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Optimal Design and Control for Minimizing the Dynamic Response of an Anisotropic Plate with Variable Thickness

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Abstract

The problem of minimizing the dynamic response of an anisotropic rectangular plate of variable thickness with minimum possible expenditure of force is presented for various cases of boundary conditions. The plate has a principal direction of anisotropy rotated at an arbitrary angle relative to the coordinate axes. The orientation angle and thickness parameter have been taken as optimization design parameters. The control problem is formulated as an optimization problem by using a performance index, which comprises a weight sum of the control objective and penalty function of the control force. Explicit solutions for the surface shape, the total elastic energy of the plate and the closed-loop distributed control force are obtained by means of Liapunov-Bellman theory. To assess the present solutions, numerical results are presented to illustrate the effect of various thickness parameters, orientation angles, aspect ratios and boundary conditions on the control process.

Keywords: Optimal control, Design, Minimization of dynamic response, Anisotropicplates, variable thickness plate.

AMS Subject Classifications: 74K20, 74H45, 49R99.

1 Introduction

Although plates with constant thickness have been widely used, the variable thickness plates have also received a lot of attention from designers and researchers. The investigation on plates with variable thickness has significance in actual engineering because such plates can enhance the material potential. Thus, the designer has to use variable thickness plates to suit some design requirements, for example, to reduce the size and weight of the structure and hence to save material, and cost requirement, to improve the distributions of stresses and displacements or to change the natural frequency of the plate away from the driving frequency. By using an appropriate thickness distribution, one may obtain a substantial increase in the structure performance over its uniform thickness counterpart. For cases where reduction of weight is of high importance, such as space structures, this type of structures may be the best choice.

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Also, the rapid development of various industrial fields requires new materials that can serve useful functions under certain conditions. In aerospace industry and many other engineering applications, the suppression of excessive vibrations occurring in large structures represents one of the most pressing and difficult problems facing structural designers. Thus, there is a need for new light materials possessing a high degree of flexibility with very low natural damping, this problem can be solved by active structural control. Therefore, The optimal control problems of dynamical systems have long been a main subject of many studies [1,2], up-to-date lists of publications in this area are given in survey articles[3,4].

Most recently, the strong interaction between structural control and design optimization has been recognized. As a result, simultaneous design and control has been the subject of several research studies with a view towards integrating optimal design and active control in a single formulation. For instance, in Refs [5,6], the design control problem was formulated as a constrained optimization problem. The mechanical behavior of a plate is strongly dependent on the fiber orientations and the plate thickness, because of this, the plate should be designed to meet the specific requirements of each particular application in order to obtain the maximum advantages of it. Accurate and efficient structural analysis, design sensitivity analysis and optimization procedures are very important to accomplish this task.

A series of publications has been concerned with the fundamental considerations of these approaches and their applications to different dynamical systems. Sloss et al. and Ledzewicz [7,8] presented a maximum principle for the optimal control of a general class of dynamical systems with distributed parameters. Moreover, some optimal distributed control results were obtained for membranes by Sadek and Adali [9], for Mindlin-Timoshenko plates by Sadek et al. [10] and for orthotropic plates by Adali et al. [11], for thick composite plates by Youssif et al. [12] and Fares et al. [13,14]. Other studies on optimal control may be found in [15-19]. Fares and his group give a series of articles in this field [20-23]. For these studies, however, there have been considerably few papers concerned with anisotropic plates with variable thickness for various cases of boundary conditions.

The problem of optimal design of plates and shells with variable thickness has received rather less attention. It has been discussed by several authors. Koiter et al. [24] have studied buckling of an axial compressed cylindrical shell of variable thickness. The non-linear analysis and optimization of shallow shell of variable thickness are treated by Zhiming [25], in particular Luong et al. [26], analyzed the stability of the elastic rectangular thin plates with sinusoidal changes in the plate thickness.

Huang et al. [27] and ÖmerCivalek [28] used a discrete method to analyze the free vibration of orthotropic rectangular plates with variable thickness. Hosseini Hashemi et al. [29] studied the vibration analysis of radially FGM sectorial plates of variable thickness on elastic foundations. De Faria and de Almeida [30] were concerned with the buckling optimization of plates with variable thickness subjected to nonuniform uncertain loads.

For the elastic analysis of plates of variable thickness, only a limited number of closed-form solutions is known. Fertis and Mijatov [31] and Fertis, and Lee [32] developed a convenient and general method to analyze variable thickness plates with various boundary conditions and loading by using equivalent flat plates. Zenkour [33] presented an exact solution for the bending of thin rectangular plates with uniform, linear, and quadratic thickness variations. Xu and Zhou [34,35] presented a three-dimensionally elasticity solution for simply supported rectangular plates with variable thickness, they used a proposed method to analyze the stress and displacement distributions of functionally graded rectangular plates with arbitrarily continuously varying thickness. More studies on plates with variable thickness may be found in [36-37].

The current work deals with design and control optimization to minimize the dynamic response of an anisotropic plate with variable thickness for various cases of boundary conditions. The plate is designed such that the principal direction of anisotropy rotated at an arbitrary angle relative to the coordinate axes. This orientation angle and the thickness non-uniformity parameter may be taken as optimization design parameters. The present control problem aims to minimize the dynamic response of a damped plate with the minimum possible expenditure of force. Control over the plate is exercised by distributed forces, which translate into force in the actual implementation of the control mechanism. The dynamic responses of the anisotropic plate comprise its deflection and velocity which constitute multiple objectives of the control problem together with the expenditure of force. The dynamic response is related to the energy of the structure, which is subject to initial disturbances. A quadratic functional of the dynamic response is specified as the control performance index. The expenditure of force is limited by attaching a functional of force to the objective functional as a penalty term. The necessary and sufficient conditions for optimal stabilization in Liapunov-Bellman sense [38] are used to determine the control force, deflections and the total elastic energy. Numerical results are given to study the influences of various thickness parameters, orientation angle, aspect ratio, boundary conditions, and thickness parameter on the control process.

2 Formulation of the Problem

Consider an anisotropic rectangular plate of length *a*, width *b* with exponentially variable thickness. The mid-plane of the plate coincides with *xy*- plane and normal to *z*-axis as shown in Figures 1 and 2. The material of the plate is assumed to possess a principal direction of elasticity rotated at an angle θ relative to *x*- direction. The upper surface of the plate is flat and subjected to transverse load $q(x, y, t)$.

