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Use of Stochastic Asset-liability Model to Find Unique Price of Asset

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Research Article

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Abstract

'Asset–liability control' is meant for managing the risk arising from changes in the relationship between assets and liabilities, due to volatile interest rate in critical situations like economic recession, inflation, etc. A stochastic asset-liability model (ALM), if adopted, and the market, though incomplete, is in equilibrium, a unique price can be obtained that is consistent both with the ALM and with the market. This paper presents a stochastic asset-liability model. A unique price, consistent with the ALM and the market, is obtained given a precise condition. The present market value of asset is also obtained with the given unique price. This classical problem considers an amount of money which an institution has in the bank that grows deterministically and a risky asset such as a stock whose value follows a geometric Brownian motion with a drift.

Keywords: Asset-Liability Control, HJB Equation, Present market value, Unique Price, Financial Institution;

1 Introduction

Asset–liability control, or ALC, is a means of managing the risk that can arise from changes in the relationship between assets and liabilities. ALC was originally pioneered by financial institutions in the 1970s as interest rates became increasingly volatile. This volatility had dangerous implications for financial institutions. Some, for example, had sold long-term guaranteed interest contracts—some guaranteed rates of around 16% for periods up to 10 years. However, when short-term interest rates subsequently fell, these institutions, such as the equitable in the US, were crippled. Prior to the 1970s, interest rates in developed countries varied little and thus losses accruing from asset–liability mismatches tended to be minimal.

Following the experience of equitable and other institutions, financial firms increasingly focused on ALC whereby they sought to manage balance sheets in order to maintain a mix of loans and deposits consistent with the firm's goals for long-term growth and risk management (Buehler &

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Anthon, 2008). They set up ALC committees to oversee the ALC process. Today, ALC has been adopted by many corporations (Dermine & Youssef, 2007; Blommestein & Kalkan, 2008), as well as financial institutions. ALC now seeks to ascertain and control three types of financial risk: Interest rate risk, credit risk (the probability of default), and liquidity risk, which refers to the danger that a given security or asset cannot be traded quickly enough in the market to prevent a loss (or make a predetermined profit).

Wang & Wang (2010) characterized the implication of liquidation costs and asset-management fee incentives for the optimal management of leverage. But ALC also now seeks to address other risks, such as foreign exchange risks and operational risks (covering areas such as fraud and legal risks, as well as physical or environmental risks). The theory of risk-sensitive control has received much attention in recent years because it provides a link between stochastic and deterministic approaches to disturbances in control systems (Fleming & Shue, 2002). The techniques that are now applied by ALC practitioners have also developed, reflecting the growth of derivatives and other complex financial instruments (Detemple & Marcel, 2008). A kind of risk-sensitive optimal control problem motivated by a kind of portfolio choice problem in certain financial market has being studied by Wang & Zhen, (2007). Using the classical convex variational technique they obtained the maximum principle for this kind of problem. ALC now includes hedging, for example, whereby airlines will seek to hedge against movements in fuel prices and manufacturers will seek to mitigate the risk of fluctuations in commodity prices. Meanwhile, securitization has allowed firms to directly address asset–liability risk by removing assets or liabilities from their balance sheets.

If a stochastic asset–liability model (ALM) is adopted, and the market, though incomplete, is in equilibrium, and the ALM is consistent with the market, then a unique price can be obtained that is consistent both with the ALM and with the market.

In this paper, we adopted a stochastic asset-liability model and obtained a unique price consistent with the market.

2 Problem Formulation

Consider a capital market with the following properties. Uncertainty is represented by a complete filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathrm{P})$ with the neutral filtration $\mathcal{F}_t = \sigma\{W(s); 0 \leq s \leq t\}$, where $(W(.))$ is a standard n-dimensional Brownian motion defined on this space with values in R^n. All agents receive information over time according to the standard (augmented) filtration $\{\mathcal{F}_t : t \in [0, \infty)\}$of W.

