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Recognition and Analyzing the Influence of Chaotic Oscillations on Markov Process Using Numerical Simulation Based on Qualitative Methods

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Author's contribution

The sole author designed, analyzed and interpreted and prepared the manuscript.

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ABSTRACT

This paper focuses on analyzing the influence of chaotic oscillations on Markov process using numerical simulation based on qualitative methods to detect different types of chaotic oscillations depending on Transition-Probability matrix of Markov process. The dissipative chaotic system whose evolution is given by set of ordinary differential equations had been considered and the results of limited probabilities of different types of chaotic processes had been calculated and verified.

Keywords: Markov process; chaotic oscillations; chaotic processes; dissipative chaotic system; perturbed chaos.

1. INTRODUCTION

Many real processes in engineering, economics, biology and medicine due to their nature are endowed with the so-called Markov property, which can be described as: the future is determined by the present [1,2]. An important aspect in the theory of Markov systems is the study of processes in which there is stabilization [12,7,18]. Under stabilization we can understand

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process' property; where at $t \to t_0$, $(t_0 \leq \infty)$ its main characteristics can accept certain values.

For Markov process, this means that there is a set of stationary probabilities, which tend over time corresponding to the probability of finding the process in their states. Such a process we can consider as a dynamic system having a stable equilibrium point [4,6,16].

Let us assume that matrix P is as a transition probability of the process with n states, which is in general, depends on time which defines system evolution.

$$P(t) = \begin{bmatrix} p_{11}(t) & p_{12}(t) & \cdots & p_{1n}(t) \\ p_{21}(t) & p_{22}(t) & \cdots & p_{2n}(t) \\ \vdots & \vdots & & \vdots \\ p_{n1}(t) & p_{n2}(t) & \cdots & p_{nn}(t) \end{bmatrix} \quad (1)$$

Then on each time step matrix $P(t)$ will look like:

$$\tilde{P}(t) = \begin{bmatrix} p_{11}(t)+\Delta_{11}(t) & p_{12}(t)+\Delta_{12}(t) & \cdots & p_{1n}(t)+\Delta_{1n}(t) \\ p_{21}(t)+\Delta_{21}(t) & p_{22}(t)+\Delta_{22}(t) & \cdots & p_{21}(t)+\Delta_{2n}(t) \\ \vdots & \vdots & & \vdots \\ p_{n1}(t)+\Delta_{n1}(t) & p_{n2}(t)+\Delta_{n2}(t) & \cdots & p_{nn}(t)+\Delta_{nn}(t) \end{bmatrix} \quad (2)$$

Where

$$\forall i \sum_{j=1}^{n} p_{ij}(t) + \Delta_{ij}(t) = 1$$

At each time step t_k the vector of unconditional probabilities $p(t) = (p_1(t), p_2(t), \dots, p_n)$ of the perturbed system can be defined as $p(t_{k)} = p(t_{k-1}\tilde{p}(t_{k-1})$ for a given initial distribution $p(0)$.

Let the perturbing factor of Markov system to be as one component of continuous chaotic process

$$X(t) = \big(x_1(t), x_2(t), \dots, x_m(t)\big) \quad (3)$$

Chaos is a special type of behavior of a deterministic system in the steady-state regime, although the evolution of this system is uniquely determined by the dynamic laws, while its dynamics is stochastic.

In this paper we consider the dissipative chaotic systems whose evolution is given by a set of ordinary differential equations.

Phase trajectories of such systems are presented in an infinite form, where lines never intersected and at $t \to \infty$ trajectory does not leave the closed area. Such systems are common in fluid dynamics, mechanics, plasma physics, etc. [3,5,11].

So, we are observing the evolution of a Markov process, which affected by the chaotic oscillations, presented in a temporal sequence of vectors:

$$p(t_k) = \big(p_1(t_k), p_2(t_k), \dots, p_n(t_k)\big) \quad (4)$$

Fig. 1 shows the components of the vector $p(t)$ of Markov system with dimension $n = 3$, which is under the influence of chaotic process. Dashed lines represent the stationary probability of the unperturbed process.