The fundamental differential equation governing the motion of the plate is given by

$$\rho h(x)\,\ddot{w} = q + M_{1,xx} + 2M_{6,xy} + M_{2,yy} \;, \tag{1}$$

$$M_i = D_{ij}^{*}\varepsilon_j^{o} \;,\; (i, j = 1, 2, 6) \;,\; \varepsilon_1^{o} = -w_{,xx} \;,\; \varepsilon_2^{o} = -w_{,yy} \;,\; \varepsilon_6^{o} = -2w_{,xy}$$

Where w is the plate deflection in *z*- direction, ρ is the constant material density, D_{ij}^{*} are the rigidities of the plate. The superposed dot denotes differentiation with respect to time and a comma followed by a variable suffix denotes partial differentiation with respect to that variable, and summation convention is used for the summation indices j.

The mean thickness h_C of the plate is constant and the thickness exponentially varies in the *x*-direction as follows:

$$h(x) = H(1-\alpha e^{-x/a}) \ , \ H = \frac{h_C \, e}{e+(1-e)\alpha} \ ,$$

This formula preserves the volume of the plate to be constant, α is the non-dimensional thickness variation parameter which determine the shape of the lower plate surface. If $\alpha = 0$ the thickness of the plate becomes constant and $h(x) = h_C$. The thickness of the plate may be zero if $\alpha \in [1, e]$ therefore, this interval must be excluded from the values of α then, one can take α to be $-\infty < \alpha < 1$. The lower surface of the plate is concave for $\alpha < 0$ Fig.1. and convex for $0 < \alpha < 1$ Fig.2.

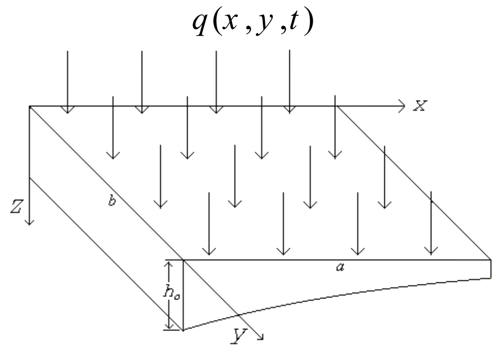

Fig. 1. The plate geometry for $\alpha < 0$

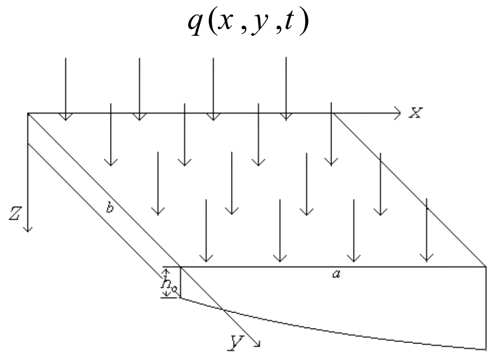

Fig. 2. The plate geometry for $0 < \alpha < 1$

In the present problem we will take various cases of boundary conditions at edges, i.e., when the plate edges are simply supported (*S*), clamped (*C*), free (*F*), or when mixed of these boundary conditions are prescribed over edges. These boundary conditions on edges perpendicular to *x*-axis (for example) take the form:

$$\begin{aligned} S &: \ w = M_1 = 0, \\ C &: \ w = w_{,x} = 0, \\ F &: \ M_1 = M_{1,x} + M_{6,y} = 0. \end{aligned} \tag{2}$$

Also, we assume that the plate is subjected to the following initial conditions:

$$w(x,y,0) = \psi(x,y) \ , \quad \dot{w}(x,y,0) = \phi(x,y) \ . \tag{3}$$

3 Optimal Control Problems

The objectives of the present study are to determine the optimal control force q^{opt} and optimal design variables θ^{opt} and α^{opt} which minimize the dynamic response of the plate in a specified time $0 \le t \le \tau < \infty$.

The total elastic energy is taken as a measure (criterion) for the dynamic response. This criterion is a function of the displacement, their spatial derivatives and velocity. The control force is introduced in the control objective by taking a performance index which compresses a weight sum of the total plate energy and a penalty functional involving the control force. Then, the mathematical formulation of the cost functional may be taken as:

$$J(q,\theta) = \xi_1 J_1 + \xi_2 J_2 + \xi_3 J_3, \tag{4}$$

$$J_1(q,\theta,\alpha) = \frac{1}{2}\int_0^{\infty}\int_0^{b}\int_0^{a} [D_{11}^* w_{,xx}^2 + 2D_{12}^* w_{,xx} w_{,yy} + D_{22}^* w_{,yy}^2 + 4D_{66}^* w_{,xy}^2 + 4(D_{16}^* w_{,xx} + D_{26}^* w_{,yy}) w_{,xy}]\,dx\,dy\,dt, \tag{5}$$

$$J_2(q,\theta,\alpha) = \frac{1}{2}\rho\int_0^{\infty}\int_0^{b}\int_0^{a} h\dot{w}^2 dxdydt \tag{6}$$

$$J_3(q,\theta,\alpha) = \int_0^{\tau}\int_0^{b}\int_0^{a} q^2(x,y,t)\,dxdydt\,, \tag{7}$$

where $\xi_i > 0$, (i = 1, 2, 3) and $\xi_1 + \xi_2 + \xi_3 = 1$ are constants weighting factors. J_1 and J_2 represent the potential and the kinetic energy of the plate, the functional J_3 is a penalty term involving the control function $q \in L^2$, where L^2 denotes the set of all bounded square integrable functions on $\{0 \le x \le a\,,\ 0 \le y \le b\,,\ 0 \le t \le \tau < \infty\}$.

Thus, the dynamic response of the plate is expressed as functionals contain w, its spatial derivatives and $\dot{w}$ given by J_1 and J_2. Then, the present multi objective control problem is to determine: firstly, the optimal control function q^{opt} from the minimization condition of the functional J and secondly, the optimal orientation angle θ^{opt} and thickness parameter α^{opt} which minimize the total elastic energy.

