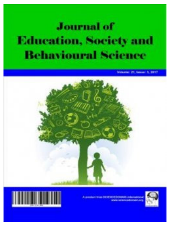

Journal of Education, Society and Behavioural Science

25(4): 1-17, 2018; Article no.JESBS.40980
ISSN: 2456-981X
(Past name: British Journal of Education, Society & Behavioural Science,
Past ISSN: 2278-0998)

Analysis of Undergraduate Physics Student's Perception of Angular Momentum in Quantum Mechanics at the University of Zambia

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Authors' contributions

This work was carried out in collaboration of all authors. Author FHM designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Authors PK and JGJ supervised and provided the guide managed the analyses and approved the study and also managed the literature searches. All authors read and approved the final manuscript.

Article Information

DOI: 10.9734/JESBS/2018/40980
Editor(s):
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(2) Pratima Parashar Pandey, Professor, Polymer composites, Nanotechnology Applied Sciences, College of Engineering & technology, Integrated Institutes of Learning, Dr. APJ Abdul Kalam Technical University, Greater Noida, India.
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(3) Subramaniam Jahanadan, Labuan Matriculation College, Malaysia.
Complete Peer review History: http://www.sciencedomain.org/review-history/25183

Original Research Article

Received 23rd March 2018
Accepted 11th June 2018
Published 19th June 2018

ABSTRACT

This study aimed at investigating the problems students face in understanding the concepts of angular momentum in classical and quantum mechanics, finding an effective way of improving understanding and learning the concepts of angular momentum in quantum mechanics, lastly document the challenges investigated and suggest some solutions to help students understand angular momentum in both classical and quantum mechanics. This study summarizes a series of investigations into how undergraduate Physics students perceive angular momentum in quantum mechanics at the University of Zambia. We investigated undergraduate physics student's perception of angular momentum in quantum mechanics by administering written questions (open-ended) to two different groups of classes. Out of these investigations we developed a Quantum Interactive Learning Tutorials (Q_u I L T_s) and peer instruction tools to assist the students build up a

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considerable knowledge structure of quantum mechanics in general and angular momentum in specific, through a guided approach technique [1]. Eleven questions were administered to a group of N = 15 students, statistics for answered questions are as follows; question one 4 out of 15 representing a percentage of 26.67%, question two only 6 out of 15 student representing 40%, question three 11 out of 15 representing a percentage of 73.33%, for question four only 1 out of 15 representing a percentage of 6.67%, question five none of the students answered, question six only 1 out of 15 students representing a percentage of 6.67% managed to answer, question seven 1 out of 15 representing a percentage of 6.67% managed to answer, question eight 11 out of 15 representing a percentage of 73.33% got the correct answer, question nine 6 out of 15 representing a percentage 40% got the right answer, question ten only three students making a percentage of 20% answered correctly, question eleven no student managed to get it right. Preliminary results show that the undergraduate physics students perception of quantum mechanics is improved after using the research-based learning tools in the lower class level quantum mechanics courses [1]. We also build up a common conceptual survey that can help lecturers better investigate undergraduate physics students understand the concept of angular momentum in particular and quantum mechanics courses in general.

Keywords: Quantum mechanics; classical mechanics; angular momentum.

ABBREVIATIONS

PER	:	*Physics Educational Research*
UNZA	:	*University of Zambia*
QM	:	*Quantum Mechanics*
CM	:	*Classical Mechanics*
AM	:	*Angular Momentum*

1. INTRODUCTION

It is noted that learning physics courses are always difficult for students at all levels from lower to higher physics courses [2]. In the lower physics courses, there is need to draw meaningful inferences from the abstract concept based highly on a complex mathematical theory and applying the fundamental principles in different situations related to physics courses.

A lot of research has been conducted investigating the differences between slow learner and fast learner students in reasoning, solving physics problems, meta-cognitive skills [3]. In general fast learners normally start solving physics problems theoretically at abstract level and then turn to the specific ones. Slow learners usually focus on the surface features and hence get lost while solving physics problems. Challenges that students face in lower physics courses have been researched [4], and many instructional techniques have been developed to assist slow learner students to acquire the knowledge as well as the ability to solve physics problems [3,5,6] and [7].

In this study, we will analyse how undergraduate physics concepts of angular momentum in quantum mechanics and the impact of using a tutorial based instruction and peer-instruction tools to improve their better understanding in future. We will discuss the findings of conceptual and misconceptions in physics education research (PER) in lower physics courses to guide the investigation of students' challenges and strategies to help them learn the concept of angular momentum better. The research into physics teaching based on the epistemology and learning physics and also educational strategies that take into account the findings of learning theories that can help students learn angular momentum in quantum mechanics.

Research shows that many students experience a lot of problems in learning quantum mechanics. The studies have shown that students have difficulties in virtually every aspect of quantum mechanics in general, be it problem-solving, describing concepts and phenomena [7].

In physics, classical mechanics and quantum mechanics are the two major sub-fields of mechanics [8]. Classical mechanics is concerned with the set of physical laws describing the motion of bodies under the action of a system of forces. The study of the movement of bodies is an ancient one, making classical mechanics one of the oldest and largest subjects in science, engineering and technology. It is also widely known as Newtonian mechanics [9].

Classical mechanics describes the motion of macroscopic objects from projectiles to parts of machinery, as well as astronomical objects, such as spacecraft, planets, stars, and galaxies.