2. DETERMINING WHAT KIND OF CHAOS DISTURB MARKOV PROCESS

To determine what kind of chaos disturb Markov process, let us consider one remarkable feature of the strange attractor which allows us to recover it based on a sequence of samples obtained from only one component of its state at a given period of time.

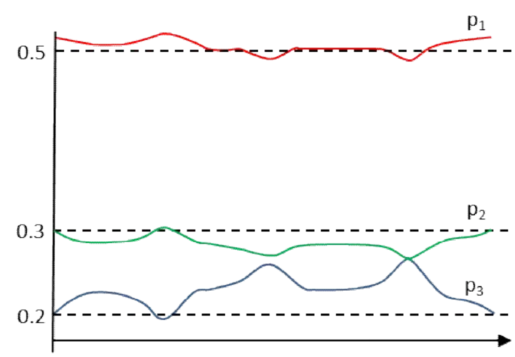

Fig. 1. The components of $p(t)$ of Markov process under the influence of chaotic oscillations

The recovery function F can be defined as follows: Consider an attractor A disposed in a compact manifold M, having the dimension of N. Then the recovery function that defines the mapping: $M \to R^{2N+1}$, can be formulated as follows:

$$F(x) = [\varphi_i(x), \varphi_i(x+\tau), \varphi_i(x+2N\tau)] \quad (5)$$

Where

$\varphi_i(x+n\tau)$ – ith component of the trajectory system;
$t > a$ – Sampling period is chosen arbitrarily.

In general, mapping of F will represent some enclosure [8,13,17]. Space R^{2N+1} is always sufficient to recover the attractor; however, such recovery can be carried out in a space whose dimension is less than $2N+1$ [4].

Recovering attractor can be implemented practically for any value of t, but still there are certain limitations:

- If the value of t is too small, then the following equality will be used:

$$\varphi_i(x + k\tau) \approx \varphi_i(x + (k+1)\tau) \quad (6)$$

And a recovered attractor is the limited area near the diagonal of the space in which recovery is carried out.

- If the value of t is too large, and the system is chaotic, then both values $\varphi_i(x + k\tau)$ and $\varphi_i(x + (k+1)\tau)$ are uncorrelated and the structure of the attractor disappears.
- If value t proves to be too close to the value of any period of the system, then that component, which is characterized by the specified period during the recovery process, will be presented inadequately.

Fig. 2 shows the phase portrait of the Chua's attractor in the space of variables $x(t)$ and $y(t)$ where Fig. 3 shows recovered values of the components of $x(t)$ at $T = 0.35$.

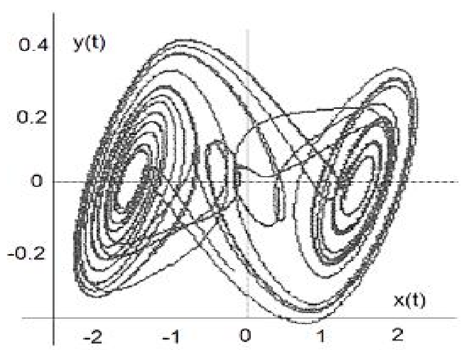

Fig. 2. Phase portrait of the Chua's attractor

Now let us consider a temporary implementation of any component of the vector $\vec{p}(t)$ of unconditional probabilities of perturbed system shown in Fig. 1.

By selecting the values of T, depending on the size of the sample interval and its dimension, we can reconstruct the phase portrait of perturbing chaotic process.

