime inherits those fractalic properties too. It also implies that the universe may be fractalic, and "solitonic atoms" and AStrings may be as tiny as Planck scale quanta and as big as cosmic strings [2] [3]. It all depends on the size of a lattice of atomic solitons [2] [3] [4] [5].

8.16. Introducing New Constants into Discrete Spacetime Models

In GR, spacetime is continuous creating some "*stumbling blocks*" noted by Einstein [1] related to dealing "*…exclusively with continuous functions of space*" making it difficult to incorporate energy quantization even with continuous quantum mechanical functions [1] [39]. In the Atomic Spacetime model, continuous spacetime can be "atomized" with solitonic atoms $cup\left(\left(x-b\right)/a\right)$ introducing parameters like lattice size/quantum width *a*, "intensity of quantum"

c and a quantum location $x = b$. If $b = na$ we've got the Einstein-Minkowski model of uniform spacetime with straight geodesics $x = x$, $y = y$, $z = z$ representable via AStrings (7.5) - (7.6) on the lattice of size a. Parameter c identifies the intensity/integral/height of solitonic atoms $cup((x-b)/a)$ ultimately contributing to spacetime energy, for example, in Friedman solution (7.7) of expanding spatially homogeneous universe. c may also be related to an energy of zero-point-field, Higgs field [44] [47], or hypothetical quantum vacuum [40] [41] in uniform space. But for curved spacetimes (**Figure 7**) created, as per GR, by matter distributions, the solitonic atom's locations $x = b$ and intensities c are variable. The denser matter, the higher would be the intensities c and dislocations b of individual atoms $cup((x-b)/a)$. Regarding the value of a major width parameter a, it may be associated with Planck's length $a \sim 1.6 \times 10^{-35}\ \text{m}$, with a "quantum of length" [38], with the size of a "loop" in Loop Quantum Gravity [18] [37], or with a length of a string in string theories [17]. Mathematically, the lesser a, the more "granular" would be the Atomic Spacetime models. But because of the fractalic properties of Atomics (§8.16) and a length in general, for macroscopic fields the actual size of solitonic atoms may be less important than the number and configuration of neighboring quanta. For example, to reproduce a parabolic value at a given point (4.12), (5.1) only 4 neighboring atoms are required (**Figure 5**). So, the configuration of atoms seems more important than their size.

8.17. Spacetime as Atomic Lattice Network

AString function (4.8) possesses an important feature—summation of AStrings pointing in one direction represents a straight line (simulating space expansion) while summation of two oppositely-directed AString kinks composes a pulse $up(x)$ (simulating atom). If we assign topological charge "+" for the right-looking $AString(x)$ kink and "−" for the left-looking $AString(-x)$ antikink, the expansion of space in right-direction can be schematically presented as $Space+ = +++++$. Kink-antikink +− pair composes "atom" $up(x)$, or schematically $OneAtom = +-$; $TwoAtoms = +-,+-$ (**Figure 3** and **Figure 4**). From partition of unity (4.4), we know that two $up(x)$ atoms join up together to compose a uniform constant field in the middle, or "molecule" ($m =' + -'+' + -' = +m-$), where charges + and − compensate each other. Grouping of more atoms and "molecules" would compose an extended quantized field $Matter = +mmmm-$. This leads to the following chain, or network, diagram:

$$Space+ = +++++;\ Atom = +-;\ TwoAtoms = +-,+-;\ AntiAtom = -+;$$

$$Molecule = Atom + AntiAtom = +m-;\ Matter = +mmmmm-. \qquad (8.9)$$

So, space can be conceptualized as a lattice/network of AStrings, and this inspired the idea first proposed in 2018 [3] that AString describes a quantum of space, or "metriant" [34]. Two opposite AStrings compose an "atom" while the joining of "atoms" composes "matter". Importantly, both "space" and "matter" are represented by one solitonic AString function upholding Einstein's idea of a deep connection between space and matter.

