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The Roles of Condition Number for the Interval Hull Solution Set in Least Squares Equation

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Abstract

The paper considers nearness to singularity for which the Co-variance matrix in the least squares equation is well known. A synchronization of the condition number with ill-conditioning is highlighted which relates the quality of approximate solution to the described system. Various theoretical lower and upper bounds to the perturbed least squares problems have been described for which, reach ability theory has strong representation. In particular, a theorem due to Rump as exemplified by Popova was revisited and examined in detail; a slight modification was made to the theorem by neglecting the second term appearing in the equation. This was found to have strong favourable appeals on the interval least squares problem. As a comparison to the computed results, a procedure described in Kramer/Rohn was used to crop the corner points solution of the linear interval system which is obtained from least squares equation based on the appropriate choice of the orthant (where there are possibilities). This leads to solving systems of linear inequalities for the interval Hull of solution set. Furthermore, the Rump/Krawczyk method was used to narrow, the computed corner point solution in order to obtain tighter approximate solution bounds of the interval Hull which may be applicable to both non-parametric and parametric interval linear equations. The loss function for the computed result obtained from Rump method for the set of data points is reported.

Keywords: Least squares problem, diffeomorphism, reachability theory, condition number, interval arithmetic, regulator in least squares problems.

1 Introduction

The paper considers condition numbers associated with solving least squares equations with some noise in the data. The Covariance matrix in least squares interval equations is often Ill-conditioned. Ill-conditioned problems are often encountered in geophysics, signal processing, and medical imaging, see [1] and the cited references therein. For example, Auto-Covariance Least squares are used in the study of Kalma filter [2]. Kalma filter has been widely applied in navigation and control of vehicles, as for example, aircraft and space craft. In its widest application areas, Kalma filter is used in the navigation system of nuclear ballistic missile

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submarines, and cruise missiles –e.g., the US Navy's Tomahawk missile, US Air Launched Cruise missile are a few typical examples for this purpose in which least squares equations are involved in their formulations [2]. Interval arithmetic has been found useful in the study of fractal mechanics where a fracture in any part of aircraft makes every other part inoperable. Computing reachable set is an essential aspect of nonlinear dynamic and control systems. Reach ability is an essential tool in the theory of validation of interval computing, since safety is always a paramount interest in scientific and engineering designs which takes into consideration scenarios of worst case error bounds, [3-5] and [6].

Reach ability problem comes in different flavours as for example, the approximate estimates for the solution to the linear system in least squares equation may be taken as ellipsoids which depends on the operator parameter. For the exhaustive discussion on theory of reach ability, the importance of topological space cannot be underscored.

The least squares problem is formulated in the form:

$$\text{Find } \hat{\beta} \in R^n \ni \quad \min_{\beta \in R^n} \|Y - T\beta\|_2 \tag{1.1}$$

Where $T \in R^{m \times n}$, $Y \in R^m, m \geq n$ and rank(T)=n.

Equation 1.1 is the minimum-length solution in which $\left\|\hat{\beta}\right\|_2$ is minimized among the infinitely many solutions that minimized $\|Y - T\beta\|_2$.

In what follows, we signify our notation by the convention: R^n, IR^n, $R^{m \times n}$, $IR^{m \times n}$ as representing real n- vector, interval n-vector, real $m \times n$ rectangular matrix and interval $m \times n$ matrix respectively. The interval T is represented as $[\underline{T}, \bar{T}]$ in the form of lower and upper bounds. The midpoint of interval matrix T as well as interval vector b is respectively defined in the form

$$T_c = 0.5(\underline{T} + \bar{T}),$$

$$b_c = 0.5\left(\underline{b} + \bar{b}\right)$$

The interval radius of interval matrix T as well as interval vector b is given by

$$\Delta T = 0.5\left(\bar{T} - \underline{T}\right), \delta b = 0.5\left(\bar{b} - \underline{b}\right)$$

Interval arithmetic operations possess strict validation of every computed result in any calculation, and any such computed result, is always perfectly correct when faithfully implemented .This process captures all possible conversion errors, rounding errors, approximation errors and are always vigorously estimated, [6].