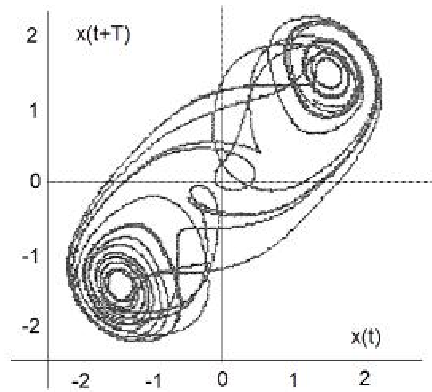

Fig. 3. Recovered values of the components of $x(t)$

Fig. 4 shows the phase portrait of the Rossler attractor in the space of variables $x(t)$ and $y(t)$ where Fig. 5 shows recovered values of the components of the vector $p_1(t)$ of unconditional probabilities of a perturbed Markov process at $t = 0.75$. Portraits were built using a length of 50 time units.

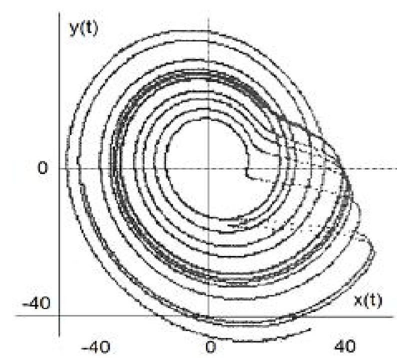

Fig. 4. Phase portrait of Rossler's attractor

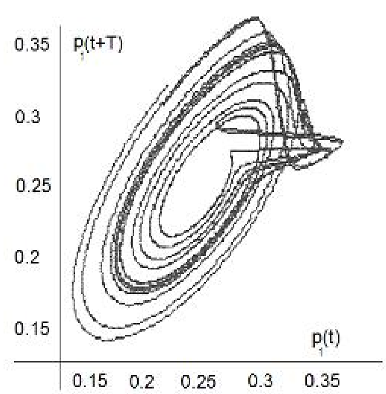

Fig 5. Recovered values of the components of the vector $p_1(t)$

Descriptive information about the properties of a dynamical system can give us calculation of the power spectrum of the process [9,19,14].

Calculations of the Fourier spectrum are also important because in the physical experiments measuring the spectrum, as a rule, is the only information about the system. Typically, the power spectra are calculated directly by the realization of data processing using fast Fourier transform algorithm [12,9,13].

In order to decrease the error in the calculation due to final realization, it is possible to use the methods of special windows; however, for qualitative analysis it may well be limited to rectangular windows, but can handle with relatively long implementation.

The need for averaging the calculated results of the spectra is more important. This procedure is adequate to the calculation of spectrum according to a certain given number of different periodograms of identical duration with the subsequent averaging of the results.

In the present work I carried out periodogram averaging to 20, each of which was built on the realization of a length of about 10 time units.

Fig. 6 shows the power spectra of $x(t)$ components of Lorenz system, and Fig. 7 shows the components of $p_1(t)$ vector of unconditional probabilities of a perturbed Markov process. Figs. 8 and 9 show respectively the corresponding spectra of Rossler system to Figs. 6 and 7.

Complete information about the probabilistic properties of the chaotic process gives density function of distribution $p(X,t)$, so by assuming that the process $X(t)$ in the system is stationary and ergodic, then we can and it is possible to considerably simplify its presence.

As a result, stationarity is excluded the dependence of the steady probability distribution with respect to the time, while ergodicity makes it possible to replace the ensemble average by averaging over time along a single realization.

The density distribution $p(X)$ of stationary ergodic process can be calculated as the limit of the relative residence time of trajectory system in the volume element of phase space corresponding to some discrete partition [10,15,20].