8.18. Spacetime as Atomic Neural Network

Representing space as a network of solitonic atoms invites another future research direction based on the analogy with Neural Networks (NN) in Artificial Intelligence and Machine Learning models [43]. A human brain is a neural network of connected nodes passing information from one node to another via neuro-electric signals modeled by the *Activation Function* [43] typically represented by a kink-like Sigmoid function. Interestingly, replacing Sigmoid with a similarly shaped AString function leads to Atomic Neural Networks (ANN) and Atomic Machine Learning theories (Eremenko, [43], 2018). But Atomic Spacetime models are also a combination of AString (§7, 8). This leads to the line of research that spacetime can be conceptualized as an Atomic Neural Network, and the opposite – that Neural Networks resemble the smaller Atomic Spacetime model. Both models are based on separated "atoms/nodes" interacting—and passing "information"—via "interaction zones" (§3). While the human brain may have around 10^{11} neurons the space network is much larger containing by some estimations 10^{98} Planck-scale "atoms" [18] [19].

The conceptual similarity of Atomic Spacetime as an Atomic Neural Network implies another core property of Neural Networks—the ability to "remember" and "learn" like a brain. It leads to the hypothesis that spacetime lattice can "host/remember" the location of matter particles by adjusting the locations, sizes, and intensities of many solitonic atoms $cup\left(\left(x-b\right)/a\right)$ in the similar way as a human brain can store the memories via "intensities" of trillions of neurons. For example, a spatial profile of a dolphin, as a network of atoms "occupying" a certain location in space, can be "remembered" in a human Neural Network by associating *similar weights* to neuron nodes. Having the same AString activation/basis function for both spacetime and ANN models enforces this similarity. AStrings and Atomic Functions on a lattice, via Atomic Quantization theorems (§5, 6), can represent ("remember") *any* analytical function with any degree of precision, and this seems relevant to both spacetime and Neural Networks models.

Also, with Machine Learning algorithms [43], Neural Networks can "learn" and "predict". Concerning spacetime, it may mean the ability of a network of "spacetime atoms" $cup\left(\left(x-b\right)/a\right)$ to "optimally adjust" ("learn") the positions to accommodate physical particles and fields. With the law of energy conservation, nature tends to create "optimal forms", and the similar particles/fields should have similar "weights" and "configurations" of solitonic atoms. So, perhaps spacetime/nature can "learn" the optimal configuration, and "apply/replicate" this "learning" to other similar particles, with only a handful of elementary particles known. It also resonates with Einsteinian GR insights that "matter" defines "spacetime", so similar "matter atoms" should create similar spacetime footprints.

Let's note that there is nothing mysterious about the analogy of spacetime lattice with neural networks of AI, and it does not mean that nature needs an artificial "brain" to function. Actually, *any* network, from tree roots to atomic structures to human brains can "remember" and "learn". It only needs abundance and a network of connected atoms as well as the *governing network principle* (usually energy conservation or least-action principles in physics) to create "optimal" fields and "optimal" networks. So, spacetime lattice, like any network, can "host", "remember", and "learn" the positions and intensities of everything in our universe which is an intriguing subject for further research.

8.19. The Shapes of Nature

Atomic Quantization theorems (§5, 6) imply that spacetime configurations/metrics (§7) can be represented as superpositions of "solitonic atoms" (4.11) made of AStrings (4.8). It means the shapes of fields affected by the fundamental force of gravity – orbits, and shapes of planets and galaxies, black hole metric, shapes of living organisms including humans, movements of rockets and projectiles, profiles of rivers and mountains, and configurations of material fields shaped by gravity, may also be composed of solitonic atoms based on AString and Atomic Functions. So, like in the Lego game, the complex shapes of nature may be the compositions of "elementary blocks" interacting to provide smoothness in between. It also means that using computer algorithms it is possible to decompose complex shapes into smaller "atomic pieces" which may lead to advanced technologies of the face and image recognition based Atomic Machine Learning algorithms [43].

9. On the Physical Interpretation of AStrings and Atomic Functions

Described Atomic Spacetime theory invites a question of what physical objects/principles AString and Atomic Functions may express. While this question could be better addressed by physicists, let's suggest some interpretations.

Atomic Quantum Field excitation. In Quantum Field Theory (QFT) [35] [46] [47] all physical particles are the excitations of corresponding quantum fields. Atomic Function $cup(x/a)$ may express "an elementary excitation" capable to form both uniform and nonuniform fields in composition. This excitation creates a small deformation of spacetime which S. Hawking often called "intricate distortions of spacetime" [16], in a similar way as some pressure excitations create ripples on the water's surface. So, atomized curved spacetime may be a reflection of the uneven distribution of an underlying pressure field related in GR to matter distribution [13] [14] [15].

Localized Atomic distribution. Atomic and AString Functions as finite functions of x, y, z may express an elementary form of *field distribution* in spacetime (**Figures 2-5**).