As a follow up in the discussion, we note that R^n itself is convex, and all interval boxes in R^n are convex. A subset $D \subseteq R^n$ is convex if for any $\beta_1, \beta_2 \in D,$ we have $t\beta_1 + (1-t)\beta_2 \in D, 0 \le t \le 1$.

The separation theorem for convex set, [7], states that a point outside a closed convex set is separated from D by a hyper plane.

It is expected that readers acquaint themselves with the concept of well known implicit function theorem which gives necessary conditions under which a continuously differentiable function has an inverse. This leads us to the concept of a diffeomorphism [8], that transforms a nonlinear system in equation 1.1 to a linear system

$$T^{/}T\beta = T^{/}Y \Rightarrow \hat{\beta} = \left(T^{/}T\right)^{-1}T^{/}Y = T^{+}Y \ , \tag{1.2}$$

Where $T^{/}$ is well defined.

It is assumed that, $X \in ID \subset IR^m$ be closed in IR^m . A function $f : ID \to IR^n$ is a diffeomorphism of ID onto its image $Y = f(ID)$ if it is one-to-one, smooth, and of full rank n.

By full rank, would imply the maximal possible rank, e.g., the case of a rectangular matrix of size $m \times n$, the full rank is given by min$\{m, n\}$.

Solving least squares problems in interval form is not without some combinatorial difficulties.

As a typical example, the Co-Variance matrix from system of equation 1.1 is sometimes nearly singular or, even singular. Thus, there is need to resolve this near singularity or even singularity problem.

Resolution of singularity in the least squares using the regulator parameter has been addressed in [9] and the cited references therein. Therefore problem 1.2 is re written in the form

$$\hat{\beta} = (T^{/}T + \tau^k I)^{-1}T^{/}Y, k \ge 2 \tag{1.3}$$

The term τ is taken to be $\frac{1}{\left\|T^{+}\right\|}$, where T^{+} is the pseudo inverse operator . Interestingly [10] proved that the best possible value for k in equation 1.3 is 2.

Hence forth, the matrix product $T^{/}T$ will now be denoted by another matrix B, and the matrix-vector product $T^{/}Y$ as a vector b. The inverse of the matrix B is the Co-Variance matrix. It will play a major role in our analysis hereafter.

In the mean time, the paper is categorized as follows: Section 2 gives the condition numbers associated with least squares problems. Section 3 discusses the perturbation error bounds for the lower and upper chain reachable set of the approximate solution to the least squares problems wherein, the residual error plays vital roles in their formulation. In section 4 the theorem due to [10], exemplified in [11] has been re-examined, modified and adopted for a useful purpose in the realization of numerical results.

2 Condition Numbers of Linear Least Squares Problem

Condition number has a very useful property in both Scientific and Engineering designs. It is also used in economics.

The quality of results computed from the least squares equation is a reflection on the condition numbers of the system given as equation 1.2. Condition number shows how the solution to the described system fits the data under study. Condition number also measures how a change in input data is propagated to a change in output process. High condition number implies high non linearity in any computed result as solution set to the system 1.1.

For a real matrix B which is not singular and whose norm $\|.\|$ is known, the condition number [12,13], was defined to be

$$K(B) = \underset{\varepsilon \to 0^{+}}{\lim} \; \underset{\|\Delta B\| \le \varepsilon \|B\|}{\sup} \frac{\left\| (B+\Delta B)^{-1} - B^{-1} \right\|}{\varepsilon \|B^{-1}\|} , \tag{2.1}$$

whereas, the traditional condition number has been known to be

$$K(B) = \|B^{-1}\|.\|B\| \tag{2.2}$$

If condition number K(B) of a matrix B is known in advance, it is possible to calculate the loss function, the digits of accuracy of solution in the variable $\hat{\beta}$ compared to precision of matrix B by taking the logarithm of K(B) .

