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A New Approach to $(\in, \in \vee q)$-Fuzzy Subgroup Theory via Soft Sets

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Abstract

In this paper, we give the notion of $(\in, \in \vee q)$-fuzzy soft group and introduce its some properties and structural characteristics. Moreover, the definition of $(\in, \in \vee q)$-fuzzy normal soft group is given and some of its basic properties are studied. Also, the theorem regarding of homomorphic image and homomorphic pre-image of a $(\in, \in \vee q)$-fuzzy soft group under a fuzzy soft function is given.

Keywords: Fuzzy soft set, Fuzzy soft group, $(\in, \in \vee q)$-fuzzy soft group.
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1 Introduction

To solve complicated problems in economics, engineering, environmental science and social science, methods in classical mathematics are not always successful because of various types of uncertainties present in these problems. There are some known approaches to describing uncertainty such as probability theory, fuzzy set theory [1], rough set theory [2], and other mathematical tools. The

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concept of fuzzy set theory was introduced by Zadeh [1]. This concept was used in topology and analysis by many researchers, see for instance ([3], [4], [5]). The concept of soft sets which is a new mathematical tool was firstly introduced by Molodtsov [6]. At present, works on the soft theory are progressing rapidly. Maji et al. [7] described the application of soft set theory to a decision-making problem. Maji et al. [8] also studied several operations on the theory of soft sets.

Some researches have studied algebraic properties of soft sets. Initially, Aktaş and Çağman [9] defined the notion of soft groups and derived their basic properties using Molodtsov's definition of the soft sets. Feng et al. [10] introduced the notions of soft semirings, soft ideals and idealistic soft semirings, and then investigated some properties of their. Acar et al. [11] defined soft rings, and introduced basic notions of soft rings. Çelik et al. [12] defined some new binary relation on soft sets, and also they investigated some new properties of soft rings. Yamak et al. [13] introduced the notion of soft hypergroupoids.

Many studies with regard to algebraic structures of fuzzy soft sets were made. Firstly, Maji et al. [14] defined fuzzy soft set, and established some results on them. Jin-liang et al. [15] defined the operations on fuzzy soft groups, and proved some results on them. Aygünoğlu and Aygün [16] gave the concept of fuzzy soft group, and defined fuzzy soft function and fuzzy soft homomorphism. Lastly, İnan and Öztürk [17] introduced the fuzzy soft ring, and $(\in, \in \vee\, q)$-fuzzy soft subring. They also studied some of their basic properties. Soft sets and fuzzy soft sets have a rich potential for applications. Some applications are presented in ([6], [7], [18], [19], [20]).

The idea of quasi-coincidence of a fuzzy point with a fuzzy set [21] has played a vital role in generating some different types of fuzzy subgroups, called α, β-fuzzy subgroups, introduced by Bhakat and Das [22]. In particular, an $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld fuzzy subgroup.

The rest of this paper is organized as follows. In the first two sections as preliminaries, we give the concepts of fuzzy subset, soft set, fuzzy soft set and $(\in, \in \vee q)$-fuzzy subgroup. In Section 3, we introduce $(\in, \in \vee q)$-fuzzy soft group and study its characteristic properties. Also, we give the definition of a $(\in, \in \vee q)$-fuzzy normal soft group and study some of its basic properties. Moreover, we give the theorem regarding of image and pre-image of a $(\in, \in \vee q)$-fuzzy soft group under a fuzzy soft function.

2 Preliminaries

Definition 2.1. [1] A *fuzzy subset* λ of G is defined as a map from G to $[0, 1]$. The family of all fuzzy subsets of G is denoted by $\mathcal{F}(G)$. The following are most popular operations on fuzzy subsets:
$\forall \lambda, \mu \in \mathcal{F}(G), \nu \in \mathcal{F}(H)$ and $x \in G, y \in H$;

$$\begin{aligned}
(\lambda \vee \mu)(x) &= \lambda(x) \vee \mu(x); \\
(\lambda \wedge \mu)(x) &= \lambda(x) \wedge \mu(x); \\
(\lambda \times \nu)(x, y) &= \lambda(x) \wedge \nu(y); \\
(\lambda \cdot \mu)(x) &= \bigvee_{x=a \cdot b} (\lambda(a) \wedge \mu(b)); \\
\lambda^{-1}(x) &= \lambda(x^{-1}); \\
f(\lambda)(y) &= \begin{cases} \bigvee_{f(a)=y} \lambda(a) & \text{if} \quad f^{-1}(y) \neq \emptyset, \\ 0 & \text{otherwise.} \end{cases} \\
f^{-1}(\nu)(x) &= \nu(f(x))
\end{aligned}$$

$\lambda \leq \mu$ if and only if $\lambda(x) \leq \mu(x)$ for all $x \in G$. For $T \subseteq G$, $\chi_T \in \mathcal{F}(G)$ is called *characteristic function* of T, and defined by $\chi_T(x) = 1$ if $x \in T$ and $\chi_T(x) = 0$ otherwise for all $x \in G$.