An atomic blob of energy. Recalling that integral of 3D Atomic Function (4.7) equal ca^3, one can treat a field representable by AFs as a *distribution of energy* composed of solitonic atoms shaped to produce smooth connections between them. To uphold QFT [40] [41], the higher the field intensity, the wider may be the atoms. This interpretation may also explain why an enclosed area of space may have quantizable energy ca^3 on a discrete lattice (as A. Einstein wanted [1], §1) and why different particles differ in sizes and energy levels.

Atomic solitons. This interpretation is related to the solitonic meaning of Atomics [2] [3] [4] [5] where solitonic atom $up(x)$ is a kink-antikink pair of two AStrings (4.8). It means that spacetime and fields may be solitonic in nature [2] [4] making "atomic solitons" the candidates for fundamental "building blocks" of nature, as hypothesized in [2] [3] [4] [5].

Solitonic atoms of quantum vacuum or Higgs field. Einsteinian GR states that matter defines the spacetime geometry without explaining via which physical carriers this occurs, for example, in a vacuum. In a modern interpretation, the vacuum is not "empty" but a physical field of virtual quantum particles popping in and out of existence [20] [40] [41]. By another interpretation, space is permeated with Higgs field which gives "matter a property of mass" [44] [47]. Finalizing the theory on the physical composition of space would clarify the role of Atomic Solitons $cup(x/a)$ which may describe the "solitonic atoms" which are those physical fields composed of.

Atomic dislocation. This interpretation follows from the solitonic theory of Frenkel-Kontorova (FK) dislocations [2] [5] [29] [30] where an absence of a "node" in some regular lattice creates a "dislocation" capable to grow, shrink, unite as well as create stable "atoms" with opposite "anti-dislocations". Interestingly, FK-dislocations [4] [29] [30] are described by solitonic kink function $\arctan(\exp(x))$, and replacing it with similar-looking $AString(x)$ (**Figure 4**) leads to "AString dislocation" [2] [4] which combined with anti-dislocation creates "solitonic atom" (4.8) $up(x) = AString(2x+1) + AString(1-2x)$ (**Figure 4(b)**). Also, in the Atomic Spacetime model (§6, 7), the atoms $cup((x-b)/a)$ are also "stretching" and "shifting" (dislocating) producing uneven spacetime. So, spacetime and matter distributions may be "dislocations" of some fields.

Particles as atomic distortions of spacetime. In this popular interpretation [16] [46] [47] particles are assumed to be some "…intricate distortions of spacetime" [16] which makes sense recalling that AStrings and Atomic Functions can be derived from GR (§6). The higher the spacetime density, the bigger—and more energetic—atomic solitons, and for every particle energy level (eq electron, boson) it may be possible to find associated "spacetime atom" with the same energy ca^3.

AString metriant. As per §8.11, the AString function offers an interpretation for a quantum (metriant) of a space/metric phenomenon from some Generalized Thermodynamics theories [34] where every field, including spacetime, should have unique conservable extensor and quantum. The presence of metriants gives a matter the "property of size" and "order of location" [34], and AString may be treated (§8.11, §2) as a "metric function" and "quantum of size".

Atomic statistical distribution. AF $up(x)$ (4.1), (4.2) has a well-known statistical meaning [6]-[12] of weighted distribution of $sync(x)=\sin(x)/x$ functions, also related to Prouhet-Thue-Morse sequence and principles of "fair game" [2] [7]-[12]. So, maybe "solitonic atoms" and spacetime are just special statistical distributions, like in the Statistical Theory of Gravitation [49] or in Quantum Mechanics where wave-particles are presumed as probabilistic distributions [18] [19] [39].

Atomic Quantum Fluctuation. Quantum Mechanics (QM) [39] and Quantum Gravity [18] [19] [37] seemingly interpret spacetime as "quantum in nature", so AF $up(x)$ may express an elementary "quantum fluctuation" capable to compose different fields, like in QFT [40] [41]. However, Einstein was knowingly uneasy [1] [16] about probabilistic QM interpretations: "*I attach only a transitory importance to this interpretation. I still believe in the possibility of a model of reality - that is to say, of a theory which represents things themselves and not merely the probability of their occurrence*". Interestingly, in this Atomic Spacetime theory, it is also possible to "quantize" the spacetime and other fields, but without QM probabilistic interpretations assuming that "solitonic atoms" $cup(x/a)$ on discrete lattice may also have discrete energies (4.7) while possessing solitonic wave-particle duality [2] [4] (§8, 9) important in QM.