The derivation of condition number in equation (2.2) may be obtained as follows: Suppose that $\hat{\beta}$ denotes the numerical approximate solution which minimizes equation 1.1. Let β^{*} be the known rigorous solution to equation 1.1. Then, following [14], it was proved that

$\mu((\beta) = \frac{\|B\beta^* - b\|}{\|\beta^*\|}$ was the size of optimal backward error to system of equation 1.2. Using definition of absolute condition number for the matrix B as

$$K^{(abs)}(B) = \limsup_{\hat{\beta} \to \beta^*} \frac{\|\hat{\beta} - \beta^*\|}{\mu(\hat{\beta})}$$, it was deduced from [14] that

$$K^{(abs)}(B) = \lim_{\hat{\beta} \to \beta^*} \frac{\|\hat{\beta} - \beta^*\| \|\beta^*\|}{\|B\hat{\beta} - b\|} = \limsup_{\delta\hat{\beta} \to 0} \frac{\|\delta\hat{\beta}\| \|\beta^*\|}{\|B\delta\hat{\beta}\|} = \|B^{-1}\| . \|\beta^*\| .$$

As a result, the relative condition number is defined to be the quantity

$$K(B) = \frac{\|B\|}{\|\beta^*\|} K^{((abs))}(B) = \|B^{-1}\| . \|B\| .$$

In what follows, given the right hand side vector b, the condition number for the solution space $\hat{\beta}$ is defined by the equation

$$K_{\hat{\beta}} = \frac{\|B^{-1}\| \|\hat{\beta}\|}{\|b\|} .$$

Let B be perturbed by ΔB for which the solution to the perturbed system of equation 1.1 exists. The perturbation in b is approximately bounded by

$$\frac{\|\delta b\| / \|b\|}{\|\Delta B\| / \|B\|} \approx K_b \leq K(B), \qquad (2.3a)$$

$$and,$$

$$\|\delta b\| \leq \frac{K(B)\|b\| . \|\Delta B\|}{\|B\|} \qquad (2.3b)$$

Assuming B is perturbed by ΔB and b is given, then the perturbation in $\hat{\beta}$ is approximately equal to:

$$\frac{\|\delta\beta\| / \|\hat{\beta}\|}{\|\Delta B\| / \|B\|} = K_\beta \leq K(B) \Rightarrow \|\delta\beta\| \approx \frac{K(B)\|\hat{\beta}\|\|\Delta B\|}{\|B\|} \qquad (2.4)$$

The product norm for the data space which holds for $\|\hat{\beta}\|_2$ as the solution space [15] is given by the equation:

$$\|(B,b)\|_F = \sqrt{\|B\|_F^2 + \|b\|_2^2} \ . \qquad (2.5)$$

As result of equation 2.5, Frobenius norm $\|B\|_F$ is defined as $\|B\|_F = \left[\sum_{i=1}^{m}\sum_{j=1}^{n}|b_{ij}|^2\right]^{\frac{1}{2}}$ while the P-norm is $\|B\|_P = \sup_{\beta\neq 0} \frac{\|B\hat{\beta}\|_P}{\|\hat{\beta}\|_P}$ for $P = 1,2,\infty$. We note in passing that both the Frobenius and P-norms satisfy certain inequalities in the form:

$$\|B\|_2 \leq \|B\|_F \leq \sqrt{n}\,\|B\|_2 \ , \qquad (2.6)$$

$$\text{Max } |b_{ij}| \leq \|B\|_2 \leq \sqrt{mn} \ \max |b_{ij}| , \qquad (2.7)$$

$$\|B\|_2 \leq \sqrt{\|B\|_1 . \|B\|_\infty} \ . \qquad (2.8)$$

Meanwhile, the reciprocal of condition number is known to be equal to the distance to the nearest singular matrix for all structured perturbations. Most importantly, [13] used the Bauer-Skeel condition number for a weighted matrix to show that

$$Cond_{BS}(B,E) = \left\| \, |B^{-1}| \ E \, \right\| , \qquad (2.9)$$

E, being defined as $|B|$. Equation (2.9) relates condition number with spectral radius by the equation:

$$Cond(B,E) = \rho(|B^{-1}|E) = \overset{\text{inf } cond_\infty}{D_1, D_2} \ (D_1 B D_2) . \qquad (2.10)$$