Definition 2.2. [21] A fuzzy subset λ of G of the form

$$\lambda(y) = \begin{cases} t(\neq 0) & \text{if} \quad y = x \\ 0 & \text{if} \quad y \neq x \end{cases}$$

is said to be a *fuzzy point* with support x and value t and is denoted by x_t.

Definition 2.3. [21] A fuzzy point x_t is said to *belong to* (resp. *be quasi-coincident with*) a fuzzy set λ, written as $x_t \in \lambda$ (resp. $x_t q \lambda$) if $\lambda(x) \geq t$ (resp. $\lambda(x) + t > 1$).

If $x_t \in \lambda$ or $x_t q \lambda$, then we write $x_t \in \vee q \lambda$.

Definition 2.4. Let λ be a fuzzy subset of G. Then, for all $t \in [0, 1]$, the set $\lambda_t = \{x \in G : \lambda(x) \geq t\}$ is called level subset of λ.

Definition 2.5. [23] Let G be a group and λ be a fuzzy subset of G. Then, λ is called a *fuzzy subgroup* of G if

(i) $\lambda(xy) \geq \lambda(x) \wedge \lambda(y)$,

(ii) $\lambda(x^{-1}) \geq \lambda(x)$

for all $x, y \in G$.

Definition 2.6. [8] Let U be a common universe set, E be a set of parameters and $A \subseteq E$. Then a pair (F, A) is called a *soft set* over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$.

Definition 2.7. [14] Let U be an inital universe set, E be a set of parameters and $A \subseteq E$. Then, a pair (F, A) is called a fuzzy soft set over U, where F is mapping given by $F : A \to \mathcal{F}(U)$.

Definition 2.8. [14] Let (F, A) and (H, B) be two fuzzy soft sets over a common universe U. Then we say that (F, A) is a *fuzzy soft subset* of (H, B) if

(i) $A \subseteq B$

(ii) $F(a) \leq H(a)$ for all $a \in A$. In this situation denoted by $(F, A)\widetilde{\subseteq}(H, B)$.

Definition 2.9. [14] For two fuzzy soft sets (F, A) and (H, B) over a common universe U , we say that (F, A) is a *fuzzy soft equal* of (H, B), denoted by $(F, A)\widetilde{=}(H, B)$, if $(F, A)\widetilde{\subseteq}(H, B)$ and $(H, B)\widetilde{\subseteq}(F, A)$.

Definition 2.10. [24] Let R be a common universe, E be a set of parameters and $A \subseteq E$.

(a) (F, A) is called a *relative null fuzzy soft* set, denoted by $\widetilde{\emptyset}_A$, if $F(a) = 0_R$ for all $a \in A$.

(b) (G, A) is called a *relative whole fuzzy soft* set, denoted by $\widetilde{\Omega}_A$, if $G(a) = R$ for all $a \in A$.

The relative whole fuzzy soft set with respect to the set of parameters E is called the *absolute fuzzy soft set* over R and simply denoted by $\widetilde{\Omega}_E$. In a similar way, the relative null fuzzy soft set with respect to the E is called the *null fuzzy soft set* over R and is denoted by $\widetilde{\emptyset}_E$.

Definition 2.11. [14] Let (F, A) and (H, B) be two fuzzy soft sets over a common universe U. Then,

(1) The $\wedge$-*intersection* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\wedge}(H, B)$ over U, where $C = A \times B$, and $Q(a, b) = F(a) \wedge H(b)$ for all $(a, b) \in A \times B$.

(2) The $\vee$-*union* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\vee}(H, B)$ over U, where $C = A \times B$, and $Q(a, b) = F(a) \vee H(b)$ for all $(a, b) \in A \times B$.

In addition to the above, we can give the following definition.