Loop Quantum Gravity (LQG) [18] [37] assumes spacetime as a network of finite "loops" similar to the "Atomic Network" model (§8.16, 8.17). Whether it is possible to interpret those "loops" as "atomic solitons" is still an open question. However, if LQG can quantize/atomize Einstein's GR equations [5] [23] like it is proposed in this work (§6, 7), the theories may converge.

Atomic String hypothesis. The hypothesis that solitonic atom $cup(x/a)$, or composing it AString kinks, may represent a new kind of string from string theory [17] [35] [36] was expressed in [2] [3] [4] [5]. Rather than consider a string as a "linearly vibrating" filament, one can hypothesize that a string may vibrate with nonlinear spatial pulse shape $cup(x/a)$ (**Figure 2** and **Figure 4**) made of two AStrings (4.8). In this case, with Atomic Quantization theorems (§5, 6), it is possible to quantize/atomize practically any smooth field as a superposition of "solitonic atoms" or AStrings. So, electromagnetic, Higgs, strong, quantum vacuum, spacetime field, matter composition, gravitational warping, and atomic lattices may all have a common atomic "building block" gluing all fields together. Because these fields have a common mathematical "ancestor", they are deeply related to each other [2] [3] [4] [5] [23]. This interpretation may contribute to a string theory [17] which also assumes a common string "ancestor" for all fields in nature.

Atomic Graviton model. The Atomic Spacetime model may offer an atomic model of a graviton – a hypothetical particle associated with gravity, spacetime ripples, and gravitational waves [48]. The clue comes from the presumption that graviton is unitless, so can be associated with also unitless deformation of spacetime. If spacetime lengths measured in meters are described with AStrings (7.4), the deformations, as spacetime shape derivatives, would be representable via Atomic Functions (AString derivatives) having a finite pulse-like shape (**Figure 2** and **Figure 3**) which can be associated with a hypothetical graviton as a pulse of a graviton field. So, Atomic Graviton may be associated with 3D "solitonic atom" (4.7) $cup\left(x/a, y/a, z/a\right)$ (**Figure 2** and **Figure 6**) with energy ca^3 confined in some finite region of space of size *a* and capable of combining with other gravitons in interaction zones (§3, 6, 7) to produce a continuous graviton field. Unlike the AString metriant model (7.4) representing the expansion of space from AStrings (§3, 6, 7), this Atomic Graviton model describes an elementary localized pulse-like "deformation/ripple/atom" of spacetime. This graviton model has the following advantages. 1) Relevance to GR – Atomic Function can be deduced from GR (§6.3) and compose gravitational field in superpositions. Differently from GR linking "matter" with "spacetime", presumably massless graviton [48] model focuses on "spacetime" only. 2) Consistency with Quantum Field Theory [35] [46] [47] where "particles" are the "excitations" of fields (graviton field in this case), because "atom" $cup\left(r/a\right)$ describes an elementary ripple of space (**Figure 4**, **Figure 5** and **Figure 7**). 3) Uniqueness and specificity – relevance to only spacetime without involving other physical quantities (like mass, spin, electric charge, etc.) and describing *unitless* deformations of curved spacetime (8.3): $\tilde{x}'\left(x\right) = \sum_k up\left(x, a_k, b_k, c_k\right)$. 4) Potentially measurable graviton energy ca^3 is associated with "size" *a* and "intensity" *c* of the graviton field atoms. 5) Being able to compose a continuous graviton field from elementary "atoms", via Atomization theorems (§6, 7). 6) Relevance to "gravitational waves" – which can be associated with traveling "distortions of spacetime" [16] [20]. 7) Combining with other physical fields (electromagnetic, weak, gravitational, Higgs) which via Atomization theorems (§6, 7) which can be also represented via Atomic Functions linking all the fields with the common "mathematical atom". Validated by physicists, this Atomic Graviton model may contribute to the gravitons theory [48].

In summary, Atomic Spacetime theory correlates and may contribute to other physical theories with the apparatus of Atomic and AString Functions [2]-[12] developing since the 1970s.