Let $B(t)$ and $S(t)$ be the amount of money an investor has in the bank and in the stock respectively. The price process $B(t)$grows deterministically at exponential rate r, with dynamics (Wang & Zhan, 2007)

$$dB(t) = r(t)B(t)dt, \tag{1}$$

where $r(t)$is the interest rate of the bank at time zero will become e^{rt} Naira at time t. The other asset is stock which follows a geometric Brownian motion process and it is marked to market at a price $S(t)$ satisfying

$$dS(t) = \mu(t)S(t)dt + \sigma(t)S(t)dW(t), \tag{2}$$

where $\mu(t)$ is the instantaneous expected rate of return and $\sigma(t)$ is the instantaneous volatility and are given by

$$\mu(t) = \int SP(S)dS \tag{2a}$$

and

$$\sigma(t) = \int \sqrt{(S-\mu)^2 P(S)dS}\ . \tag{2b}$$

$P(S)$ is the probability distribution of the future market value of asset given as (Wright, 1938)

$$P(S) = \frac{\theta}{\sigma^2} exp\left[\int_{-\infty}^{S} \frac{\mu(x)}{\sigma^2(x)} dx\right], \tag{2c}$$

(where θ is a constant chosen so that $\int_{-\infty}^{+\infty} P(S)dS = 1$.

A Naira invested at time t with $X(t) \triangleq \mu t + \sigma W(t)$ (where $W(t)$ is the standard Brownian motion as defined above), will become $e^{X(t)}$ at time t.

The proportional costs (the transaction cost like commission for buying or for sales of stock) involves withdrawing m Naira from the bank to buy $m\lambda$ $(0 < \lambda < 1)$ worth of stock and the sale of n Naira of worth of stock to get $n\alpha$ $(\alpha \geq 0)$ in cash. The investor receives incentive fees at a constant rate λ, as a fraction of the current liquidation value of the fund (Wang &Wang, 2010). Goetzmann & Ross (2003), argue that a control (management) fee provides the greatest incentives to the fund manager when investors are likely to remain for the long term and when asset volatility is low.

Let $C(t)$ be the institution's net cash flow at time t and $L(t)$ the market value institution's liabilities at time t after cash flow then payable. The dynamics of the control system (as in Rodriguez-Pedraza, 2005) is governed by

$$dS(t) = S(t)\left[\left(\mu(t) + \frac{\sigma^2(t)}{2}\right)dt + \sigma(t)dW(t)\right] + dC(t) - dL(t), \tag{3}$$

$$dB(t) = r(t)B(t)dt - (1+\alpha)dC(t) + (1-\lambda)dL(t) \tag{4}$$

with initial conditions $S(0) = S_0$ and $B(0) = B_0$. $C(t)$ and $L(t)$ are control processes in the context of stochastic control. To obtain the optimal strategy, we look for a pair of processes of bounded variation $C(t), L(t) \geq 0$ such that the controls are feasible if

(i) $E_x\{C(t)\}, E_x\{L(t)\} < \infty$ for all t.
(ii) The system of equations (3) and (4) has a unique nonnegative solution for $B(t)$ and $S(t)$ for all $t \geq 0$.

Rodriguez- Pedraza, (2005) defined a wealth process $h(t)$ as:

$$h(t) = (1-\lambda)S(t) + B(t), \tag{5}$$

such that in differential terms we have

$$dh(t) = (1-\lambda)S(t)\left[\left(\mu(t) + \frac{\sigma^2(t)}{2}\right)dt + \sigma(t)dW(t)\right] + r(t)B(t)dt + (\lambda+\alpha)dC(t). \tag{6}$$

Or in integral form;

$$h(t) = h_0 + \int_0^t \left[r(s)B(s) + (1-\lambda)\left(\mu(s) + \frac{\sigma^2(s)}{2}\right)\right] ds$$
$$+(1-\lambda)\sigma(t)\int_0^t dW(s) + (\lambda+\alpha)C(t). \quad (7)$$

The processes $C(.), L(.)$ and hence $h(.)$ are right continuous with left limit at each $t \geq 0$. Assume $\lambda \to 1$ and $\alpha \to 0$, that is a situation where m money is withdrawn from the bank and the fraction of the current value of the stock liquidated with no stock sold (a situation where transaction cost is made up of the money in the bank), we have (4) become;

$$dh(t) = r(t)B(t)dt + dC(t) \quad (8)$$

or in integral form

$$h(t) = h_0 + \int_0^t r(s)B(s)dt + C(t). \quad (9)$$

This means that the net wealth is made up of institution's net cash flow plus the money in the bank with no market price of liability. In practice this is not always real as there are assets and liabilities to be managed.