Definition 2.12. [18] Let (F, A) and (H, B) be two fuzzy soft sets over a common universe U. Then,

(1) The *extended union* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\cup}(H, B)$ over U, where $C = A \cup B$ and

$$Q(c) = \begin{cases} F(c) & \text{if} \quad c \in A \backslash B \\ H(c) & \text{if} \quad c \in B \backslash A \\ F(c) \vee H(c) & \text{if} \quad c \in A \cap B \end{cases}$$

for all $c \in C$.

(2) The *restricted union* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\cup}_{\Re}(H, B)$ over U, where $C = A \cap B$, and $Q(c) = F(c) \vee H(c)$ for all $c \in C$.

(3) The *extenden intersection* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\cap}_{\varepsilon}(H, B)$ over U, where $C = A \cup B$, and

$$Q(c) = \begin{cases} F(c) & \text{if} \quad c \in A \backslash B \\ H(c) & \text{if} \quad c \in B \backslash A \\ F(c) \wedge H(c) & \text{if} \quad c \in A \cap B \end{cases}$$

for all $c \in C$.

(4) The *restricted intersection* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\cap}(H, B)$ over U, where $C = A \cap B$, and $Q(c) = F(c) \wedge H(c)$ for all $c \in C$.

(5) The *cartesian product* of fuzzy soft sets (F, A) and (H, B) is defined as the fuzzy soft set $(Q, C) = (F, A)\widetilde{\times}(H, B)$ over U, where $C = A \times B$, and $Q(a, b) = F(a) \times H(b)$ for all $(a, b) \in A \times B$.

Now, we define a binary operation on fuzzy soft sets in the following way:

Suppose that $\oplus$ is a binary operation on $\mathcal{F}(E)$, and $\otimes$ is a binary operation on $\mathcal{F}(U)$. Then for any two fuzzy soft set (F, A) and (H, B) over U, $(F, A) \oplus_{\otimes} (H, B)$ is defined as the fuzzy soft set (Q, C) where $C = A \oplus B$ and

$$Q(c) = \begin{cases} F(c) & \text{if} \quad c \in A \backslash B \\ H(c) & \text{if} \quad c \in B \backslash A \\ F(c) \otimes H(c) & \text{if} \quad c \in A \cap B \\ 0_U & \quad \text{otherwise} \end{cases}$$

for all $c \in C$. Here we describe a general binary operation on fuzzy soft sets obtained as a special case of the some binary relations such as extended union, restricted union, extended intersection,restricted intersection, restricted sum, extended sum, restricted product and extended product for the $\oplus \in \{\cup, \cap\}$ and $\otimes \in \{\vee, \wedge\}$.

Definition 2.13. Let $\{(H_i, B_i) \mid i \in \Lambda\}$ be a family of fuzzy soft sets over a common universe U. Then,

(1) The intersection of the family (H_i, B_i), denoted by $\widetilde{\bigcap}_{i\in\Lambda}(H_i, B_i)$, is a fuzzy soft set (H, B) defined as:

$$B = \bigcap_{i\in\Lambda} B_i, \quad (\forall b \in B) \quad H(b) = \wedge_{i\in\Lambda} H_i(b).$$

(2) The $\wedge$-intersection of the family (H_i, B_i) , denoted by $\widetilde{\bigwedge}_{i\in\Lambda}(H_i, B_i)$, is a fuzzy soft set (H, B) defined as:

$$B = \prod_{i\in\Lambda} B_i, \quad (\forall (b_i)_{i\in\Lambda} \in B) \quad H((b_i)_{i\in\Lambda}) = \wedge_{i\in\Lambda} H_i(b_i).$$

(3) The $\vee$-union of the family (H_i, B_i), denoted by $\widetilde{\bigvee}_{i\in\Lambda}(H_i, B_i)$, is a fuzzy soft set (H, B) defined as:

$$B = \prod_{i\in\Lambda} B_i, \quad (\forall (b_i)_{i\in\Lambda} \in B) \quad H((b_i)_{i\in\Lambda}) = \vee_{i\in\Lambda} H_i(b_i).$$

Definition 2.14. [19] Let (F, A) and (H, B) be two fuzzy soft sets over G and K, respectively. Let $\phi : G \to K$, $\psi : A \to B$ be two functions. Then we say that the pair (ϕ, ψ) is a *fuzzy soft function* from (F, A) to (H, B) denoted by $(\phi, \psi) : (F, A) \to (H, B)$ if the following condition is satisfied: $\phi(F(x)) = H(\psi(x))$ for all $x \in A$. If ϕ and ψ are injective (resp. surjective, bijective), then (ϕ, ψ) is said to be injective (resp. surjective, bijective).