Besides this, many specialisations within the subjects deal with gases, liquids, solids and other specific sub-topics. Classical mechanics also provides extremely accurate results as long as the domain of study is restricted to large objects and the speeds involved do not reach the speed of light.

Quantum mechanics reconciles the macroscopic laws of physics with the atomic nature of matter; it describes how systems, big and small evolve with time. Quantum phenomena (the microscopic world) cannot be explained in classical ways. Students have more challenges in mastering and understanding fundamental concepts of quantum mechanics [1].

The mathematical descriptions can be overwhelming to students in understanding quantum mechanics, and they may not have the opportunity to focus on the conceptual framework and build a coherent knowledge structure of quantum mechanics [10]. Research has shown that lack of mathematical descriptions related to the context of quantum mechanics can hinder conceptual learning [11]. The conceptual aspect of quantum mechanics can lead the students making significant mathematical errors that they could not do in a linear algebra course.

Physics education research (PER) refers both to the methods currently used to teach physics and to an area of pedagogical research that seeks to improve those methods [12]. One primary goal of physics education research is to develop educational techniques and strategies that will help students learn physics more effectively and help instructors to implement these techniques [12].

Deficiencies in Zambian education and the capabilities of students are gaining increased attention at all levels of educational systems, from elementary schools to schools of professional training [13]. Therefore students fail to achieve sound and useful learning of complicated subject matter. It has also been identified that the goal of science education is to impart new schemata to replace the student's extant ideas that differ from the scientific theories being taught [14].

Cognitive structures are the underlying mental processes people use to make sense of information. Other names for cognitive structures are mental structures, mental tools and patterns of thought. Common types of cognitive structures are parallel thinking structures, symbolic representation structures and logical reasoning structures, [15].

Lecturers are surprised that despite their best efforts, students do not grasp fundamental ideas covered in class. Best students give correct answers just by memory, when asked more closely, these students reveal their failure to understand the underlying concepts fully. Some physics students believe learning consist of memorising formulas provided by the lecturer, while others may believe it entails applying and modifying their own understandings [16].

Misconceptions are not only to be observed in today's children or students, but even scientists and philosophers also developed many misconceptions in the past. Ideas developed without the knowledge of the subject are not wrong but are described as alternative, original or pre-concepts. Misconceptions can be categorised as follows: the Preconceived notion, Non-scientific beliefs, Conceptual misunderstandings, Vernacular misconceptions**,** Factual misconceptions.

Recent research done on students conceptual misunderstanding of natural phenomena indicates that new concepts cannot be learned if alternative models that explain a phenomena that already exist in the learners mind [17]. A few examples are “preconceptions” which acknowledge that the “incorrect” conceptions may also be useful. Smith, DiSessa and Roschelle have described a number of assumptions made in misconceptions research, summarized as [18]:

- Students have misconceptions,
- Misconceptions originate in prior learning (in classroom or some other part of the world),
- Misconceptions can be stable and widespread among students, and misconceptions can be strongly and resistant to change,
- Misconceptions interfere with learning,
- Misconceptions must be replaced,
- Instructions must confront misconceptions,
- Research should identify misconceptions

Physics education research is the systematic investigation of the teaching and learning of physics. Its goals include the improvement of student understanding of physics. Correlations between students' epistemological beliefs of quantum mechanics and their understanding of the concepts have been found. However, the changes are not clear-cut [19] and [20].

2. METHODOLOGY

In this study we used the open-ended questions to collect the data. We used both qualitative and quantitative methods of questioning to enhance the understanding of the nature of students difficulties in the concepts of angular momentum in both classical and quantum mechanics. Open-ended questions have become a major tool for physics education researchers in evaluating student conceptual understanding [21]. Qualitative questions have little or no symbolic manipulation or formulas. They require students to draw meaningful inferences upon their qualitative resources and reasoning ability [22].

2.1 Research Site and Participants

This research was conducted at the University of Zambia (UNZA) School of Natural Sciences, Department of Physics. UNZA has two groups of students studying physics, the first group pursues a Bachelor of Science degree (BSc) with physics as major and the other group a Bachelor of Science with education (BSc Ed). Both groups learn classical and quantum mechanics, besides the students from these programs learn a lot of physics courses hence they were targets of this research as participants. The target participants were 13 fourth years both BSc Ed and BSc, while 27 third years. The two programs take four academic years to complete their undergraduate. The following are the courses taken by the participants by the end of the four-year program.

- Mathematical methods (M111,M112, M211, M212,M911, M912)
- Introduction to Chemistry (C101 & C102)
- Introduction to Physics (*P 191* and *P 192, P 198*)
- Electricity and Magnetism *(P 261)*
- Electromagnetic Theory (P 361)
- Properties of Matter and Thermal Physics (231)
- Classical Mechanics I & II (*P 251* and *P 252*)
- Atomic Physics (*P 212*)
- Nuclear Experimental Techniques (P411)
- Optics (*P 272*)
- Introduction to Electronics, Digital Electronics and Analogue Electronics (*P341and P441*)
- Quantum Mechanics I (*P 351*)
- Quantum Mechanics II (P455)
- Computational Physics
- Statistical mechanics (P332)
- Physics of Renewable Energy sources and the Environment (P458)

To gain more insights into this research, we required participant who have done P351 or are learning P455 and have done classical mechanics, electricity and magnetism, atomic physics, and mathematical methods as pre-requisite courses. The coverage of P351 and P455 are based on the content of the prescribed books namely, Introduction to Quantum Mechanics 2nd edition by Bransden and Joachain (2000) and Quantum physics by Gaiorowicz (1971), the recommended text is Quantum Mechanics, Institute of physics publisher, (1993). The course have three (3) hours of lectures, one (1) hour tutorial and three (3) hour laboratory session per week. In the course of quantum mechanics the students will cover the many chapters see appendix.