Let $$r(t) = \mu_0(h(t))$$
(10)

as in Fleming & Sheu, (2002), then (8) becomes;

$$dh(t) = b\big(h(t)\big)dt + dC(t), \quad (11)$$

where $C(t)$ is now assume a standard m-dim Brownian motion with

$$b\big(h(t)\big) = r(t)B(t), \quad (12)$$

such that B is a stable matrix. That is

$$\sum B_{ij}\, u_i u_j \leq \; -c_0|u|^2, \quad (13)$$

for all $u = (u_1, \ldots, u_m) \in R^m$ for some $c_0 > 0$. $|.|$ is the Euclidean norm. For the choices of U, the $U = R^N$ corresponds to no investment control constraints. The $U = \{(u_1, \ldots, u_N); u_i \geq 0, i = 1, \ldots, N\}$ corresponds to no shortselling constraint.

By (11) we have assumed that there are m economic factors $h_1(t), \ldots, h_m$ which determine the performance of the market (Bielecki & Pliska, 1999). Fleming & Sheu (2002) described the dynamics for $r(t), S_i(t), i = 1, \ldots, N$ by

$$\frac{dS_i(t)}{S_i(t)} = \mu_i(h(t))dt + \sigma_B^{(i)} dC(t) + \sigma_I^{(i)} d\bar{C}(t), \quad (14)$$

$\bar{C}(t)$ is a $\bar{m}$-dim Brownian motion and is independent of $C(.)$; $\sigma_B^{(i)}, \sigma_I^{(i)}$ are m-dim, $\bar{m}$-dim constant vectors and

$$\mu_i(h) = A^{(i)}h + a_i, i = 0,1,2, \dots, N. \tag{15}$$

$A^{(i)}$ is an m-dim vector and $a_i \in R$ is a constant.

Remark:

(i) If $\lambda \to 0$ and $\alpha \to 1$,then there has been no money withdrawn from the bank and no new stock acquired.

(ii) If $\lambda \to 0$ and $\alpha \to 0$, then there is no transaction costs- no money has been withdrawn and no new stock acquired. This is the analysis in Merton (1969, 1971) with no transaction costs and no consumption.

2.1 The control problem

Given the pair $(C, L) \in \mathfrak{V}(x, y)$, $\mathfrak{V}(x, y)$ a class of all such pair,x is the initiated position of the riskless asset (CASH BOUND) with differential form given by (1) whose unique solution for the value $B_0 = 1$ is

$$B_t = exp\left(\int_0^t r(s)ds\right), \tag{16}$$

and y the initiated position of the share price $S(t)$ of the risky asset (STOCK) which follows the differential form (2) and has a unique solution with initial condition $S(0)$ as

$$S(t) = S(0)exp\left(\sigma(t)W(t) - \frac{\sigma^2(t)}{2} + \int_0^t \mu(s)ds\right). \tag{17}$$

Note: under the risk-neutral measure $r(t) = \mu(t)$.

The present value of the institution's assets-liabilities to be controlled is given by

$$V(x, y) = \sup_{(C,L)\in\mathfrak{V}} \mathfrak{J}(x, y; C, L), \tag{18}$$

and the pair $(C, L) \in \mathfrak{V}(x, y)$ the optimal for the problem (18), where

$$\mathfrak{J}(x,; C) = E\int_t^\infty \exp\left(-\int_0^t B(s)ds\right) U_1(C)ds \tag{19}$$

and

$$\mathfrak{J}(y,; L) = E\int_t^\infty \exp\left(-\int_0^t B(s)ds\right) U_2(L)ds, \tag{20}$$

with initial endowment $x \geq 0, y \geq 0$, over the class $\mathfrak{V}(x, y)$ of assets / liabilities pair $(C, L) \in \mathfrak{V}(x, y)$ for which

$$E\int_t^\infty exp\left(-\int_0^t B(s)ds\right) U(C, L)\, dt < 0. \tag{21}$$

In what follows, we define a new state variable

$$h(t) = \zeta(t)Z(t), \tag{22}$$

where

$$Z(t) \triangleq exp\left\{-\sum_{i=1}^{d}\int_0^t \theta_i(s)dW_i(s) - \frac{1}{2}\int_0^t \|\theta(s)\|^2 ds\right\}. \tag{23}$$