In this definition, If ϕ is homomorphism from G to K, then (ϕ, ψ) is said to be a *fuzzy soft homomorphism*, and that (F, A) is *fuzzy soft homomorphic* to (H, B). The latter is denoted by $(F, A) \sim (H, B)$. If ϕ is an isomorphism from G to K and ψ is a bijective mapping from A onto B, then we say that (ϕ, ψ) is a *fuzzy soft isomorphism* and that (F, A) is *fuzzy soft isomorphic* to (H, B). The latter is denoted by $(F, A) \simeq (H, B)$.

Definition 2.15. [19] Let (F, A) and (H, B) be two fuzzy soft sets over G and K, respectively. Let (ϕ, ψ) be a fuzzy soft function from (F, A) to (H, B) . Then,

(1) The image of (F, A) under the fuzzy soft function (ϕ, ψ), denoted by $(\phi, \psi)(F, A)$, is a fuzzy soft set over K defined by $(\phi, \psi)(F, A) = (\phi(F), B)$, where

$$\phi(F)(y) = \begin{cases} \bigvee_{\psi(x)=y} \phi(F(x)) & \text{if} \quad y \in Im\psi \\ 0_{R_2} & \text{otherwise} \end{cases}$$

for all $y \in B$.

(2) The pre-image of (H, B) under the fuzzy soft function (ϕ, ψ), denoted by $(\phi, \psi)^{-1}(H, B)$, is the fuzzy soft set over G defined by $(\phi, \psi)^{-1}(H, B) = (\phi^{-1}(H), A)$, where

$$\phi^{-1}(H)(x) = \phi^{-1}(H(\psi(x)))$$

for all $x \in A$.

Definition 2.16. [15] Let G be a group and (F, A) be a fuzzy soft set over G. Then (F, A) is called a *fuzzy soft group* over G if $F(a)$ is a fuzzy subgroup of G for all $a \in A$.

Definition 2.17. [22] A fuzzy subset λ of G is said to be an $(\in, \in \vee q)$-*fuzzy subgroup* of G if for all $x, y \in G$ and $t, r \in (0, 1]$,

(i) $x_t, y_r \in \lambda \Rightarrow (xy)_{M(t,r)} \in \vee q\lambda$, where $M(t, r)$ denote min(t, r)

(ii) $x_t \in \lambda \Rightarrow (x^{-1})_t \in \vee q\lambda$

Remark 2.18.

(I) The condition (i) in Definition 2.17 is equivalent to

(i′) $\lambda(xy) \geq M(\lambda(x), \lambda(y), 0.5)$ for all $x, y \in G$, and the condition (ii) in Definition 2.17 is equivalent to

(ii′) $\lambda(x^{-1}) \geq M(\lambda(x), 0.5)$ for all $x \in G$.

(II) The necessary and sufficient condition for a fuzzy subset λ to be a $(\in, \in \vee q)$-fuzzy subgroup of G is $\lambda(xy^{-1}) \geq M(\lambda(x), \lambda(y), 0.5)$ for all $x, y \in G$.

Definition 2.19. [22] An $(\in, \in \vee q)$-fuzzy subgroup λ of G is said to be $(\in, \in \vee q)$-*fuzzy normal subgroup* if for any $x, y \in G$ and $t \in (0, 1]$,
$x_t \in \lambda \Rightarrow (yxy^{-1})_t \in \vee q\lambda$.

Lemma 2.20. Let f be a homomorphism from G to K. Then,

(1) If λ is a $(\in, \in \vee q)$-fuzzy subgroup of G, then $f(\lambda)$ is a $(\in, \in \vee q)$-fuzzy subgroup of K,

(2) If ν is a $(\in, \in \vee q)$-fuzzy subgroup of K, then $f^{-1}(\nu)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G.

Lemma 2.21. Let λ and μ be two $(\in, \in \vee q)$-fuzzy subgroups of G. Then,

(1) $\lambda \cap \mu$ is a $(\in, \in \vee q)$-fuzzy subgroup of G,

(2) $\lambda \wedge \mu$ is a $(\in, \in \vee q)$-fuzzy subgroup of G,

(3) $\lambda \times \mu$ is a $(\in, \in \vee q)$-fuzzy subgroup of G.

3 $(\in, \in \vee q)$-Fuzzy Soft Groups

Let G and H be groups, A is any non-empty set and λ be a fuzzy subset of G.