4 Solution Procedures

To solve the equation (1) under conditions (2) and (3) one can assume the displacement function w and the control function q in the following form of double series:

$$w = \sum_{m,n} W_{mn}(t) X_m(x) Y_n(y)\,, \qquad q = \sum_{m,n} Q_{mn}(t) X_m(x) Y_n(y)\,, \tag{8}$$

Where W_{mn} and $Q_{mn}$are unknown functions of time, X_m (x) and Y_n (y) are continuous or thonormedeigen functions which satisfy the boundary conditions given in (2) and represent approximate shape of the deflected upper surface of the free vibrating plate. These functions for different cases of boundary conditions (on x-axis for example) take the following forms [39]:

$$SS : X(x) = \sin \mu_m x\,, \qquad \mu_m = m\pi / a\,.$$

$$CC: X(x) = \sin\mu_m x - \sinh\mu_m x - \eta_m\left(\cos\mu_m x - \cosh\mu_m x\right),$$
$$\eta_m = \left(\sin\mu_m a - \sinh\mu_m a\right)/\left(\cos\mu_m a - \cosh\mu_m a\right), \quad \mu_m = \left(m + 0.5\right)\pi / a.$$
$$CS: X(x) = \sin\mu_m x - \sinh\mu_m x - \eta_m\left(\cos\mu_m x - \cosh\mu_m x\right),$$
$$\eta_m = \left(\sin\mu_m a + \sinh\mu_m a\right)/\left(\cos\mu_m a + \cosh\mu_m a\right), \quad \mu_m = \left(m + 0.25\right)\pi / a.$$
$$CF: X(x) = \sin\mu_m x - \sinh\mu_m x - \eta_m\left(\cos\mu_m x - \cosh\mu_m x\right),$$
$$\eta_m = \left(\sin\mu_m a + \sinh\mu_m a\right)/\left(\cos\mu_m a + \cosh\mu_m a\right), \quad \mu_1 = 1.875/a, \quad \mu_2 = 4.694/a,$$
$$\mu_3 = 7.855/a, \quad \mu_4 = 10.996/a \quad \text{and} \quad \mu_m = \left(m - 0.25\right)\pi / a \quad \text{for} \quad m \geq 5.$$

According to Galerkin's technique, substitute formulae (8) into equation (1), then, multiply both sides of the resulting equation by $X_m(x)Y_n(y)$, and integrate over the domain of the solution, we get:

$$\ddot{W}_{mn} + \omega_{mn}^2 W_{mn} = \frac{I_{13}}{\rho H I_0} Q_{mn}, \tag{9}$$

$$\omega_{mn}^2 = \frac{1}{\rho H I_0}[d_{11}^*\left(I_1 + 6I_6 + 6I_{10} + 3I_{14}\right) + d_{12}^*\left(2I_3 + 6I_8 + 6I_{12} + 3I_{16}\right) + 2d_{26}^*\left(2I_4 + 3I_9\right) + \tag{10}$$
$$+ 2d_{16}^*\left(2I_2 + 6I_{11} + 9I_7 + 3I_{15}\right) + 4d_{66}^*\left(I_3 + 3I_8\right) + d_{22}^* I_5], \qquad d_{ij}^* = B_{ij}^* H^3/3$$

where the fundamental elastic constants B_{ij}^* as follows

$$\begin{bmatrix} B_{11}^* \\ B_{12}^* \\ B_{22}^* \\ B_{16}^* \\ B_{26}^* \\ B_{66}^* \end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 & 2c^2s_2 & 2s^2s_2 & 4c^2s^2 \\ c^2s^2 & 1-2c^2s^2 & c^2s^2 & -c_2s_2 & c_2s_2 & -4c^2s^2 \\ s^4 & 2c^2s^2 & c^4 & -2c^2s_2 & 2s^2s_2 & 4c^2s^2 \\ -c^3s & -\tfrac{1}{2}\,c_2s_2 & cs^3 & c^2\left(1-4c^2\right) & s^2\left(4c^2-1\right) & -c_2s_2 \\ -cs^3 & -\tfrac{1}{2}\,c_2s_2 & c^3s & s^2\left(4c^2-1\right) & c^2\left(1-4c^2\right) & -c_2s_2 \\ c^2s^2 & -2c^2s^2 & c^2s^2 & -c_2s_2 & c_2s_2 & \left(s^2-c^2\right)^2 \end{bmatrix} \begin{bmatrix} B_{11} \\ B_{12} \\ B_{22} \\ B_{16} \\ B_{26} \\ B_{66} \end{bmatrix}$$

where: $c = \cos\theta$, $s = \sin\theta$, $c_2 = \cos 2\theta$ and $s_2 = \sin 2\theta$.

Substituting the relations (8) into expressions (5) and (6), we can easily get:

$$J_1 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int_0^{\infty} e_1 W_{mn}^2\,dt, \; J_2 = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\int_0^{\infty} e_2 \dot{W}_{mn}^2\,dt, \tag{11}$$

Where $e_1 = \frac{1}{2}\left(d_{11}^{*}I_{17} + 2d_{12}^{*}I_3 + d_{22}^{*}I_{18} + 4d_{66}^{*}I_{19} + 4d_{16}^{*}I_{20} + 4d_{26}^{*}I_{21}\right)$, $e_2 = \frac{1}{2}\rho H I_0$,

$$\left(I_0, I_{13}\right) = \int_0^b \int_0^a \left(h_1(x), 1\right) X^2 Y^2 dxdy \ , \ h_1(x) = 1 - \alpha e^{-x/a}$$

$$\left(I_1, I_2, I_3, I_4, I_5\right) = \int_0^b \int_0^a \left(X_{,xxxx}Y, X_{,xxx}Y_{,y}, X_{,xx}Y_{,yy}, X_{,x}Y_{,yyy}, XY_{,yyyy}\right) h_1^3(x) XY dxdy \ ,$$

$$\left(I_6, I_7, I_8, I_9\right) = \int_0^b \int_0^a \left(X_{,xxx}Y, X_{,xx}Y_{,y}, X_{,x}Y_{,yy}, XY_{,yyy}\right) h_{1,x} h_1^2(x) XY dxdy \ ,$$

$$\left(I_{10}, I_{11}, I_{12}\right) = \int_0^b \int_0^a \left(X_{,xx}Y, X_{,x}Y_{,y}, XY_{,yy}\right) h_{1,x}^2 \, h_1(x) XY dxdy \ ,$$

$$\left(I_{14}, I_{15}, I_{16}\right) = \int_0^b \int_0^a \left(X_{,xx}Y, X_{,x}Y_{,y}, XY_{,yy}\right) h_{1,xx} \, h_1^2(x) XY dxdy \ ,$$

$$\left(I_{17}, I_{18}, I_{19}, I_{20}, I_{21}\right) = \int_0^b \int_0^a \left(X_{,xx}^2 Y^2, X^2 Y_{,yy}^2, X_{,x}^2 Y_{,y}^2, X_{,xx} X_{,x} Y_{,y} Y, X_{,x} X Y_{,yy} Y_{,y}\right) h_1^3(x) dxdy \,. \quad (12)$$

Substituting expressions (7) and (11) into (4), the functional J takes the form:

$$J = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \int_0^{\infty} \left(e_1 W_{mn}^2 + e_2 \dot{W}_{mn}^2 + e_3 Q_{mn}^2\right) dt, \qquad e_3 = I_{13}. \quad (13)$$

Since the system of Eq. (9) is separable, hence the functional (13) depends only on the variables found in (m, n)th equation of the system. With the aid of this condition, the problem is reduced to a problem of analytical design of controllers [38,40] for every m , n = 1, 2, . . . , ∞.