Then $\{Z(t), \mathcal{F}_t; 0 \le t \le T\}$ is a martingale with probability measure (Karatzas et al., 1987)

$$P(A) \triangleq E[Z(T)I_A], A\epsilon\mathcal{F}_t.$$

It follows that

$$dZ(t) = -Z(t)\theta^T(t)dW(t).$$

Lemma 1.

For each $\theta\epsilon R$ the $\{Z(t)\}_{t\geq 0}$ is a positive martingale relative to the Brownian filtration.

Proof .Using the elementary properties of conditional expectation and the fact that the random variable $W(t+s) - W(t)$ is independent of the σ-algebra $\mathcal{F}_s$, we have

$$E(Z(t+s)|\mathcal{F}_s) = Z(s)$$

as required.

A fundamental related process useful in the sequel is

$$\zeta(t) \triangleq Z(t)\int_0^t (\mu(u) - r(u))du, \quad 0 \le t \le T, \tag{24}$$

and it can be verified that (24) satisfies the stochastic linear equation

$$d\zeta(t) = (\mu(t) - r(t))\zeta dt - \zeta\theta^T(t)dW(t) \tag{25}$$

with

$$\theta(t) \triangleq \frac{\mu(t) - r(t)}{\sigma(t)}, \ 0 \le t \le T. \tag{26}$$

Theorem For the pair $(C, L) \in \mathfrak{V}(x, y)$ we have

$$V(x, y; C(t), L(t)) = e^{-\mu t}\varphi(t; C(t), L(t)). \tag{27}$$

From (19) and (20) we have

$$\begin{aligned} V(x, y; C, L) = E&\begin{bmatrix} \int_t^\infty \{exp\left(-\int_0^t \mu(u)du\right)U_1(C)ds \\ -\left(exp\left(-\int_0^t \mu(u)du\right)U_2(L)ds\right)\} \end{bmatrix} \\ &= E\left[\int_t^\infty exp\left(-\int_0^t \mu(u)du\right)U_1(I_1(\zeta Z))ds\right] \\ &\quad -E\left[\int_t^\infty exp\left(-\int_0^t \mu(u)du\right)U_2(I_2(\zeta Z))ds\right] \end{aligned}$$

$$= E\left[\int_t^{\infty} e^{-\mu t}\left(\left(U_1(I_1(\zeta Z))\right) - \left(U_2(I_2(\zeta Z))\right)\right) ds\right]$$
$$= e^{-\mu t}\varphi(t; C(t), L(t)).$$

Where

$$\varphi(t; C(t), L(t)) = E\left[\int_t^{\infty}\left(\left(U_1(I_1(\zeta Z))\right) - \left(U_2(I_2(\zeta Z))\right)\right) ds\right],$$

and

$$C = I_1(\zeta Z),\ L = I_2(\zeta Z).$$

The Hamilton-Jacobi-Bellman stochastic dynamic equation corresponding to (27) is given by

$$sup_{C,L\geq 0}\left\{V_t + \mu(t)S(t)V_S + \frac{1}{2}\sigma^2(t)S^2(t)V_{SS} + \vartheta V_k + (V(C) - \vartheta V(L))e^{-\mu t}\right\}. \quad (28)$$

If

$$V = \varphi(C(t), L(t), t)e^{-\mu t} \quad (29)$$

and

$$V_k = ae^{-\mu t}. \quad (30)$$

Then by Onah & Ugbebor (1999)

$$\frac{1}{2}\sigma^2 \sum_{i=1}^{n} S_i^2(t)\,\varphi_{S_iS_i} + \alpha\sum_{i=1}^{n} S_i(t)\varphi_{S_i} - \mu\varphi = Q. \quad (31)$$