Definition 3.1. Let (F, A) be a fuzzy soft set over G. Then, (F, A) is said to be a $(\in, \in \vee q)$-*fuzzy soft group* over G if $F(x)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G for all $x \in A$.

Example 3.2. Let $G = \{g_1, g_2, g_3, g_4\}$ be a group, $A = \{e_1, e_2, e_3\}$ and $F : A \to \mathcal{F}(G)$, $F(e_i) = \lambda_i : G \to [0, 1]$ be a set-valued function defined by

	g_1	g_2	g_3	g_4
$\lambda_1(g_i)$	0.6	0.4	0.7	0.4
$\lambda_2(g_i)$	0.7	0.5	0.8	0.5
$\lambda_3(g_i)$	0.5	0.3	0.6	0.3

Obviously (F, A) is a fuzzy soft set over G. Also, we see that since $\forall x, y \in G$ and $\forall 1 \leq i \leq 3$, $\lambda_i(xy^{-1}) \geq M(\lambda_i(x), \lambda_i(y), 0.5)$ condition hold, $F(x)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G for all $x \in A$. Therefore, (F, A) is a $(\in, \in \vee q)$-fuzzy soft group over G.

Theorem 3.3. If (F, A) and (H, B) be two $(\in, \in \vee q)$-fuzzy soft groups over G. Then, $(F, A)\widetilde{\cap}_\varepsilon(H, B)$ is a $(\in, \in \vee q)$-fuzzy soft group over G.

Proof. Let $(F, A)\widetilde{\cap}_\varepsilon(H, B) = (Q, C)$, where $C = A \cup B$ and $Q : A \cup B \to \mathcal{F}(G)$ is a mapping. Clearly, if $c \in A \backslash B$, then $Q(c) = F(c)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G, and if $c \in B \backslash A$, then $Q(c) = H(c)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G. If $c \in A \cap B$, then $Q(c) = F(c) \wedge H(c)$ is a $(\in, \in \vee q)$-fuzzy subgroup of G since the intersection of two $(\in, \in \vee q)$-fuzzy subgroups is a $(\in, \in \vee q)$-fuzzy subgroup over G. Hence, $(F, A)\widetilde{\cap}_\varepsilon(H, B)$ is a $(\in, \in \vee q)$-fuzzy soft group over G.

Theorem 3.4. If (F, A) and (H, B) be two $(\in, \in \vee q)$-fuzzy soft groups over G. Then, $(F, A)\widetilde{\cap}(H, B)$ is a $(\in, \in \vee q)$-fuzzy soft group over G.

Proof. It is similar to the proof of Theorem 3.3.

Theorem 3.5. Let (F, A) and (H, B) be two $(\in, \in \vee q)$-fuzzy soft groups over G. If $A \cap B = \emptyset$, then $(F,A)\widetilde{\cup}(H,B)$ is a $(\in,\in\vee q)$-fuzzy soft group over G.

Proof. Let $(F,A)\widetilde{\cup}(H,B)=(Q,C)$, where $C=A\cup B$ and $Q:A\cup B\to\mathcal{F}(G)$ is a mapping. Since $A\cap B=\emptyset$, it follows that either $c\in A\backslash B$ or $c\in B\backslash A$ for all $c\in A\cup B$. If $c\in A\backslash B$, then $Q(c)=F(c)$ is a $(\in,\in\vee q)$-fuzzy subgroup of G, and if $c\in B\backslash A$, then $Q(c)=H(c)$ is a $(\in,\in\vee q)$-fuzzy subgroup of G. Hence, (Q,C) is a $(\in,\in\vee q)$-fuzzy soft group over G.

Theorem 3.6. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G. If $F(c)\leq H(c)$ or $H(c)\leq F(c)$ for all $c\in A\cap B$, then $(F,A)\widetilde{\cup}_{\Re}(H,B)$ is a $(\in,\in\vee q)$-fuzzy soft group over G.

Proof. Let $(F,A)\widetilde{\cup}(H,B)=(Q,C)$, where $C=A\cup B$. If $c\in A\backslash B$ or $c\in B\backslash A$ for all $c\in A\cup B$, then it is clear that $Q(c)$ is a $(\in,\in\vee q)$-fuzzy subgroup of G from the proof of Theore 3.5. If $F(c)\leq H(c)$ or $H(c)\leq F(c)$ for all $c\in A\cap B$, then $Q(c)=F(c)\vee H(c)$ is a $(\in,\in\vee q)$-fuzzy subgroup of G for all $c\in A\cap B$. Hence, (Q,C) is a $(\in,\in\vee q)$-fuzzy soft group over G.