2.2 Statistical Method Used for Data Analysis

For analysing the data for this research, we used Microsoft Excel. Microsoft Excel is an advanced statistical program that performs a variety of statistical analysis, including the computation of correlation and reliability coefficient.

We have used open-ended questions in this research. Hence the data for students' scores in these questions were either binary or continuous numbers. Student's responses to open-ended questions, after normalisation, student's scores were any number between zero and three. We made our data compatible by converting the continuous data to categorical data, and hence we were able to perform a t-test. The questionnaire in the appendix will give much information, we divided the scores into four codes and assigned the same number to all the systems in a given question. If the respondent did not attempt, the problem the code assigned was 0. If a respondent tried, but the answer given was wrong the code assigned was 1. Any respondent who answered almost correct the code assigned was 2. Lastly but not the least if the response was proper the assigned system was 3.

Now we used an advanced statistical program Microsoft Excel that performs a variety of statistical analysis, we were able to find the mean for all the variables and then the standard deviation, and compare the two. Descriptive statistics were obtained, which gave information about the normality of the population. Variables like age and responses were compared even year of study against their respective answers.

3. RESULTS

3.1 Question One (1)

This question was administered to a group of N = 15 students, and it was found that only 4 out of 15 representing a percentage of 26.67% answered correctly. It was found that the concept involved in angular momentum in classical mechanics presented a lot of challenges to students. Students could not distinguish between scalar and vector quantities; students presented significant difficulties in the calculation of the magnitude of angular momentum.

The failure rate of students which was 73.33% was based on the confusion between the algebra behind the definition of the dot and cross product of two vectors. The dot and vector products are topics which are covered at the second year in P 251(classical mechanics I) and M 211(mathematical method III). Students show more difficulties with the formalism of understanding of the physical concept of angular momentum regarding abstract mathematical symbols.

3.2 Question Two (2)

The primary investigation was to write the cross product of a position vector ($\vec{r}$) and the linear momentum of a particle mass m in 3-D. This question was administered to the same group $N=15$, and only six students representing 40% managed to get it correct.

The failure rate of 60% shows that the students do not have a good background of solving the determinant of dimension three by using any of the methods which are taught in mathematics from secondary schools to Universities. The students presented severe challenges in calculating a determinant in 2-D using Cramer's plan, it was also observed that students confused between scalars and vectors quantities. They were unable to put arrows on the unit vectors, $\vec{i},\vec{j},\vec{k}$.

3.3 Question Three (3)

The primary investigation in this question was to write the expression of linear momentum of the particle of mass in m classical mechanics. 11 out of 15 representing a percentage of 73.33% answered the question correctly. The failure rate of 26.67% was related to just lack of fundamental knowledge of physics.

3.4 Question Four (4)

The primary investigation in this question was to differentiate the linear momentum in classical mechanics and linear momentum operator in quantum mechanics. The mathematical background of this question is based on the introduction of the imaginary number in the sophisticated analysis which is a vector operator in 3-D applied only to a scalar field.1 out of 15 representing a percentage of 6.67% managed to get the question correct.

The failure rate of 93.33% was based on the lack of good mathematical background and its related manipulation and skills. Another difficulty students had was the confusion between the linear momentum at the macroscopic scale and microscopic worlds in 3-D related between a scalar and vector quantities as mentioned earlier.

3.5 Question Five (5)

This question was investigating student’s skills in establishing the relationship between the linear momentum operator in quantum mechanics and the angular momentum in classical mechanics. The problem was administered to a group of $N=15$, and none of the students answered correctly. The failure rate of 100%, explains that students face challenges in figuring out the proper representation of linear momentum operators in 3-D, in the determination of the Laplacian operator in the same 3-D. Student’s found it difficult to calculate the second partial derivatives in 3-D. The Laplacian operator is taught in the second year in classical mechanics II (P 252).

3.6 Question Six (6)

The investigation of this question was to write the vector angular momentum in classical physics $\vec{L}$, regarding respective components of linear momentum operator in quantum mechanics. This definition will lead to the establishment of angular momentum vector in quantum mechanics. From this angular momentum vector, students were supposed to derive the respective component $\vec{L}$.

The question was administered to a group $N=15$, and only 1 out of 15 students are representing a percentage of 6.67% managed to get right. The failure rate of 93.33% students is based on the angular momentum in classical mechanics; it is also noted that students have challenges on deriving the expression of linear momentum operators in quantum mechanics. Students lack well-structured mathematical skills; students had difficulties in solving the cross product of two vector quantities $\vec{r}$ and $\vec{p}$ in classical mechanics, which leads to the calculation of the vector angular momentum in classical mechanics. It has been noted that students have challenges in deriving the expression of linear momentum operators in quantum mechanics. This is due to the lack of mathematical skills involved when calculating quantum mechanics.