3 The Present Value and the Unique Price

Let

$$\mu(C, L) = inf\ \{t \geq 0, S(t) \in \Omega, (C, L) \in \mathfrak{B}\}$$

be the markov time with finite expectation then,

$$V(S(t)) = E^x \int_t^{\infty} \varphi\big(X(t)\big)e^{-\mu s}ds,\ \text{a.s.} \quad (32)$$

Since $S(t)$ is the solution of (2). The differentiation of the function $\varphi(X(t))e^{-\mu s}$ in the Ito's sense gives;

$$dV\big(S(t)\big)e^{-\mu s} = AV\big(S(t)\big)dt + V_s\big(S(t)\big)e^{-\mu t}dW(t)$$

where

$$A = \frac{1}{2}\sigma^2 S^2(t)V_{SS} + \alpha S(t)V_S - \mu V. \quad (33)$$

We therefore have;

$$V\big(S(t)\big)e^{-\mu s} - V\big(S(t)\big) = -E^x\int_t^{\mu}\big[\varphi\big(S(x)\big)e^{-\mu s}ds + S(x)V_S\big(S(x)\big)e^{-\mu x}dW(x)\big],\text{a.s.}$$

For $n = 1$, the time homogeneous form of (31) is equivalent to (33) such that

$$\frac{1}{2}\sigma^2 S^2(t)V_{SS} + \alpha S(t)V_S - \mu V = V_t, \quad (34)$$

where $V_t = q(C(S(t)), L(S(t))$. For a relatively short period where no transaction has been made $L(S(t)) \to 0$ as $t \to 0$, so that

$$V_t = q(C(S(t)). \quad (35)$$

The rate of change of the worth of the institution now depends on the price S, therefore (35) becomes

$$V' = S. \quad (36)$$

3.1 Present value and unique price for $\alpha \neq \mu$.

Since the differentiation of V on the left hand side of (35) is with respect to S, we then have (33) become

$$\frac{1}{2}\sigma^2 S^2 V'' + \alpha S V' - \mu V = -S. \quad (37)$$

We then obtain the present market value for $\alpha \neq \mu$ as (see appendix)

$$V(S) = kS^{\lambda_1} + \frac{S}{\mu - \alpha} \quad (38)$$

and the unique price $\hat{S}$ as

$$\hat{S} = \frac{\lambda_1(\mu - \alpha)}{\lambda_1 - 1}\bar{S}. \quad (39)$$

3.1.1 Present value and unique price for $\alpha = \mu$.

Under risk-neutral measure, $\alpha = \mu$ so that (34) can be written as

$$\frac{1}{2}\sigma^2 S^2(t)V_{SS} + \mu S(t)V_S - \mu V = V_t, \quad (40)$$

with

$$V(0,0) = 0 \quad (41)$$

and

$$V_S(S,t) = 0\ . \quad (42)$$

We therefore have the present value as

$$V = V_0 exp\left\{\frac{\hat{S}\lambda\sigma^2 - (2\hat{S}^2 + \sigma^2)\mu t}{\sigma^2}\right\}, \quad (43)$$

withλ the characteristic root given as

$$\lambda = \pm\alpha^{1/2}. \quad (44)$$

And the unique price (see appendix)

$$\hat{S} = \left(4\alpha^{1/2}\sigma^{-2}t\right)^{1/3} exp\{\mu t\}. \quad (45)$$

4 Conclusion

In this paper, we have considered a stochastic control problem where the present market value and the unique price consistent with the market are obtained. It is remarkable to note that for $\alpha = \mu = \frac{1}{2}$,$\lambda_1$ of A3 is unity and the unique price $\hat{S}$ of (36) is finite. For $0 < \alpha < \mu$, $\mu - \alpha > 0$, if $S^{\lambda_1} \to \infty$ as $S \to \infty$, then $V(S) \to +\infty$, thus the institution asset grows without bound. But for $S^{\lambda_1} \to 0$ as $S \to \infty$ and $\alpha > \mu$, $\alpha - \mu < 0$ then $V(S) \to -\infty$, this is liability. Furthermore (using (40), the market value is an asset if $\widehat{S\lambda}\sigma^2 \gg \left(2\hat{S}^2 + \sigma^2\right)\mu t$ or liability if $\widehat{S\lambda}\sigma^2 \ll \left(2\hat{S}^2 + \sigma^2\right)\mu t$. The asset growth rate is given by $V^{'} = (\mu - \alpha)^{-1}$.