Theorem 3.7. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G. Then, $(F,A)\widetilde{\wedge}(H,B)$ is a $(\in,\in\vee q)$-fuzzy soft group over G.

Proof. Let $(F,A)\widetilde{\wedge}(H,B)=(Q,C)$, where $C=A\times B$ and $Q(\alpha,\beta)=F(\alpha)\cap H(\beta)$ for all $(\alpha,\beta)\in A\times B$. Since $F(\alpha)$ and $H(\beta)$ are $(\in,\in\vee q)$-fuzzy subgroups of G for all $(\alpha,\beta)\in A\times B$, then the intersection $F(\alpha)\wedge H(\beta)$ is also $(\in,\in\vee q)$-fuzzy subgroup of G. Hence, we find that (Q,C) is a $(\in,\in\vee q)$-fuzzy soft group over G.

Theorem 3.8. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G. If $F(a)\leq H(b)$ or $H(b)\leq F(a)$, for all $(a,b)\in A\times B$, then $(F,A)\widetilde{\vee}(H,B)$ is a $(\in,\in\vee q)$-fuzzy soft group over G.

Proof. It is similar to the proof of Theorem 3.6.

Theorem 3.9. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G_1 and G_2, respectively. Then, $(F,A)\widetilde{\times}(H,B)$ is a $(\in,\in\vee q)$-fuzzy soft group over over $G_1\times G_2$.

Proof. Let $(F,A)\widetilde{\times}(H,B)=(Q,C)$, where $Q=A\times B$ and for all $(a,b)\in A\times B$, we have $Q(a,b)=F(a)\times H(b)$. Since (F,A) and (H,B) are $(\in,\in\vee q)$-fuzzy subgroups over G_1 and G_2, respectively, then $F(a)$ and $H(b)$ are $(\in,\in\vee q)$-fuzzy soft groups over G_1 and G_2 for all $(a,b)\in A\times B$. Also, for all $x\in G_1$ and $y\in G_2$, we write $(F(a)\times H(b))(x,y)=F(a)(x)\wedge H(b)(y)$. Hence, we obtain $Q(a,b)=F(a)\times H(b)$ is a $(\in,\in\vee q)$-fuzzy subgroup of $G_1\times G_2$ for all $(a,b)\in A\times B$. Consequently, (Q,C) is a $(\in,\in\vee q)$-fuzzy soft group over over $G_1\times G_2$.

Definition 3.10. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G. Then, (F,A) is called $(\in,\in\vee q)$-*fuzzy soft subgroup* of (H,B) if

(1) $A\subset B$

(2) $F(a)$ is a $(\in,\in\vee q)$-fuzzy subgroup of $H(a)$ for all $a\in A$.

Theorem 3.11. Let (F,A) and (H,B) be two $(\in,\in\vee q)$-fuzzy soft groups over G. If $F(x)\leq H(x)$ for all $x\in A$, then (F,A) is a $(\in,\in\vee q)$-fuzzy soft subgroup of (H,B).

Proof. It is straightforward.

Theorem 3.12. Let (F,A) be a $(\in,\in\vee q)$-fuzzy soft group over G, and $\{(H_i,B_i)\mid i\in I\}$ be a non-empty family of $(\in,\in\vee q)$-fuzzy soft subgroups of (F,A), where I is an index set. Then,

(1) $\widetilde{\bigcap}_{i\in I}(H_i,B_i)$ is a $(\in,\in\vee q)$-fuzzy soft subgroup of (F,A),

(2) $\widetilde{\bigwedge}_{i\in I}(H_i, B_i)$ is a ($\in, \in \vee q$)-fuzzy soft subgroup of (F, A),

(3) If $B_i \cap B_j = \emptyset$ for all $i, j \in I$, then $\widetilde{\bigvee}_{i\in I}(H_i, B_i)$ is a ($\in, \in \vee q$)-fuzzy soft subgroup of (F, A).

Proof. From Definitions 2.13 and Theorems 3.4, 3.5, the proofs of (1), (2) and (3) can be achived similarly.

Theorem 3.13. Let (F, A) and (H, B) be two ($\in, \in \vee q$)-fuzzy soft groups over G, and (F, A) be a ($\in, \in \vee q$)-fuzzy soft subgroup of (H, B). If f is a homomorphism from G to K, then $(f(F), A)$ and $(f(H), B)$ are both ($\in, \in \vee q$)-fuzzy soft groups over K and $(f(F), A)$ is a ($\in, \in \vee q$)-fuzzy soft subgroup of $(f(H), B)$.