To minimize the functional J, we apply Liapunov-Bellman theory, that gives the minimization condition in the form:

$$\min_{Q=Q^{opt}} \left(\frac{\partial V_{mn}}{\partial W_{mn}} \dot{W}_{mn} + \frac{\partial V_{mn}}{\partial \dot{W}_{mn}} \ddot{W}_{mn} + \bar{J}_{mn} \right) = 0 \,, \quad (14)$$

where V_{mn} is a Liapunov function and may be chosen in the form:

$$V_{mn} = \varphi_{mn} W_{mn}^2 + 2\varepsilon_{mn} W_{mn} \dot{W}_{mn} + \eta_{mn} \dot{W}_{mn}^2, \quad (15)$$

$\bar{J}_{mn}$ is the integrand of (13), φ_{mn}, ε_{mn} and η_{mn} are parameters chosen according to the condition that the Liapunov function V_{mn} must be positive definite. Then, from expressions (13)-(15), we can obtain the optimal control force in the form:

$$Q_{mn}^{opt} = \frac{-I_{13}}{\rho H \, I_0 \, e_3} \left(\varepsilon_{mn} W_{mn} + \eta_{mn} \dot{W}_{mn} \right), \quad (16)$$

Substituting equations (9) and (16) into (14), and equating the coefficients of $W_{mn}^2, \dot{W}_{mn}^2$ and $W_{mn}\dot{W}_{mn}$ by zero, we get a system of equations, its general solution is:

$$\varepsilon_{mn} = -e_4\left(\omega_{mn}^2 - \sqrt{\omega_{mn}^4 + e_1/e_4}\right), \qquad \eta_{mn} = \sqrt{\left(2\varepsilon_{mn} + e_2\right)e_4}\,,$$
$$\varphi_{mn} = \varepsilon_{mn}\left(\omega_{mn}^2 + \eta_{mn}/e_4\right), \qquad where \qquad e_4 = I_0^2\rho^2 H^2 e_3 / I_{13}^2. \tag{17}$$

The signs before the square roots are chosen according to the condition on Liapunov functions. On the other hand, we can rewrite the equation (9) as follows:

$$\ddot{W}_{mn} + \alpha_{mn}\dot{W}_{mn} + \Omega_{mn}^2 W_{mn} = 0\,, \qquad \alpha_{mn} = \eta_{mn}/e_4\,, \qquad \Omega_{mn}^2 = \omega_{mn}^2 + \varepsilon_{mn}/e_4\,. \tag{18}$$

The solution of equation (18) where $2\Omega_{mn} > \alpha_{mn}$, is given by:

$$W_{mn}^{opt} = e^{-\alpha_{mn}t/2}\left[\beta_{mn}\cos\left(\nu_{mn}t\right) + \gamma_{mn}\sin\left(\nu_{mn}t\right)\right], \qquad \nu_{mn}^2 = \Omega_{mn}^2 - \tfrac{1}{4}\alpha_{mn}^2, \tag{19}$$

where β_{mn} and γ_{mn} are unknown coefficients which may be obtained from the initial conditions (3) by expanding it in series, then:

$$\gamma_{mn} = \frac{\alpha_{mn}\beta_{mn} + 2\bar{A}_{mn}}{2\nu_{mn}}, \qquad \left(\beta_{mn}, \bar{A}_{mn}\right) = \frac{4}{ab\,\omega_{mn}^2}\int_0^b\int_0^a \left(\psi, \phi\right) XY\,dx\,dy\,. \tag{20}$$

Substituting from equation (20) into (19) we get the optimal deflection as

$$W_{mn}^{opt} = \frac{2e^{-\alpha_{mn}t/2}}{ab\,\omega_{mn}^2}\left[2\cos\left(\nu_{mn}t\right)\int_0^b\int_0^a \psi XY\,dx\,dy + \frac{\sin\left(\nu_{mn}t\right)}{\nu_{mn}}\int_0^b\int_0^a \left(\alpha_{mn}\psi + 2\phi\right)XY\,dx\,dy\right] \tag{21}$$

Differentiate (21) with respect to *t* we get

$$\dot{W}_{mn}^{opt} = \frac{e^{-\alpha_{mn}t/2}}{ab\,\omega_{mn}^2}\left[4\cos\left(\nu_{mn}t\right)\int_0^b\int_0^a \phi XY\,dx\,dy - \sin\left(\nu_{mn}t\right)\int_0^b\int_0^a \left(4\nu_{mn}\psi + \frac{\alpha_{mn}^2}{\nu_{mn}}\psi + 2\frac{\alpha_{mn}}{\nu_{mn}}\phi\right)XY\,dx\,dy\right] \tag{22}$$

Substituting equation (21) and (22) into (13) and (16), we obtain the total elastic energy and the optimal control force for various boundary conditions.

5 Numerical Results and Discussion

In this section, numerical results for the optimal deflections w^{opt}, optimal force q^{opt} and the total elastic energy J^{opt} are presented for orthotropic plate. In this case, the engineering constants are introduced instead of the elastic constants from the relations:

$$B_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad B_{12} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \quad B_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \quad B_{66} = G_{12}, \quad B_{16} = B_{26} = 0,$$

Where E_i are Young's moduli; ν_{ij} are Poisson's ratios and $G_{ij}$are shear moduli. The Poisson's ratios and Young's moduli are related by the reciprocal relations $\nu_{ij}E_j = \nu_{ji}E_i \ ,(i,\ j = 1,\ 2)$
The initial conditions (3) may be taken in the form:

$$w\ (x,y,0) = 5X\ (x)Y\ (y), \quad \dot{w}\ (x,y,0) = 0.$$

In all calculations, unless otherwise stated, the following parameters are used,

$$E_1/E_2 = 12.67, \ \ G_{12}/E_2 = 1.5, \ \nu_{12} = 0.22, \ \rho = 0.057\,Ib/in^3 = 1578\ kg/m^3$$
$$\xi_1 = 0.1, \qquad \xi_2 = 0.2, \qquad \xi_3 = 0.7, \qquad h_c = 50/ab$$

Which are typical of carbon fiber reinforced plastic. The four letters of the boundary conditions (*SSCC*, *SCCF*, … ,etc.) with its order from left to right indicate the kind of fixing at the plate edges *x=0, x=a, y=0* and *y=b,* respectively.