D_1, D_2 , appearing in equation 2.11 is diagonal scaling and ρ is the spectral radius. It follows from reasoning deduced in [12] that

$$\frac{1}{\rho\left(\left|B^{-1}\right|E\right)} \leq \delta(B,E) < \frac{\left(3+2\sqrt{2}\right)}{\rho\left(\left|B^{-1}\right|E\right)} \ . \tag{2.11}$$

The expression $\delta(B,E)$ in equation 2.11 is the familiar singular value of (B,E) .Equation 2.11 showed the extent to which a matrix that is not strongly regular can become singular if an attempt is made to increase the radius of this matrix by a factor of $\left(3+2\sqrt{2}\right)$. In any case, [12] warned that the factor $\left(3+2\sqrt{2}\right)$ cannot be replaced by 1 since computationally**,** $\delta(|B|,E) = \dfrac{n}{\rho\left(\left|B^{-1}\right|\,|B|\right)}$.

3 Backward Perturbation Error Bound for Least Squares Equation

The smallest Frobenius perturbation matrix ΔB for the matrix B that makes a given non-zero approximate solution $\hat{\beta}$ into a solution space of the perturbed problem $\min_{\hat{\beta}} \left\| b - (B+\Delta B)\beta\hat{\beta} \right\|_2$ was estimated [16,17] to be

$$\chi_F^{(LS)}(\hat{\beta}) = \left(\frac{\|r\|_2^2}{\left\|\hat{\beta}\right\|_2^2} + \min\{0,\lambda\} \right)^{\frac{1}{2}} \tag{3.1}$$

Where,

$$\lambda = \lambda_{\min}\left(B - \frac{rr^T}{\left\|\hat{\beta}\right\|_2^2} \right) \tag{3.2}$$

The number λ is the smallest singular value of B. The lower bound for equation 3.1 was found to be

$$\frac{1}{\sqrt{2}}\chi_F^{(LS)}(\hat{\beta}) \leq \chi_2^{(LS)} \leq \chi_F^{(LS)}(\hat{\beta}) \tag{3.3}$$

More complicated upper error bound for the least squares solution set to problem 1.1 was later found in [18] to be

$$\left(\chi_2^{(LS)}(\hat{\beta})\right)^2 = H \frac{\left(r^T B \hat{\beta}\right)^2}{\left\|\hat{\beta}\right\|_2^2 \left(\left\|B\hat{\beta}\right\|_2^2 + \|r\|_2^2\right)} \tag{3.4}$$

Wherefrom, it was defined that:

$$H = \frac{2}{1+\sqrt{1-\eta}} \ , \tag{3.5}$$

$$\eta = \frac{4\left(r^T B \hat{\beta}\right)^2}{\left(\left\|B\hat{\beta}\right\|_2^2 + \|r\|_2^2\right)^2} \tag{3.6}$$

Two conditions were adduced for least squares problem to be always ill-conditioned in [19], namely, either b could be nearly orthogonal to the column space of B, or, B itself is ill-conditioned.

In what follows, assuming that $BB^{/}\Delta B = \Delta B,$ and $B^{/}B(\Delta B)^{/} = (\Delta B)^{/}$. Let the range of $((\Delta B)^{/}) \subseteq range(B^{/})$ and $range(\Delta B) \subseteq$ range (B) , then via truncated Taylor Series expansion, the perturbed Covariance matrix $(B + \Delta B)^{/}$ is approximated [20] by the equation

$$(B + \Delta B)^{-1} = B^{-1} - B^{-1}(\Delta B)B + 0(\varepsilon^2)\ . \tag{3.8}$$

Since it was known that

$$\hat{\beta} + \delta\hat{\beta} = (B + \Delta B)^{-1}(b + \delta b) = \hat{\beta} + B^{-1}\delta b - B^{-1}(\Delta B)Bb + 0(\varepsilon^2), \tag{3.8}$$

and,

$$\delta b = -\frac{\|b\|}{\|B\|\left\|\hat{\beta}\right\|}\Delta B\hat{\beta}\ , \tag{3.9}$$