3.7 Question Seven (7)

This question was to find the difference between the discrete and continuous distribution of physical quantities. Quantization is just the restriction of a subset, usually distinct, of the possible values for a variable. There is quantisation of angular momentum in the unit of Planck constant (ħ) some valuables can be quantised in some situations but not others. The energy of a free particle is not quantised, why restricting the range of motion of the particle in between impenetrable walls results in quantisation of its momentum. This question was administered to a group $N=15$, and only 1 out of 15 representing a percentage of 6.67% managed to answer it correctly. The failure rate of 93.33% was due to difficulties in understanding the concepts of discrete distributions as results of the quantisation of electric charge in units of electron charge. The interaction between particles proceeded only through the photon the electromagnetic interaction. Students present more challenges in the perception of the smallest possible amount of quantity.

3.8 Question Eight (8)

The investigation of this research question was to identify the nature of two operators and hence calculate the commutator of two operators. This question was administered to a group $N=15$, and 11 out of 15 representing a percentage of 73.33% got the correct answer. The failure rate of 26.67% was due to the confusion in finding the product of two commuting operators.

3.9 Question Nine (9)

The investigation of this question was to find the relationship between commuting operators. The problem was administered to a group of $N=15$ students and 6 out of 15 representing a percentage 40% got the right answer. The failure rate of 60% is due to the lack of axiomatic algebraic laws on commutativity offered from secondary school to tertiary education.

3.10 Question ten (10)

The investigation of this question was based on the perception of how students understood the concept commutator of two operators $\vec{x}$ and $\vec{p}$ .This question was administered to a group of $N=15$ students, and only three students making a percentage of 20% answered correctly and the failure rate of 80%.

3.11 Question Eleven (11)

The investigation was based on student understanding of cross product of two vector fields $\vec{r}\ and\ \vec{p}$ students understanding of commutator of commutation relations between position vector of the particle and its linear momentum.

This question was administered $N=15$, and no student managed to get it right.

The failure rate of 100% based on the lack of scientific abstracts behind this concept. We have found that these concepts are very challenging for students. A misconception often originates from the partial understanding of other physics courses on the visible world, which presents negative repercussion in quantum mechanics.

The mean for most of the variables in the questionnaire is more than the standard deviation, which implies that the data used is meaningfully reliable as shown Table 1.

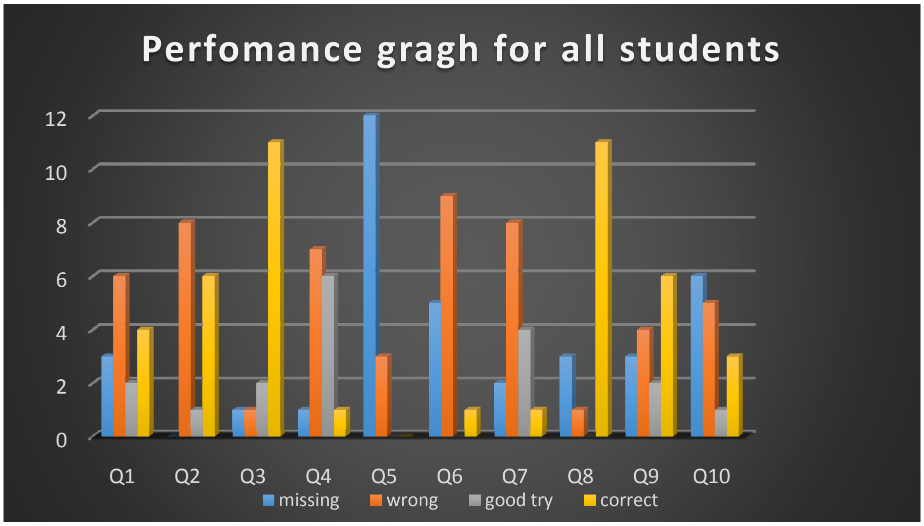

Fig. 1. Summary of the response of all the students from question 1 to 10

Table 1. Descriptive statistics

	DAMCM	CPTVQ	LMCM	LMOQM	DMO	CAMITO	UTQQM	CTO	KTCO
Mean	1.467	1.867	2.533	1.467	0.2	0.8	1.267	2.267	1.73
Median	1	1	3	1	0	1	1	3	2
Mode	1	1	3	1	0	1	1	3	3
Standard deviation	0.623	0.5485	0.5077	0.4116	0.2293	0.4290	0.4424	0.7088	0.6772

Key: DAMCM- Definition of Angular Momentum in Classical Mechanics
CPTVQ – Cross Product of Two Vector Quantities
LMCM – Linear Momentum Classical Mechanics
LMOQM – Linear Momentum Operator in Quantum Mechanics
DMO – Derivatives of Momentum Operators
CAMITO – Components of Angular Momentum In Terms of Operators
UTQQM – Understanding of the Term Quantized in Quantum Mechanics
CTO – Commutator of Two Operators
KTCO – Knowledge about Two Commuting Operators

Table 2. t- test: Two sample assume equal variables, year of study in relation to DAMCM

	Year of study	DAMCM
Variance	0.2381	1.2667
Mean	1.6667	1.4667
Critical value	1.701131	
t- value at 5%	0.631453	

Note: The degrees of freedom is 28

Table 3. t-test: Year of study about CPVTQ

	Age	CPVTQ
Mean	1	1.4667
Variance	0.7143	1.2667
Critical value	1.7011	
t-value	-1.2841	

In this context, the data may be reasonably assumed to come from a normally distributed population. This justifies the usage of the t-distribution for comparisons of variables.