Table 1 gives values of the optimal orientation angle θ^{opt} by degrees, at which the total elastic energy J takes minimum values for different cases of boundary conditions and different values of thickness parameters α and aspect ratio *a/b*. It is shown that, the orientation angle θ which gives the minimum energy strongly depends on the aspect ratio of the plate and the boundary conditions, butvery weakly on α, this may be due to that the thicknesses of the plate is small with respect to its other dimensions and the taken formula of the plate variable thickness preserves the volume of the plate to be constant. From this table one can notice also that for long plate ($a/b \geq 2$) the optimal orientation angle becomes $\theta^{opt} \approx 90^0$.

Tables 2 and 3 give values of w^{opt}, q^{opt} and J^{opt} for different values of α and *a*/*b* for some boundary conditions. These tables show that – in general – values of w^{opt}, q^{opt} and J^{opt} increase with the increasing of *a*/*b*, also, for *SSSS*, *SSCS* and *CCCC* the values of w^{opt}, q^{opt} and J^{opt} increase with the increasing of α for concave plate and conversely decrease with the increasing of α for a convex plate.

Figs. 3, 4 and 5 give curves of J^{opt} against *t* with different values of *a*/*b* and different values of α and θ. It shows clearly that – for all cases – at α^{opt} and θ^{opt} the *J*-curves is the lowest one, which confirm that the present method is effective.

Figs. 6, 7 and 8, give curves of J^{opt} against E_1/E_2 with different values of *a*/*b* and different values of α and θ. It show clearly that whatever the values of α and θ the *J*-curves at α^{opt} and θ^{opt} usually the lowest one. This confirm the effectiveness of the proposed design procedure.

Figs. 9, 10 and 11, give curves of q^{opt} against E_1/E_2 with different values of a/b and different values of α and θ. These Figures show that the present control approach not only plays an efficient role in minimizing the dynamic response of the plate, but also, it contributes significantly in decreasing the expenditure of the control force.

Table 1. Values of optimal orientation angle θ^{opt} (by degrees) which minimize the total elastic energy against α and *a/b*, for various boundary conditions

α	***a/b***	***SSSS***	***SSCS***	***CCSS***	***CSCS***	***CCCC***	***SSCF***
	1	45.0	56.5	0	36.7	0	11.1
-1.00	1.5	61.2	90.0	42.4	90.0	90.0	12.0
	2	90.0	90.0	90.0	90.0	90.0	90.0
	1	45.0	56.9	0	38.8	0	9.5
-0.75	1.5	61.7	90.0	43.0	90.0	90.0	10.3
	2	90.0	90.0	90.0	90.0	90.0	90.0
	1	45.0	57.3	0	40.9	0	7.3
-0.50	1.5	62.2	90.0	43.7	90.0	90.0	7.9
	2	90.0	90.0	90.0	90.0	90.0	90.0
	1	45.0	57.8	0	43.0	0	4.2
-0.25	1.5	62.9	90.0	44.3	90.0	90.0	4.6
	2	90.0	90.0	90.0	90.0	90.0	90.0
	1	45.0	58.3	0	45.0	0 ; 90	0
0	1.5	63.5	90.0	44.9	90.0	90.0	0
	2	90.0	90.0	90.0	90.0	90.0	90.0
	1	45.0	58.6	0	46.8	90.0	0
	0.251.5	64.0	90.0	45.3	89.6	90.0	0
	2	90.0	90.0	90.0	89.9	90.0	90.0
	1	45.0	58.5	0	48.2	90.0	0
	0.501.5	63.7	90.0	45.0	88.9	90.0	0
	2	90.0	90.0	90.0	89.6	90.0	88.9
	1	45.0	57.0	0	48.7	0	0
0.75	1.5	61.5	90.0	42.8	87.4	90.0	0
	2	90.0	90.0	90.0	89.3	90.0	88.1
	1	45.0	55.4	0	48.3	0	0
0.95	1.5	59.1	90.0	37.5	83.2	90.0	0
	2	90.0	90.0	90.0	88.9	90.0	87.5

Table 2. Values of $max|w|$, $max|q|$ and J against α and a/b for some boundary conditions, $\theta = 45^o$

α	a/b	SSSS			SSCS			CCSS		
		$max|w|$	$max|q|$	J	$max|w|$	$max|q|$	J	$max|w|$	$max|q|$	J
	1	0.00209	0.00364	0.00020	0.00453	0.00915	0.00120	0.00393	0.00908	0.00100
-1.00	1.5	0.00960	0.00556	0.00076	0.01800	0.01355	0.00395	0.02338	0.01636	0.00493
	2	0.02516	0.00663	0.00179	0.04209	0.01587	0.00815	0.06953	0.02081	0.01305
	1	0.00211	0.00366	0.00020	0.00458	0.00921	0.00122	0.00398	0.00913	0.00102
-0.75	1.5	0.00971	0.00559	0.00077	0.01816	0.01359	0.00399	0.02358	0.01645	0.00498
	2	0.02540	0.00665	0.00181	0.04239	0.01592	0.00822	0.06992	0.02088	0.01316
	1	0.00214	0.00368	0.00021	0.00463	0.00927	0.00124	0.00403	0.00918	0.00103
-0.50	1.5	0.00982	0.00563	0.00078	0.01833	0.01364	0.00403	0.02377	0.01654	0.00504
	2	0.02565	0.00668	0.00183	0.04267	0.01597	0.00829	0.07028	0.02095	0.01328
	1	0.00217	0.00370	0.00021	0.00468	0.00933	0.00125	0.00408	0.00922	0.00105
-0.25	1.5	0.00993	0.00566	0.00080	0.01846	0.01368	0.00407	0.02394	0.01662	0.00510
	2	0.02587	0.00671	0.00185	0.04289	0.01601	0.00835	0.07053	0.02102	0.01340
	1	0.00218	0.00372	0.00021	0.00471	0.00936	0.00126	0.00411	0.00927	0.00107
0	1.5	0.00999	0.00569	0.00080	0.01853	0.01370	0.00410	0.02401	0.01669	0.00515
	2	0.02598	0.00674	0.00187	0.04295	0.01603	0.00839	0.07052	0.02108	0.01348
	1	0.00217	0.00372	0.00021	0.00468	0.00934	0.00126	0.00409	0.00928	0.00107
0.25	1.5	0.00994	0.00569	0.00081	0.01840	0.01368	0.00409	0.02383	0.01669	0.00516
	2	0.02581	0.00675	0.00187	0.04259	0.01600	0.00835	0.06988	0.02110	0.01347
	1	0.00210	0.00369	0.00021	0.00453	0.00916	0.00123	0.00395	0.00922	0.00105
0.50	1.5	0.00961	0.00563	0.00079	0.01780	0.01353	0.00398	0.02308	0.01650	0.00506
	2	0.02496	0.00670	0.00183	0.04124	0.01583	0.00812	0.06775	0.02100	0.01321
	1	0.00187	0.00353	0.00019	0.00406	0.00856	0.00111	0.00353	0.00894	0.00095
0.75	1.5	0.00860	0.00545	0.00071	0.01609	0.01304	0.00363	0.02095	0.01605	0.00468
	2	0.02250	0.00650	0.00167	0.03758	0.01532	0.00746	0.06215	0.02053	0.01232
	1	0.00149	0.00313	0.00015	0.00326	0.00782	0.00091	0.00282	0.00823	0.00078
0.95	1.5	0.00689	0.00499	0.00058	0.01317	0.01189	0.00302	0.01730	0.01499	0.00396
	2	0.01830	0.00612	0.00137	0.03127	0.01441	0.00629	0.05249	0.01958	0.01064