Appendix

The solution of (36) is obtained by the method of variation of parameter as

$$V(S) = kS^{\lambda_1} + \frac{S}{\mu-\alpha}, \quad \text{A1}$$

form which we have

$$\frac{dV(S)}{dS} = k\lambda_1\hat{S}^{\lambda_1} + \frac{\hat{S}}{\mu-\alpha} = 0, \quad \text{A2}$$

where

$$\lambda_1 = -\left(\alpha - 1/2\right) + \left\{\left(\alpha - 1/2\right)^2 + 2\mu\right\}^{1/2} \quad \text{A3}$$

is the positive characteristic root.

Under equilibrium condition the unique price $\hat{S}$ must be equal to expected unit cost $\bar{S}$ of the risky stock. We therefore have

$$V\left(\hat{S}\right) = k\lambda_1\hat{S}^{\lambda_1} + \frac{\hat{S}}{\mu-\alpha} = \bar{S}. \quad \text{A4}$$

Solving for k in A2 and A4 and equating results gives the unique

$$\hat{S} = \frac{\lambda_1(\mu-\alpha)}{\lambda_1-1}\bar{S},$$

as required.

To remove the effect of the discount rates μ in (37) we make the following transformations;

$$V = \varphi e^{-\mu t} \quad \text{A5}$$

and

$$\hat{S} = e^{-\mu t}S. \quad \text{A6}$$

Equation (37) now becomes

$$\frac{1}{2}\sigma^2\hat{S}^2\varphi_{SS} + \varphi_t = 0. \quad \text{A7}$$

The absence of μ is nice since it is difficult to estimate in practice. We assume a solution $\varphi = h\left(\hat{S}\right)w(t)$ by the method of separation of variables and substitute into (37) to get

$$\frac{\sigma^2\hat{S}^2h^{''}}{2h} = \frac{w^{'}}{w} = \alpha, \quad \text{A8}$$

which gives

$$w' = \frac{-2\alpha w}{\sigma^2 \hat{S}^2} \qquad \text{A9}$$

and

$$h'' = \alpha h, \qquad \text{A10}$$

with solutions

$$w = V_0 exp\left\{-\frac{2\alpha t\hat{S}^2}{\sigma^2}\right\} \qquad \text{A11}$$

and

$$f(\hat{S}) = exp\{\lambda\hat{S}\}. \qquad \text{A12}$$

Hence

$$\varphi(\hat{S}, t) = V_0 exp\left\{\frac{-2\alpha t\hat{S}^2 + \sigma^2\lambda\hat{S}}{\sigma^2}\right\}. \qquad \text{A13}$$

To obtain the unique price we have $\varphi_{\hat{S}} = 0$ (using A13) so that

$$\lambda + 4\sigma^{-2}\hat{S}^{-3}\alpha = 0. \qquad \text{A14}$$

Equating (A14) and (A6) and solving for $\hat{S}$ we have as required,

$$\hat{S} = \left(4\alpha^{1/2}\sigma^{-2}t\right)^{1/3} \exp\{\mu t\}.$$

We have used the negative characteristic root $\lambda = -\alpha^{1/2}$ so that $\hat{S}(t)$ will remain admissible economically.
Using (A5) and (A13) we have the present market value for $\alpha = \mu$

$$V = V_0 exp\left\{\frac{\hat{S}\lambda\sigma^2 - (2\hat{S}^2 + \sigma^2)\mu t}{\sigma^2}\right\}.$$

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Definitions, Acronyms, Abbreviations

Let X_t be the p-component state vector of the stochastic model at time t. The model defined the conditional distribution:

$$F_{X_t}\left(x \middle| X_{t-1}\right)$$

An ALC defines the following variables as functions of X_t:

C(t) = the institution's net cash flow at time *t*;

$V_a(t)$= the market value at time t of an investment in asset category $a = 1,..., A$ per unit investment at time $t - 1$;

f_{t+1} = the amount of a risk-free deposit at time $t + 1$ per unit investment at time t.

We denote by *L(t)* the market value of the institution's liabilities at time *t* after the cash flow then payable.