Therefore the H_o: no relationship between year of study and variable 1 (DAMCM).

H: there is a relationship between year of study and variable1 (DAMCM).

Since the t-statistics is less than the critical value we fail to reject the null hypothesis and conclude that there is no relationship between year of study and the variables in the questionnaire.

And now comparing the age and the variables:

H_o: no relationship between age and variable1 (CPVTQ)

H: There is a relationship between age and variable1 (CPVTQ).

Since the t-statistics is less than the critical value we fail to reject the null hypothesis and conclude that there is no relationship between age and the variables in the questionnaire.

The data presented shows that students are always able to recognise the basic concepts of angular momentum in both classical and quantum mechanics problems, and often fail to use the mathematical background beyond the development of these concepts. There is a correlation between student's accurate knowledge of angular momentum and their ability to solve angular momentum related problems in the context of quantum mechanics.

4. DISCUSSION

Instructional material developed like tutorials by physics education researchers attempt to bridge the gap between the abstract quantitative formalism of quantum mechanics and the qualitative understanding necessary to explain and predict diverse physical phenomena [23]. Q_uILTs are based on the investigation of student difficulties in learning quantum mechanics and to help them build links between the abstract formalism and conceptual aspects of quantum mechanics.

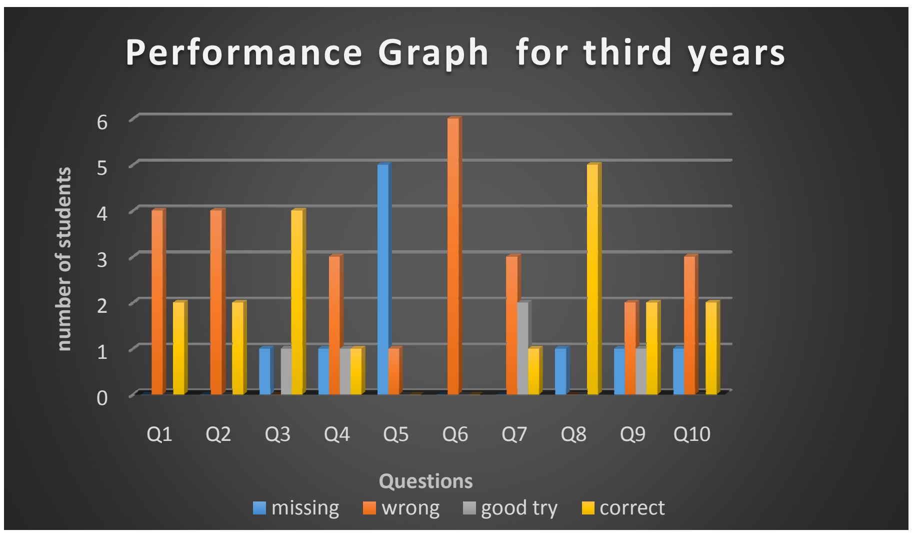

Fig. 2. Summary of response given by third years

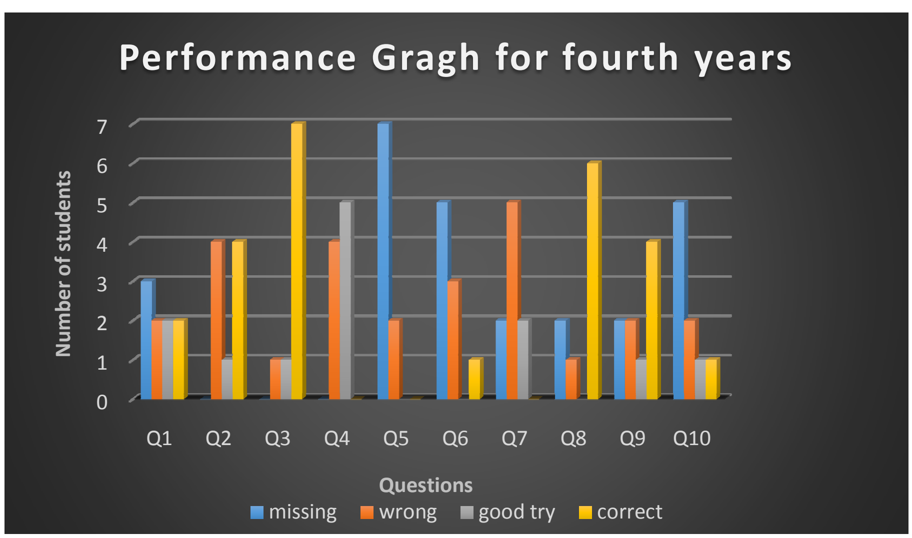

Fig. 3. Summary of responses given by fourth years

The worksheet contains materials on both basic Angular momentum concepts and mathematical skills related to the formalism of these concepts. The tutorial will be developed basing on the results of the research.

4.1 Worksheets Related to Angular Momentum

A useful worksheet progresses from simple to complex ideas of Angular Momentum. Firstly definition of conceptual themes of Angular Momentum. We come up with the initial "learning tutorial" based upon quantum mechanics syllabus and specifically angular momentum and the difficulties faced by students.