Table 3. Values of $max|w|$, $max|q|$ and J against α and a/b, for some boundary conditions, $\theta = 45^o$

α	a/b	CSCS			CCCC			SSCF		
		$max\|w\|$	$max\|q\|$	J	$max\|w\|$	$max\|q\|$	J	$max\|w\|$	$max\|q\|$	J
	1	0.01092	0.02304	0.00768	0.00917	0.02643	0.00625	0.10688	0.08202	0.05910
-1.00	1.5	0.04945	0.03774	0.02873	0.03931	0.04124	0.02202	0.98924	0.09714	0.65068
	2	0.12157	0.04497	0.06241	0.09073	0.04931	0.04447	3.50815	0.25056	3.34322
	1	0.01093	0.02313	0.00776	0.00925	0.02658	0.00633	0.11657	0.08635	0.06620
-0.75	1.5	0.04920	0.03778	0.02882	0.03947	0.04136	0.02218	1.09676	0.10241	0.77259
	2	0.12067	0.04500	0.06244	0.09089	0.04941	0.04469	3.81378	0.27893	3.90381
	1	0.01092	0.02319	0.00784	0.00933	0.02673	0.00641	0.13076	0.09227	0.07714
-0.50	1.5	0.04879	0.03779	0.02888	0.03961	0.04148	0.02234	1.26159	0.12867	0.98436
	2	0.11936	0.04500	0.06238	0.09098	0.04951	0.04491	4.26060	0.32166	4.81575
	1	0.01086	0.02324	0.00790	0.00940	0.02687	0.00649	0.15353	0.10193	0.09615
-0.25	1.5	0.04812	0.03776	0.02886	0.03968	0.04159	0.02250	1.54701	0.17828	1.42549
	2	0.11743	0.04494	0.06214	0.09092	0.04959	0.04509	4.97775	0.39274	6.51314
	1	0.01071	0.02328	0.00793	0.00944	0.02697	0.00655	0.19627	0.11803	0.13655
0	1.5	0.04702	0.03764	0.02869	0.03961	0.04166	0.02260	2.16526	0.29750	2.72160
	2	0.11448	0.04478	0.06157	0.09053	0.04963	0.04518	6.32472	0.53200	10.4937
	1	0.01039	0.02326	0.00788	0.00937	0.02696	0.00656	0.30700	0.14970	0.27210
0.25	1.5	0.04518	0.03732	0.02821	0.03920	0.04161	0.02256	4.55844	0.81456	12.3438
	2	0.10983	0.04449	0.06038	0.08945	0.04957	0.04503	9.83322	0.91209	25.7894
	1	0.00973	0.02306	0.00764	0.00907	0.02665	0.00643	1.36372	0.68295	4.53910
0.50	1.5	0.04200	0.03653	0.02711	0.03804	0.04124	0.02217	8.28156	2.19868	48.3825
	2	0.10218	0.04402	0.05797	0.08688	0.04921	0.04428	45.2337	4.91632	581.564
	1	0.00845	0.02236	0.00699	0.00822	0.02546	0.00594	0.31549	0.81392	0.96737
0.75	1.5	0.03642	0.03466	0.02477	0.03510	0.03998	0.02087	1.50683	0.70383	2.62244
	2	0.08916	0.04268	0.05317	0.08086	0.04801	0.04201	9.43027	1.34756	30.3433
	1	0.00671	0.02074	0.00596	0.00676	0.02266	0.00502	0.11130	0.76113	0.48256
0.95	1.5	0.02918	0.03268	0.02128	0.03005	0.03700	0.01835	0.66695	0.56847	1.06720
	2	0.07237	0.04046	0.04614	0.07059	0.04609	0.03763	3.27667	0.68469	5.19852

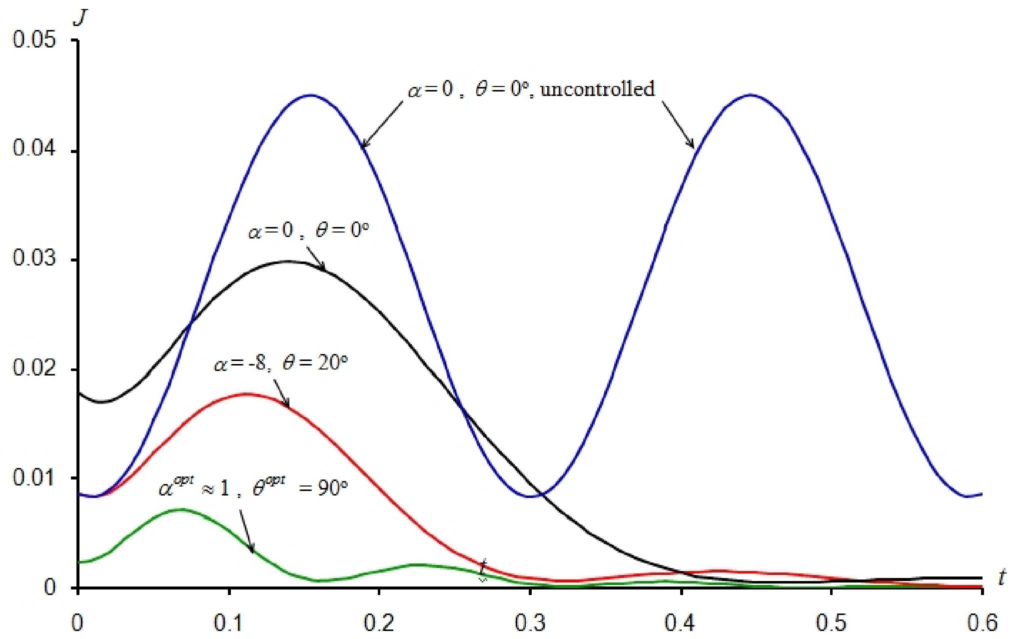

Fig. 3. Values of J against t for different values of α and θ, SSSS, a/b = 2, E_1/E_2 = 12.67