Proof. Since f is a homomorphism from G to K, $f(F(x))$ and $f(H(y))$ are ($\in, \in \vee q$)-fuzzy subgroups of K for all $x \in A$, $y \in B$. Therefore, $(f(F), A)$ and $(f(H), B)$ are both ($\in, \in \vee q$)-fuzzy soft groups over K. If (F, A) is a ($\in, \in \vee q$)-fuzzy soft subgroup of (H, B), then $F(x)$ is a ($\in, \in \vee q$)-fuzzy subgroup of $H(x)$, and $f(F(x))$ is a ($\in, \in \vee q$)-fuzzy subgroup of $f(H(y))$ for all $x \in A$. We obtain $(f(F), A)$ is a ($\in, \in \vee q$)-fuzzy soft subgroup of $(f(H), B)$.

Definition 3.14. Let (F, A) be a fuzzy soft set over G. Then, (F, A) is said to be a ($\in, \in \vee q$)-*fuzzy normal soft group* over G if $F(x)$ is a ($\in, \in \vee q$)-fuzzy normal subgroup of G for all $x \in A$.

Theorem 3.15. Let (F, A) be a ($\in, \in \vee q$)-fuzzy soft group over G, and $\{(H_i, B_i) \mid i \in I\}$ be a non-empty family of ($\in, \in \vee q$)-fuzzy normal soft subgroups of (F, A), where I is an index set. Then,

(1) $\widetilde{\bigcap}_{i\in I}(H_i, B_i)$ is a ($\in, \in \vee q$)-fuzzy normal soft subgroup of (F, A),

(2) $\widetilde{\bigwedge}_{i\in I}(H_i, B_i)$ is a ($\in, \in \vee q$)-fuzzy normal soft subgroup of (F, A),

(3) If $B_i \cap B_j = \emptyset$ for all $i, j \in I$, then $\widetilde{\bigvee}_{i\in I}(H_i, B_i)$ is a ($\in, \in \vee q$)-fuzzy normal soft subgroup of (F, A).

Proof. It is similar to the proof of Theorem 3.12.

Theorem 3.16. Let (F, A) is a ($\in, \in \vee q$)-fuzzy soft group over G and (F, A) is fuzzy soft isomorphic to (H, B), then (H, B) is a ($\in, \in \vee q$)-fuzzy soft group over K.

Proof. Since (F, A) is ($\in, \in \vee q$)-fuzzy soft isomorphic to (H, B), that is $G \simeq K$ and $A \simeq B$, then $f(F(x)) = H(g(x))$ for all $x \in A$. So $H(y) = H(g(x)) = f(F(x))$ for all $y \in B$. From Lemma 2.20., we obtain $f(F(x))$ is a ($\in, \in \vee q$)-fuzzy subgroup of K and $H(y)$ ($\in, \in \vee q$)-fuzzy subgroup of K. Thus, (H, B) is a ($\in, \in \vee q$)-fuzzy soft group over K.

Theorem 3.17. Let (F, A) and (H, B) be two ($\in, \in \vee q$)-fuzzy soft groups over G and K, respectively. Let (ϕ, ψ) be a fuzzy soft group homomorphism from (F, A) to (H, B). Then,

(1) If $\phi : G \to K$ is a homomorphism of groups and ψ is a bijective mapping, then $(\phi(F), B)$ is a ($\in, \in \vee q$)-fuzzy soft group over K,

(2) If $\phi : G \to K$ is a homomorphism of groups, then $(\phi^{-1}(H), A)$ is a ($\in, \in \vee q$)-fuzzy soft group over G.

Proof. (1) Let $y \in B$. Since ψ is bijective, then there exist a unique $x \in A$ such that $\psi(x) = y$. Since $F(x)$ is a ($\in, \in \vee q$)-fuzzy subgroup of G_1, and ϕ homomorphism, then $\phi(F(x))$ is a ($\in, \in \vee q$)-fuzzy subgroup of G_2. Since ψ is bijective mapping, then $\phi(F)(y) = \phi(F(x))$ is a ($\in, \in \vee q$)-fuzzy subgroup over G_2. Consequently, $(\phi(F), B)$ is a ($\in, \in \vee q$)-fuzzy soft group over G_2.