The material is focused on helping students develop basic quantum mechanical concepts, to use these concepts to interpret different quantum automatic system and to relate and use multiple representations in describing the quantum system.

Unfortunately, for many reasons, we were not able to asses this worksheet statistically. For example, we did not have a proper pre-test and post-test administered. However, more than 75% of the students found the worksheet useful although they were not tested after seeing the worksheet.

These are some comments from students after seeing the worksheet:

S1: "*most staff makes sense now*."
S2: "*we take things for granted as students making simple mistakes."*
S3: "angular momentum and quantum mechanics is not difficult at all if every topic had a worksheet."
S4: "this worksheet has made me see what I never used to see in this topic."

While most of the students found this worksheet helpful, this worksheet breaks down a complex problem into a few simple ones and guides students through to find derivatives regarding the angular momentum operators and the components of the angular momentum vector.

5. LIMITATIONS OF THE WORKSHEET

There were several limitations in this worksheet that make the assessment of their usefulness very difficult. The most severe was the limitation on the amount of contact time with the students we could spend on this worksheet during the term. Secondly, we did not have a control group for this study; these courses are offered only once per academic year with only two classes in any given term. Also, this worksheet was not the only instruction students had received on this topic. Lack of reliable pretest and post-test data for this worksheet is another limitation in doing a better assessment at this point.

6. CONCLUSION

This study was set out to document the student's perception of Angular Momentum in quantum mechanics at the University of Zambia. The target participants were 40 but out of these only 15 were willing to help with the research because it was purely voluntary.

It was found that the concept involved in angular momentum in classical mechanics presented a lot of challenges to most students. The analysis was done on each question, as tabulated in Fig. 1, from question one to eleven the performance percentages were as follows in order, 26.7%, 40%, 73%, 6.7%, 0%, 6.7%, 6.7%, 7.3%, 40%, 20%, 0%. The analysis also showed that there was no relation between age of students and the response was given.

Students presented significant difficulties in the calculation of the magnitude of angular momentum. Students showed more problems with the formalism of understanding of the physical concept of angular momentum regarding abstract mathematical symbols. Another difficulty students had was the confusion between the linear energy at macroscopic and microscopic worlds in 3-D. It was noticed that on most questions students were incompetent to make proper derivations.

After a one on one discussion with the students, when they had already been exposed to the worksheet, the students acknowledged that to understand angular momentum in Quantum Mechanics one should first understand it in classical mechanics.

Most of the times it is the misconceptions that students have which enables them to make unnecessary mistakes. "One of the misconceptions is the *angular momentum measurements, given a system with definite values of* L_z, *there exists a definite value of* L_x, *but that when its value has measured the* *outcome of the measurements cannot be predicted. This misconception is reinforced by the "rotating vector model" often invoked in modern physics courses. This is not to say that the rotating vector model must never be used, only that it must be presented along with cautions concerning its limitations. To combat this misconception, it is desirable to use the term "expected value" rather than " mean value", the latter suggests that there is only one, correct, value, which is the subject to experimental error"* [24].

It is useful to review and think about possible misconceptions before teaching a class or laboratory in which new material is introduced. Some mistakes can be uncovered by asking students to sketch or describe some object or phenomenon. Strategies for helping students to overcome their mistakes are based on research about how they learn [25] and [26]. The key to success is ensuring that students are constructing or reconstructing a correct framework for their new knowledge. As a lecturer encourage students to test their conceptual frameworks by discussing with other students and thinking about the evidence and possible tests.

Another way to help students understand concepts and develop metacognitive skills in problem-solving is for the instructor to act as a model and moderator, by taking the lead. Instead of presenting a solution, the lecturers must describe their cognitive and metacognitive processes while solving a problem and the learners will want to imitate and in turn, their metacognitive skill will improve. Logical reasoning is also crucial, these structures use general thinking strategies to process and generate information systematically. They include deductive and inductive reasoning, analogical and hypothetical thinking, cause-effect relationships, analysis, synthesis, evaluation, problem framing, and problem-solving [15].

This study has opened up many questions for further research, such as "is it only angular momentum which students have a challenge with?".

7. RECOMMENDATIONS

- Lecturers should not leave students as independent learners with considerably cognitive and meta-cognitive skills but should guide them so that they do not have difficulties to build up the knowledge of physics from their pre-requisite knowledge acquired at lower and intermediate levels.
- Apart from the mathematics courses, students have the introduction of another mathematics course, having the content of the difficulties students usually face when making calculations of quantum mechanics problems.
- The proposed instructional material in this study, be used by both students and lecturers.
- The problems students faced, investigated in this study, should be made known to all students taking quantum mechanics as a remedy for making quantum mechanics easy.
- Student needs to move to the quantum theory of probability, amplitude and densities when describing microscopic entities, where these probability densities are expectations values.

ACKNOWLEDGEMENTS

Dr. George Kasali for his suggestions, time and support. I would like to thank Dr. H.V Mweene the current dean of Natural sciences at the University of Zambia for his help making the research possible to be conducted at the University of Zambia. I am grateful to the head of Department of Physics at UNZA Dr. Hatwambo for his assistance and Dr. Sibanda for his patience and help.