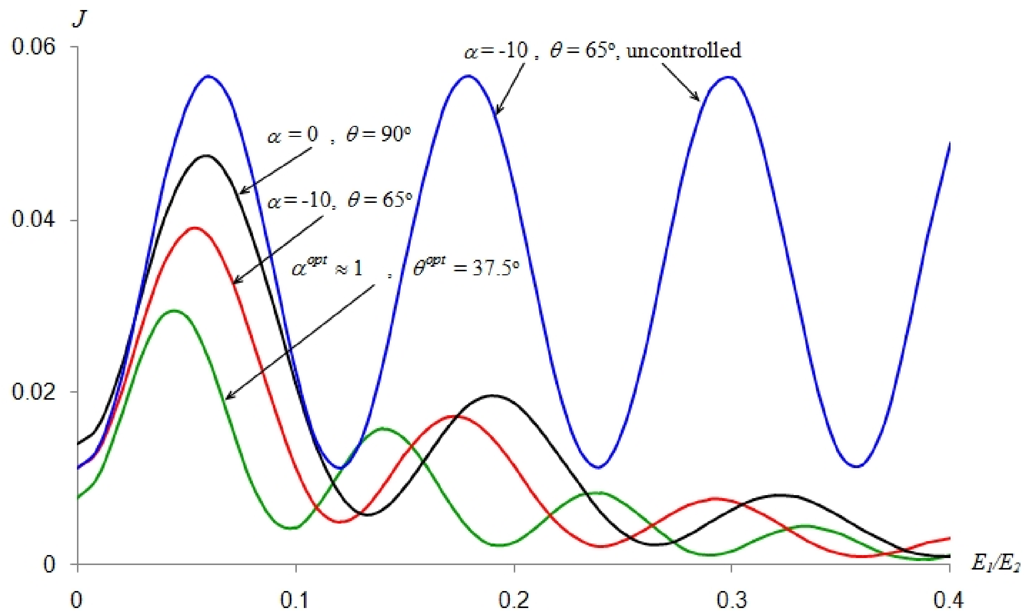

Fig. 4. Values of J against t for different values of α and θ, *CCSS*, a/b=1.5, E_1/E_2 = 12.67,

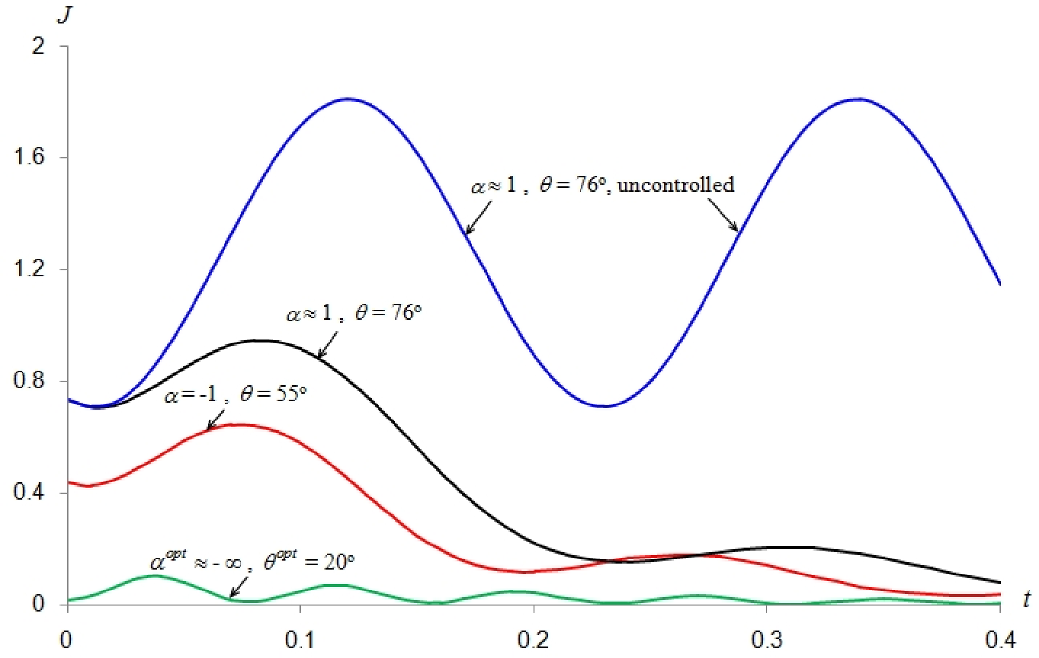

Fig. 5. Values of J against t for different values of α and θ, *SSCF* , a/b =1 , E_1/E_2 = 12.67,

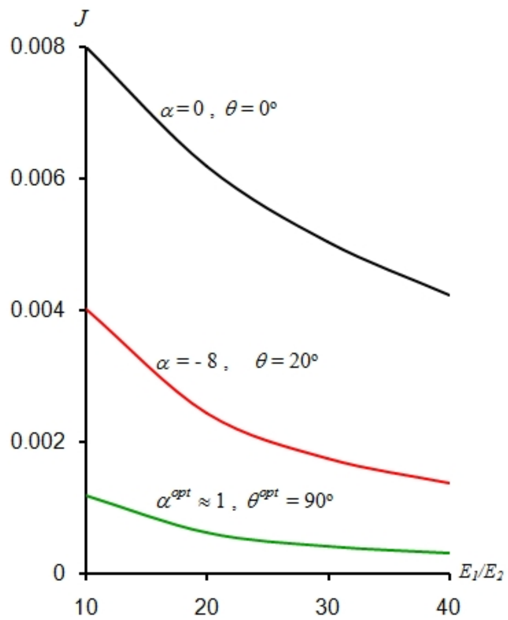

Fig. 6. Values of J against E_1/E_2 for different values of α and θ, SSSS , a/b =2