It follows that $\delta\hat{\beta}$ **is** approximated by the equation

$$\delta\hat{\beta} = -B^{-1}\Delta B\hat{\beta}\left(1+\frac{\|b\|}{\|B\|\|\hat{\beta}\|}\right). \quad (3.10)$$

Thus the condition number for the solution set is

$$K(B,b) = \|B^{-1}\|\|B\| + \|B^{-1}\|.\frac{\|b\|}{\|\hat{\beta}\|} \approx \|B^{-1}\|\|B\| + \|B^{-1}\|\frac{\|B\hat{\beta}\|}{\|\hat{\beta}\|} \quad (3.11)$$

4 Computability of Reachable Sets

As said earlier at the beginning of the paper, computability set is a very important issue in nonlinear dynamic and control theory, [21,22]. Reach-ability problem comes in different flavours, depending on the nature of algorithms in use. For instance, given a diffeomorphism which transforms a nonlinear system to a linear system, the approximate estimates for the solution set may be considered in the form of ellipsoids which depends on the operator. Various values of parameters will lead to a family of ellipsoids whose intersection is the exact reach set.

As it were, one can apply ellipsoid on linear system to obtain the reach set where the inverse operator becomes very necessary to approximate the nonlinear system 1.1 for the convex bodies. The only worry one has is ‘’ which is the best operator to be used as a tool for this objective?’’. Thus we need the best selection of interval Hull $\sum(B,b)$ as solution set to the given problem 1.1.

After transforming system 1.1 into an equivalent linear system1.2 an introduction of some kind of affliction as data noise into the parameters describing the interval linear system was established which hereafter, will be referred to as parametrised linear interval system in the form:

$$B(p).\beta = b(p) \quad (4.1)$$

Where $B(p)\in R^{m\times n}$ and $b(p)\in R^{n}$ all depend on parameter vector $p\in R^{k}$, and p varies within a range $[p]\in IR^{k}$, the set of solution to all $B(p).\beta = b(p),\ p\in[p]$.Because of [10], the solution set is given by the equation

$$\sum .^{p} = \sum \left(B(p), b(p), [p]\right) = \left\{ \hat{\beta} \in R^{n} \,\middle|\, B(p).\hat{\beta} = b(p) \right\}, (\, for \;\; some \;\; p \in [p]) \quad (4.2)$$

Where,

$B(p)$ and b(p) are defined by

$$b_{i,j}(p) = b_{i,j}{}^{(0)} + \sum_{v=1}^{m} p_v b_{i,j}{}^{(v)} \quad , \text{(i,j=1,1,…,n)}$$

$$b_i(p) = b_i^{(0)} + \sum_{v=1}^{m} p_v b_i{}^{(v)} \quad .$$

The following theorem is useful for adoption in our work.

Theorem 4.1[11]. Let $B(p).\beta = b(p)$ with $B(p) \in R^{n\times n}, b(p) \in R^{n}, p \in R^{k}$ be a parametrised linear system, where $B(p), b(p)$ are given. Assuming $R \in R^{n\times n}, [W] \in IR^{n}, \beta \in R^{n}$, define $[N] \in IR^{n}, [C] \in IR^{n\times n}$ by

$$[N]_i = R.\left(b_v{}^{(0)} - B_v{}^{(0)}\bar{\beta}\right) + \sum_{v=1}^{k} [p_v]\left(Rb^{(v)} - RB^{(v)}.\bar{\beta}\right).$$

$$[C(p)]) = I - R.B_v{}^{(0)} - \sum_{v=1}^{k} [p_v]\left(R.B^{(v)}\right).$$

Define $[V] \in IR^{n}$ by means of the following iteration enclosure

$$1 \le i \le n : [V_i] = \{[N] + [C].[U]\}_i$$

$$[U] = \left(V_1, V_2, ..., V_{i-1}, W_i, ..., W_n\right)',$$

If, $[V] \subset [W]$, then R and every matrix $B(p), p \in [p]$ are regular, and for every $p \in [p]$, the unique solution

$$\hat{\beta} = B^{-1}(p).b(p) \;, \quad (4.3)$$

satisfies $\hat{\beta} = \beta_c + [V]$, with $[\Delta] = \{[C].[V] \in IR^{n}\}$ the radius of Krawczyk operator and the solution set $\sum .^{p}$ as defined by equation 4.2, the following inner and outer estimation hold true

$$[\beta_c - \inf([N]) + \sup([\Delta]), \beta_c + \sup([N]) + \inf([\Delta]) \subseteq [\inf(\sum .^{\hat{\beta}}), \sup(\sum .^{\hat{\beta}})] \quad (4.4)$$