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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APPENDIX A

Physics syllabus

P351: QUANTUM MECHANICS I

Rationale

Quantum mechanics is the most fundamental theory of physics and has the widest range of applicability. It is the cornerstone of any physics degree programme. Therefore a good introduction to the subject is essential for physics students. A thorough grasp of it is necessary for almost all postgraduate work. P351 aims to introduce the basics of quantum mechanics to the student in a way that lays a foundation for further study of the subject and for its use in other areas of physics.

Objectives

At the end of this course, the student should have learned:

1. The failings of classical physics and the need for quantum concepts
2. How quantum mechanics resolves problems that confounded classical mechanics
3. The basic principles of quantum mechanics
4. How to apply quantum mechanics in simple situations
5. The student should be ready for more advanced concepts in quantum mechanics.

Course Content:

Origins of quantum theory: Wave-particle duality; black-body radiation; Compton effect; photoelectric effect; atomic spectra and Bohr model; the correspondence principle; the de Broglie hypothesis and electron diffraction.

Wave packets and the uncertainty relations: The wave function; superposition; wave packets and group velocity; the Heisenberg uncertainty principle and examples.

The one-dimensional Schrödinger equation and applications: Time dependent and time-independent Schrödinger equation; eigenvalues and eigenfunction; one-dimensional examples; normalization; quantum mechanical tunnelling; the harmonic oscillator.

Basic postulates and formalism of quantum mechanics: The wave function; dynamical variables and operators; Hermitian operators; expansion in eigenfunctions; commuting observables; compatibility.

Angular momentum in quantum mechanics: Angular momentum operators; eigenvalues and eigenfunctions; experimental demonstration of angular momentum quantization; general solution of the eigenvalue problem; matrix representation; spin and Pauli exclusion principle.

Three-dimensional Schrödinger equation: The wave equation; separation in Cartesian coordinates; separation in spherical polar coordinates; the radial equation; application to the hydrogen atom; degeneracy.

Time Allocation:

Lectures: 3 hrs/week, Tutorial: 1 hr/week
Lab: Associated laboratory of 3 hrs/week

Assessment:

Continuous assessment: 40%, Final Examination: 60%
Pre-requisites: P251, P252, P261, P212, M211, M212

Prescribed Texts

1. B. H. Bransden and C. J. Joachain, Introduction to Quantum Mechanics, ELBS, 1989, ISBN: 0-582-44498-5

2. S. Gasiorowicz, Quantum Physics, J. Wiley & Sons, 1971, ISBN: 0 471 29281-8
Recommended Texts:

1. A. I. M. Rae, Quantum Mechanics, Institute of Physics Publisher, 1993, ISBN: 13:9780750302173

P455: QUANTUM MECHANICS II

Rationale

This course is essentially a continuation of P351, Quantum Mechanics I. Further topics are introduced in order to further develop the theory and to give the students an improved mastery of the subject. Together with P351, P455 aims to give students all the elements of elementary quantum mechanics.

Objectives

At the end of this course, the student should be able to:

1. Use approximation methods to solve quantum systems that are close to exactly solved systems.
2. Add two angular momenta.
3. Construct a description of systems of non-interacting bosons or fermions. The basic principles of quantum mechanics
4. Employ basic theory to solve problems involving the interaction of radiation with matter.

Course Content

Approximation Methods in Quantum Theory; perturbation theory; time-independent theory; non-degenerate theory; degenerate theory; the Stark effect; time dependent theory; harmonic perturbations; transition probabilities; the variation method; the Wentzel-Kramers-Brillouin (WKB) approximation; tunnelling; the helium atom.

The Harmonic Oscillator: Treatment by algebraic methods.

Angular Momentum: angular momentum and rotations; general angular momentum; matrix representation; addition of angular momentum.

Many-particle Systems: systems of identical particles; bosons and fermions; spin-1/2 particles in a box; the Fermi gas.

The Interaction of Quantum Systems with Radiation: the electromagnetic field and its interaction with radiation; perturbation theory for harmonic perturbations and transition rates; spontaneous emission; selection rules for electric dipole transitions; line intensities, widths and shapes; the spin of the photon and helicity; photoionisation; photodisintegration.

Time Allocation:

Lectures: 3 hrs/week: Tutorial: 1 hr/week

Assessment:

Continuous assessment: 30%: Final Examination: 70%

Pre-requisites: P351
Prescribed Texts:

1. B. H. Bransden and C. J. Joachain, Introduction to Quantum Mechanics, Longman, 1989.

ISBN: 13-978-0582 444 980

APPENDIX B

Questionnaire

Title

"An analysis of undergraduate students perception of angular momentum in quantum mechanics at the University of Zambia."

Objective

To probe how students understand some conceptual themes in angular momentum at the University of Zambia.

Student information

You have been selected to assist with information concerning our research. Confidentiality is fully guaranteed for all your responses. Your sincere consideration to this request will be highly appreciated.

Questions

Age	
sex	
Program	

1. (i) How can you define angular momentum in classical physics?

   (ii) Use the cross product of the two vector quantities $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{p} = p_x\vec{i} + p_y\vec{j} + p_z\vec{k}$ to find the vector $\vec{L}$.

   (iii) Write down the expression of linear momentum in classical physics?

   (iv) Write down the expression of linear momentum operator in quantum mechanics

   (v) From $\widehat{p}_x = -i\hbar\frac{\partial}{\partial x}$

   $$\widehat{p}_y = -i\hbar\frac{\partial}{\partial y}$$

   $$\widehat{p}_z = -i\hbar\frac{\partial}{\partial z}$$

   Derive any x, y and z-derivations in terms of the momentum operators.