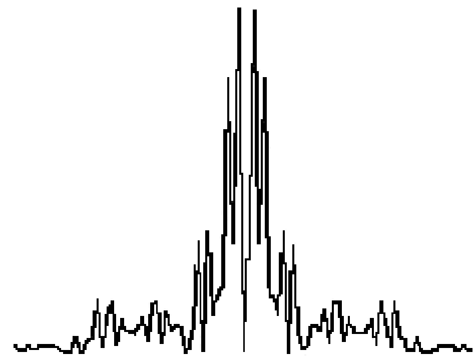

Fig. 6. The power spectra of $x(t)$ components of Lorenz system

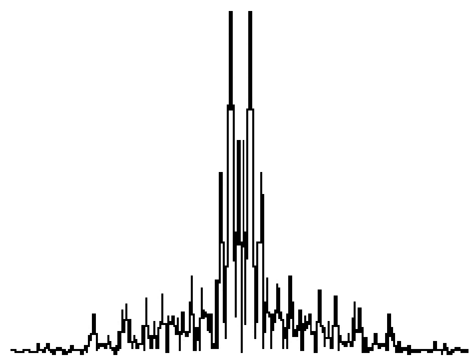

Fig. 7. The components of $p_1(t)$ vector of unconditional probabilities of a perturbed Markov process

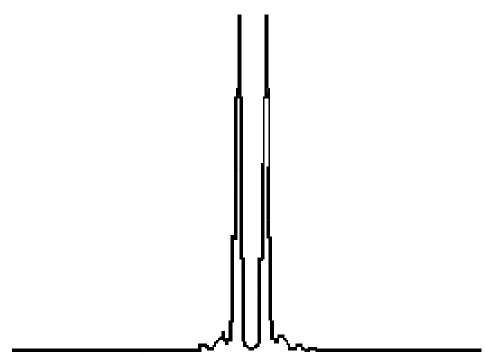

Fig. 8. Corresponding spectra of Rossler system to Fig. 6

Fig. 10 shows the components distribution density $x(t)$ of Lorenz system and Fig. 11 shows the components of the vector $p_1(x)$ of unconditional probabilities of the perturbed Markov process. Density function is based on the realizations of length 100 time units and the range of variation in the argument was broken into 80 equal intervals.

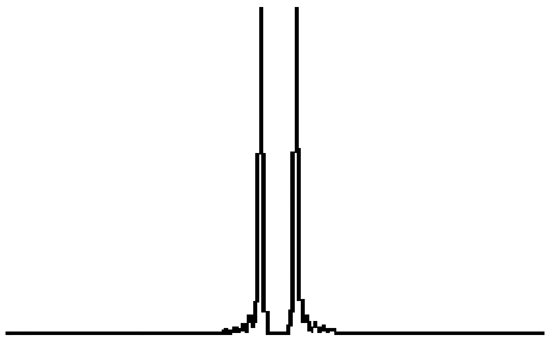

Fig. 9. Corresponding spectra of Rossler system to Fig. 7

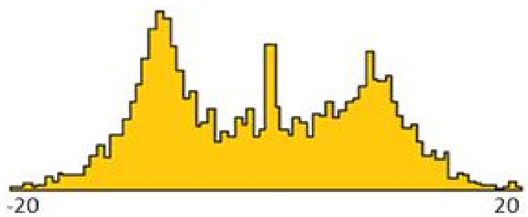

Fig. 10. The components distribution density $x(t)$ of Lorenz system

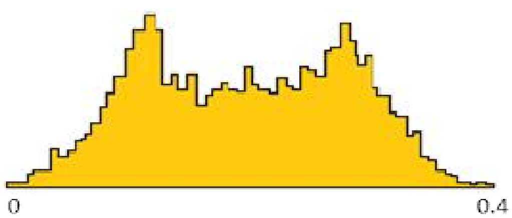

Fig. 11. The components of the vector $p_1(x)$ of unconditional probabilities of the perturbed Markov process

3. CONCLUSION

Summing up, we can say that the “perturbed chaos" of unconditional probabilities of a Markov process acquire the topological structure inherent in this type of chaotic system. Using techniques such as the construction of the phase portrait, power spectrum and density distribution can be adequately establishing the type of chaotic process.

COMPETING INTERESTS

Author has declared that no competing interests exist.

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