Because of the fruitfulness of above detailed discussions so far, we are inspired to invoke the procedures described in [23] as a tool to crop the corner point solution of $\sum(B,b)\bigcap O_k$, where O is the sign vector $s=(s_i)\in S^n$, $(s_i \in \{-1,+1\}$, i=1(1)n, it corresponds to the signs of the components of an interior point of O called the orthant, [23,24] for example. The exact solution to the parameterised linear interval system [23] of equation 4.1 obtained from least squares system 1.1 can be represented in the form:

$$\sum(B,b)\cap O=\bigcap_{i=1}^{n}\left(\underset{-}{B_i}\cap \bar{B_i}\right)\bigcap O. \tag{4.5}$$

The $\underset{-}{B_i},\bar{B_i}$ appearing in equation 4.5 are half spaces computable systems of linear inequalities and they are dependent on the choice of orthant O (with 2^n $possibilities)$. Following ideas expressed in [23,24,25],exact solution set of the linear interval system of equation 4.1 assuming iterative methods are used will be in the form:

$$\sum(B,b)=\bigcup_{k=1}^{2^n}\left[\ \left(\bigcap_{i=1}^{n}\left(\underset{-}{B}_i^k\cap \bar{B}_i^k\right)\cap O_k\right)\ \right] \tag{4.6}$$

The expression in the right hand side bracket [.], may either be ϕ or a convex polytope. As a result, we are led to the following theorem, an extension of Oettli-Prager theorem [4,6] which narrows closely what is discussed in equations 4.5 and 4.6.

Theorem4.2,[26]. Let $B=[B_c-\Delta B, B_c+\Delta B]$ be an $n\times n$ interval matrix, $b=[b_c-\delta, b_c+\delta b]$ an interval n-vector, where ΔB and δb are as defined in section 1. Let Z be a subset of S_n having the following properties:

(i) $\text{sgn}(\beta_0)\in Z$ for some $\beta_0\in\sum(B,b)$,
(ii) for each $z\in Z$ the systems of inequalities:

$$(QB_c-I)O_z\geq|Q|\Delta B, \tag{4.7}$$

$$(QB_c-I)O_{-z}\geq|Q|\Delta B\ , \tag{4.8}$$

have matrix solutions Q_z and Q_{-z}, respectively.

(iii) if $z\in Z$, $Q_{-z}b_c-|Q_{-z}|\delta\leq Q_z b_c+|Q_z|\delta$, and
$(Q_{-z}b_c-|Q_{-z}|\delta)_j(Q_z b_c+|Q_z|\delta)_j\leq 0$

for some j, then $z - 2z_j e_j \in Z$.

Then, there holds good that interval matrix B is regular, and furthermore, the interval Hull solution set

$$\sum(B,b) \subseteq \bigcup_{z \in Z_0} [\underline{\hat{\beta}}_z, \overline{\hat{\beta}}_z], \tag{4.9}$$

converges on the dense subset of equation 4.6, where

$$Z_0 = \left\{ z \in Z \;\middle|\; \sum(B,b) \cap R_z^n \neq \phi \right\} \tag{4.10}$$

Let us note that an improvement on the computed results from methods 4.5 and 4.10 can be further tightened using Rump/Krawczyk method [23]

$$Rb_c + (I - RB)[U] \subset \mathrm{int}(U) \text{ , } ([U \supset \sum(B,b)) \tag{4.11}$$

5 Numerical Example

Problem 1.1:
As an illustration, the Planetary elliptical orbit of ten observations of its position in the (X,Y) plane is considered based on the sample data taken from [27] as displayed in Table 1.