10. Discussion on Atomic Spacetime Theory and "Atomic Theory" of A. Einstein

In conclusion, let's recall again A. Einstein's 1933 paper [1] cited in §1 where he envisaged a "*perfectly thinkable*" "*atomic theory*" with "*simplest concepts and links between them*" resolving "*stumbling blocks*" of theories operating *"…exclusively with continuous functions of space"* and explaining how a finite region of space can have quantized energy levels. The described Atomic Spacetime theory may be that theory sought by A. Einstein. Indeed, the theory is "atomic" assuming atomizing/quantizing spacetime field with finite Atomic String Functions. Atomic Functions, as "solitonic atoms" made of two AStrings

(4.8), are "*simple concepts*". "*Links between them*", in the form of overlapping superpositions (4.9), (4.12), (7.1), describe both flat and curved spacetime and other physical fields. The theory also overcomes "*stumbling blocks*" of theories dealing "*…exclusively with continuous functions of space"* [1]*;* here, atomized spacetime is both discrete and continuous (§7, 8.14). Also, Einstein's *"…region of three-dimensional space at whose boundary electrical density vanishes everywhere*" [1] naturally leads to a finite Atomic Function (**Figure 2**) with discrete energy (integral (4.7)) levels ca^3. Atomic Spacetime theory supports another Einstein's quote: "*I have deep faith that the principle of the universe will be beautiful and simple"*. This "principle" may be realized with the model of spacetime and field composition from overlapping Atomic Solitons described by simple AString metriant functions.

11. Conclusion and Future Research Directions on an Atomic Unified Theory and Atomic String Theory

Atomization Theorems (§5, 6) provide a theoretical foundation for applications of AStrings and Atomic Functions in many physical theories being researched further [2] [3] [4] [5] [23] [42]. The common feature of these theories is the unified description of fields as superpositions of flexible overlapping solitonic atoms $cup\left((x-b)/a\right)$ made of two AStrings $up(x) = AString(2x+1) - AString(2x-1)$ $= AString'(x)$. It means that mathematically the field distributions are just complex lattices/networks of flexible AStrings. Moreover, due to properties (4.1), and (4.8) of Atomics to have derivatives expressed via the functions themselves, not only the fields but their derivatives expressing fields deformations and curvatures, would also be some AStrings combinations. This invites the hypothesis raised in [2] [3] [4] [5] whether AString mathematically describes some fundamental solitonic object from which everything is made, and having a common "ancestor", different fields may be deeply related to each other [5] [23] [42].

The obvious candidate for a "particle of everything" is a new kind of string [2] [3] [4] from string theory [17] [35] [36]. Rather than consider a string as a "vibrating" filament, one can hypothesize that a string with length a may vibrate with compact nonlinear $cup(x/a)$ "solitonic atom" spatial shape composed of two AStrings (**Figure 4**) with intensity/amplitude c and energy/integral ca (ca^3 in 3D). Two neighboring strings overlap and produce either constant or variable smooth field while continuous space x "emerges" between strings $x \equiv aAString\left(x-\frac{a}{2}\right) + aAString\left(x+\frac{a}{2}\right),\ x \in \left[-\frac{a}{2}, \frac{a}{2}\right]$ and extends by adding more strings $x \equiv \sum_k aAString\left((x-ka)/a\right)$ (§3, 7). Then, electron, quark, or Higgs "particles" may be the spatial excitation of strings with their own intensity, energy, and size (§9). Because these fields have a common "string ancestor", they become deeply related to each other, with the preservation of energy during exchanges.

Interestingly, this Atomic String model assumes the existence of a "string with length" *a* embedding the concepts of a "length", "dimensionality", and "lattice". It looks like every particle (electron, boson) apart from unique physical characteristics (eq electric charge) includes some "quanta of space", or metriants (§8.11), as A. Veinik [34], call them in the 1980s. Like the Higgs field giving matter "a property of mass" [44] [47], metriants "give the matter the property of size" and "order of location" [34]. AString function not only offers a mathematical model for the metriant (§3, 7, 8.11) but also allows the building of "solitonic atoms" $up(x) = AString(2x+1) - AString(2x-1)$ capable to compose different fields in superposition. This concept of AString metriants as "common blocks" of fields has synonyms like "quantum of length", "elementary distortion of spacetime", and "ripple/excitation" of spacetime used by other authors [16] [20] [38] [46] [47].

Hopefully, verified by physicists and string theorists, these string and metriant models-under-research [2] [3] [4] [5] [23] [42] [51] may contribute to spacetime physics, quantum field theories, and unified theories of nature.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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