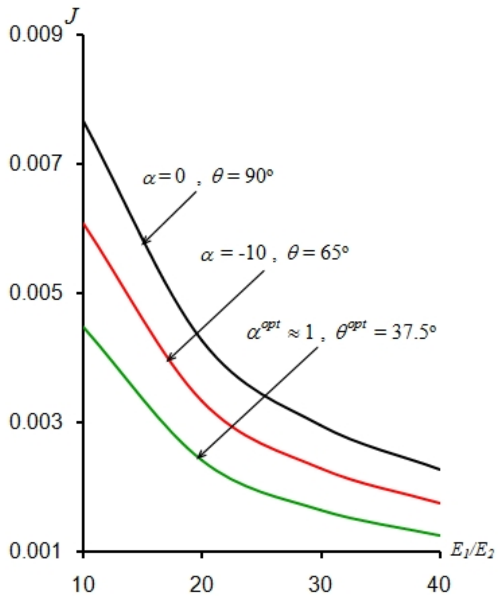

Fig. 7. Values of J against E_1/E_2 for different values of α and θ, *CCSS* , a/b =1.5,

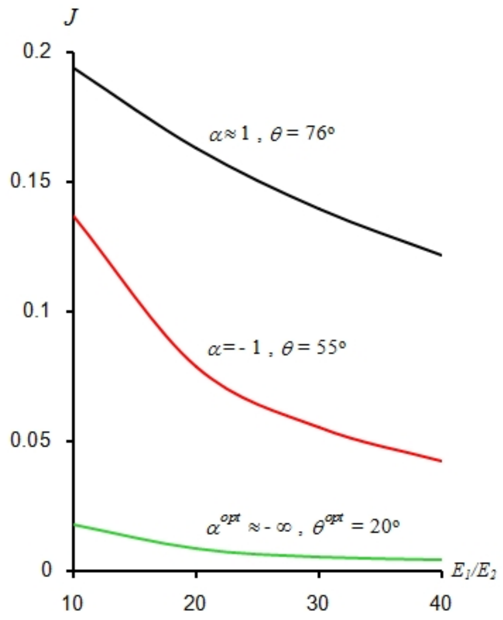

Fig. 8. Values of *J* against E_1/E_2 for different values of α and θ, *SSCF*, a/b =1,

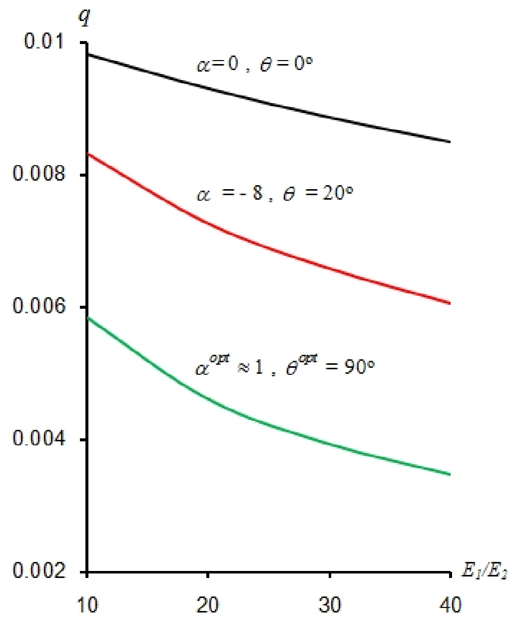

Fig. 9. Values of *q* against E_1/E_2 for different values of α and θ, SSSS, *a/b* =2

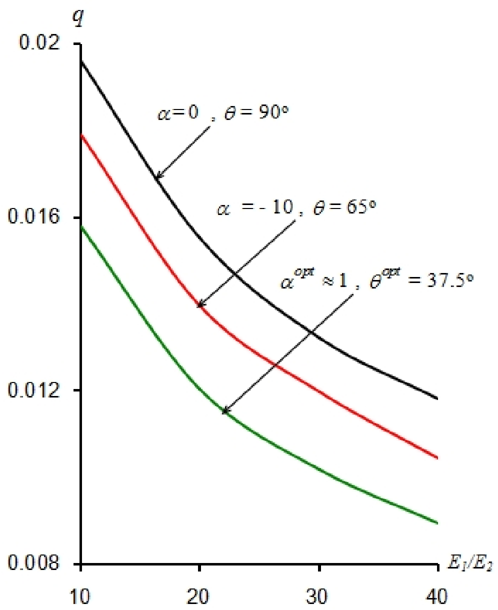

Fig. 10. Values of *q* against E_1/E_2 for different values of α and θ, *CCSS*, *a/b* =1.5,

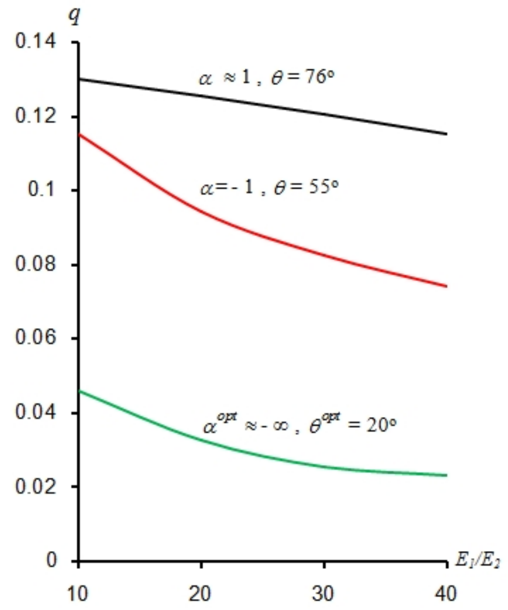

Fig. 11. Values of *q* against E_1/E_2 for different values of α and θ, *SSCF*, a/b =1,

6 Conclusion

Design and active control procedures are applied to minimize the dynamic response of anisotropic rectangular plates with exponentially variable thickness for various cases of boundary conditions. Furthermore, it is noticed that these procedures contribute significantly in decreasing the expenditure of the used control force and the time needed for the damping process. The effects of the orientation angle, boundary conditions, aspect ratios and thickness parameter on the control process, are considered. Comparative examples are given to show the advantages of the present structural control and optimization approach. It is found that the vibrations of the plate can be reduced substantially at a given terminal time by exercising closed-loop control. Also, it is noticed that $\alpha^{opt} \approx 1$ for all boundary conditions discussed unless for *SSCF*, $\alpha^{opt} \approx -\infty$. Also it is found that the partially optimal design over the thickness parameter is less effective than that over the fibers orientation angle. But, the optimal design over both the ply orientation angle and the thickness parameter is the most efficient and has exterior influence on the dynamic response. For each case of boundary conditions, E_1/E_2 can play a significant role to enhance the design process so, the plate may be tailored using E_1/E_2 to improve its performance. There is a suitable optimal design for every plate to improve its performance. The numerical results clarify the influences of various thickness parameters, orientation angle, aspect ratio and boundary conditions, on the control process.

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Competing Interests

Authors have declared that no competing interests exist.

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