Table 1

X	Y
1.02	0.39
0.95	0.32
0.87	0.27
0.77	0.22
0.67	0.18
0.56	0.15
0.44	0.13
0.30	0.12
0.16	0.13
0.01	0.15

Take interval uncertainty vector as data noise to be $[p] = 0.0005[e_i]$, where $e_i = \{-1,1\}_{i=1}^m$ for the dependent variable Y_i and independent variable X_i where i=1,2,3,…,m.

Using cubic polynomial fit, result for the Least squares problem 1 assuming without noise in the data set is computed with MATLAB 2007 Windows version as

$$\hat{\beta} = (0.1550, -0.2578, 0.4694, -0.0295)'.$$

Similarly, the following results are displayed in Tables 2 and 3 as approximate solutions to the given problem 1.1 with the presence of noise as parameter in the data set.

Table 2. Showing results computed from problem1.1 based on theorem [10]

Results from modi-Fied rump/ popova Method [10,11] for $[\hat{\beta}]$
[0.1549994,0.15500067]
[-0.25780655,-0.2577921]
[0.46940375,0.46940565]
[-0.02950075,-0.0294949]

Table 3. Showing results computed from method 4.5 and improved results with rump/krawczyk method

Corner point solution with Cropping from method 4.5 for $[\hat{\beta}]$	Improved results with rump/Krawczyk method [10] for $[\hat{\beta}]$
[0.1553,0.1553]	[0.1552891,0.1553112]
[-0.2611,-0.2620]	[-0.2617332,-0.2614201]
[0.4772,0.4803]	[0.4787567,0.4790004]
[-0.0367,-0.0342]	[-0.0355532,-0.0353243]

As a consequence of what has been computed as approximate solution for $\hat{\beta}$, the value of $\frac{\left\| \beta_c - \hat{\beta} \right\|}{\left\| \hat{\beta} \right\|}$ was evaluated and the loss function for modified method [10,11] found to be [6.8446, 6.8446].

In addition, the condition number for the approximate solution without noise in the data using equation 3.11 was found to be 1.1684×10^4.

Polynomial fit of order three was used for modified method [10,11] and is given in the form:

$$Y = \beta_0 + \beta_1 X + \beta_2 X^2 + \beta_3 X^3$$
$$= [0.1549994, 0.15500067] + [-0.25780655, -0.2577921]X + [0.46940275, 0.46940565]X^2 +$$
$$[-0.02950075, -0.0294949]X^3.$$

To demonstrate the faithfulness of our implementation to the described method, we take as an example, arbitrary value of X =0.12 and computed Y = [0.1308, 0.1308] for modified method [10,11]. This coincides within the values of the vector Y from the given data set. The same analogy goes for Table 3 as given in equation 4.5. All results were computed using MATLAB Version 2007.

6 Conclusion

The paper considered the roles of condition number in the studies of least squares interval equations using polynomial fit of order three. In the paper, possible areas of applications of least squares were highlighted, for instance, extension to the backward perturbation errors in least squares problems also discussed. A diffeomorphism was introduced into system 1.1 which relates the reachability theory for the computable set in the resulting linear interval system from which, an ellipsoid may be applied. Here, in the paper, a section of the theorem due to [10], as exemplified by Popova in [11], was adopted which provides validated bounds to the linear interval system obtained from the Least squares equation. The loss function was computed based on the approximate results from Table 2.

As can be expected, the computed results in Table 2 approximated sufficiently close to the theoretical floating point results obtained in the absence of noise in the data using MATLAB 2007.

In addition, theoretical approximation technique for cropping corner points solution in the sense of [23,24,25] was adopted in the paper to advance the approximate solution. This involves solving systems of inequalities. The Rump/Krawczyk method was used to narrow further the computed approximate result as showed in Table 3. Further insight was given on how to estimate Y based on the given value of input data in order to further ascertain the correctness or otherwise of computed approximate solution from the described numerical method.

Competing interests

Author has declared that no competing interests exist.

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