(2) Since $\psi(x) \in B$ for all $x \in A$ and (H, B) is a ($\in, \in \vee q$)-fuzzy soft group over G_2, then $H(\psi(x))$ is a ($\in, \in \vee q$)-fuzzy subgroup of G_2 for all $x \in A$. Moreover its homomorphic inverse

image $\phi^{-1}(H(\psi(x)))$ is also a $(\in, \in \vee q)$-fuzzy subgroup of G_1 for all $x \in A$. Hence $(\phi^{-1}(H), A)$ is a $(\in, \in \vee q)$-fuzzy soft group over G_1.

4 Conclusion

In this study, we have introduced the notion of $(\in, \in \vee q)$-fuzzy soft groups. We have also defined the notion of $(\in, \in \vee q)$-fuzzy normal soft groups, and gave $(\in, \in \vee q)$-fuzzy soft group homomorphism. Moreover, we have investigated various properties of $(\in, \in \vee q)$-fuzzy soft groups. As future work, the application of the newly defined notions to the study of some algebraic structures (such as semigroups and modules) could be further explored.

Competing Interests

The author declares that no competing interests exist.

References

[1] Zadeh LA. Fuzzy sets. Information and Control. 1965;8:338-353.

[2] Pawlak Z. Rough sets. Int. J. Inf. Comp. Sci. 1982;23:341-356.

[3] Grabiec M. Fixed points in fuzzy metric space. Fuzzy Sets and Systems. 1988;27:385-389.

[4] Heipern S. Fuzzy mappings and fixed point theorems. J. Math. Anal. Appl. 1988;83:566-569.

[5] Kaleva O, Seikkala S. On Fuzzy metric spaces. Fuzzy Sets and Systems. 2008;12:215-229.

[6] Molodtsov D. Soft set theory - first results. Comput. Math. Appl. 1999;37:19-31.

[7] Maji PK, Roy AR, Biswas R. An application of soft sets in a decision making problem. Comput. Math. Appl. 2001;44:1077-1083.

[8] Maji PK, Biswas R, Roy AR. Soft set theory. Comput. Math. Appl. 2003;45:555-562.

[9] Aktaş, H. and Çağman, N. Soft sets and soft group. Information Science. 2007;177:2726-2735.

[10] Feng F, Jun YB, Zhao X. Soft semirings. Comput. Math. Appl. 2008;56:2621-2628.

[11] Acar U, Koyuncu F. Soft sets and soft rings. Comput. Math. Appl. 2010;59:3458-3463.

[12] Çelik Y, Ekiz C, Yamak S. A new view on soft rings. Hacettepe J. Math. and Stat. 2011;40(2):273-286.

[13] Yamak S, Kazancı O, Davvaz B. Soft hyperstructure. Comput. Math. Appl. 2007;62:797-803.

[14] Maji PK, Biswas R, Roy AR. Fuzzy soft sets. J. Fuzzy. Math. 2001;9(3):589-602.

[15] Jin-liang L, Rui-xia Y, Bing-xue Y. Fuzzy soft sets and fuzzy soft groups. Chinese Control and Decision Conference; 2008. doi:10.1109/CCDC.2008.4597801:2626-2629.

[16] Aygünoğlu A, Aygün H. Introduction to fuzzy soft groups. Comput. Math. Appl. 2009;58:1279-1286.

[17] İnan E, Öztürk MA. Fuzzy soft rings and fuzzy soft ideals. Neural Comput. Appl. 2011;21(1):1-8.

[18] Ali MI, Shabir M. Comments on De Morgan's law in fuzzy soft sets. International Journal of Fuzzy Mathematics. 2010;18(3):679-686.

[19] Çelik Y, Ekiz C,d Yamak S. Applications of fuzzy soft sets in ring theory. Ann. Fuzzy Math. Inform. 2013;5(3):451-462.

[20] Roy AR, Maji PK. A fuzzy soft set theoretic approach to decision making problems. J. Comput. Appl. Math. 2007;203:412-418.

[21] Ming PP, Ming LY. Fuzzy topology 1. Neighbourhood structures of a fuzzy point and Moore-Smith convergence. J. Math. Anal. Appl. 1980;(76):571-579.

[22] Bhakat SK, Das P. $(\in, \in \vee q)$- fuzzy subgroup. Fuzzy Sets Syst. 1996;80:359-368.

[23] Mordeson JN, Malik DS. Fuzzy Commutative algebra. World Scientific Publishing Co; 1998. Singapure.

[24] Ali MI, Feng F, Liu X, Min WH, Shabir M. On some new operations in soft set theory. Comput. Math. Appl. 2009;57:1547-1553.

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