   (vi) Write down the components of the angular momentum $\vec{L}$ in terms of operators $\widehat{p}_x, \widehat{p}_y\ \widehat{p}_z$

   (vii) What do you understand by the term quantized?

   (viii) Consider two operators A and B.

   Write down the commutator [A,B] of the two operators

   (ix) if [A,B] = 0, what do you say about A and B.

   (x) If $\widehat{p}_x = -i\hbar\frac{\partial}{\partial x}$, find the $\lfloor x, \widehat{p}_x \rfloor$

   (xi) The angular momentum is defined by: $\vec{L}=\vec{r}\times\vec{p}$ use the commutations relations between $\vec{r}\ and\ \vec{p}$ to find the following commutation relations $[L_x, L_y]$, $[L_y, L_Z]$ and $[L_Z, L_X]$.

(a) Deduce the value of L^2

(b) Find $[L^2, L_X]$ and $[L^2, L_y]$

2. (i) indicate the extent to which you are familiar with angular momentum (show by ticking)

1	Very easy
2	Somewhat easy
3	Not sure if easy
4	Difficult

Explain your answer.

(ii) Are there any teaching aids (materials and apparatus) you would recommend for teaching angular momentum?

(iii) What other suggestions would you give for angular momentum to be taught and understood by students easily?

APPENDIX C

WORKSHEET

FINDING ANGULAR MOMENTUM IN CLASSICAL MECHANIC

1. Identify which ones are vectors and scalars $\vec{L}, L, \vec{p}, r, \hbar, \vec{r} \quad and \quad p$.

2. When the rotation axis is the z-axis, the z-component of the angular momentum $L_{s,z}$ about the point s is; $L_{s,z} = I_s\,\omega_z$ the variables $L_{s,z}\,, I_s\,, \omega_z$ stand for?

3. State conservation law of the angular momentum for an isolated system?

4. Use the cross product of the two vector quantities $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{p} = p_x\,\vec{i} + p_y\,\vec{j} + p_z\,\vec{k}$ gives vector $\vec{L}$.

(a) What is vector $\vec{L}$.
(b) Find vector $\vec{L}$.

5. Write an expression relating angular momentum and moment of inertia.

6. Derive $\left(\vec{F}\right)_{ext} = \left(\frac{d\vec{p}}{dt}\right)_{sys}$?

7. When all internal forces are directed along the line connecting the two interacting objects, then the internal torque about the point s is zero. Is this statement true or false?

8. Show that the angular momentum for the system of particles is the vector sum of the individual angular momenta?

9. Write down the expression of linear momentum in classical mechanics?

10. Write down an expression of angular momentum in classical mechanics?

11. Write in classical mechanics, how the kinetic energy of a particle of mass μ can be written in terms of linear momentum

FINDING ANGULAR MOMENTUM IN QUANTUM MECHANICS

1. The imaginary number $i^2 = -1$, is this statement true or false?

2. What is $\hbar$?

3. Write the expression of time –independent Schrödinger wave equation?

4. The basic interpretation of the time independent of the Schrödinger equation (T I S E) is that if the energy *E* of the particle is measured, then the result will be one of the eigenvalues of the above equation.

(a) Is this statement true or false, give a reason for your answer?
(b) The location of the particle will be described by?

5. What does the non-zero commutator of $x \; and \; p_x$ signify?

6. Can you state Heisenberg uncertainty principle in quantum mechanics?

7. If we introduce the spherical polar coordinates, the classical kinetic energy can be divided into radial and angular parts. Write an expression of the angular kinetic energy?

8. From the Schrödinger representation of quantum mechanics, the classical linear momentum is replaced by the operator $\vec{p} = -i\vec{\nabla}$ write an expression for the quantum mechanical kinetic energy?

9. Write down the expression of linear momentum operator in quantum mechanics?

10. The solution of the angular part of the Laplace and Schrödinger equation for central force problems are spherical harmonics denoted $Y_\ell^m(\theta,\varphi)$. In this case $Y_\ell^m(\theta,\varphi)$ represents the eigenfunctions of what?

11. What are hermitian operators?

12. Derive any x, y and z-derivatives in terms of the momentum operators from $\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$ $\hat{p}_y = -i\hbar\frac{\partial}{\partial y}$ and $\hat{p}_z = -i\hbar\frac{\partial}{\partial z}$

13. Write down the components of the angular momentum $\vec{L}$ in terms of operators $\hat{p}_x, \hat{p}_y\ \hat{p}_z$

14. Consider two operators A and B. Write down the commutator [A, B] of the two operators?

15. Show that L^2 *and* L_z are commutative operators?

16. If [A, B] = 0, what do you say about A and B.

17. The representation of eigenvectors and eigenvalues is often more convenient using spherical coordinates: write an expression representing angular momentum operators in spherical coordinates?

18. The angular momentum is defined by: $\vec{L} = \vec{r} \times \vec{p}$ use the commutations relations between $\vec{r}$ *and* $\vec{p}$ to find the following commutation relations $[L_x, L_y]$, $[L_y, L_Z]$ and $[L_Z, L_X]$.

Peer-review history:
The peer review history for this paper can be accessed here:
http://www.sciencedomain.org/review-history/25183