



On Neutrosophic Z-algebras

Sahar Jaafar Mahmood ^a, Adel Salim Tayyah ^a and Dhirgam Allawy Hussein ^{b*}

^a Department of Multimedia, College of Computer Science and Information Technology, University of Al-Qadisiyah, P.O.Box-88, Al Diwaniyah, Al-Qadisiyah, Iraq.
^b Directorate of Education in Al-Qadisiyah, Diwaniyah, Iraq.

Authors' contributions

This work was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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Abstract

This study presents the notion of neutrosophic Z-algebra and neutrosophic pseudo Z-algebra explores some of its properties. Also studied are the neutrosophic Z-ideal, neutrosophic Z-sub algebra, and neutrosophic Z-filter. Several properties are discovered, and some findings from the study of homomorphism are discussed.

Keywords: Neutrosophic Z-algebra; neutrosophic pseudo Z-algebra; neutrosophic Z-sub algebra; neutrosophic Z-ideal; neutrosophic Z-filter.

1 Introduction

Smarandache established the area of philosophy known as neutrosophy, which has a many implementations in the real world and in mathematics, particularly in algebra [1].also gave more information about neutrosophy see [2,3]. making use of neutrosophic theory Kandasamy and Smarandache [4] in 2004 suggested a set-based algebraic structure of neutrosophic numbers of the type $\mathcal{N} = \mathcal{Z} + \uparrow \mathcal{J}$ that they dubbed \mathcal{J} -Neutrosophic Algebraic Structure. ,where $\mathcal{Z}, \uparrow \in \mathbb{R}$ or \mathbb{C} , and \mathcal{J} which means indetermined or uncertain thus that $\mathcal{J}^2 = \mathcal{J}$, is referred to as literal indeterminacy, here \mathcal{Z} is referred to as the \mathcal{N} 's determinate portion, and $\uparrow \mathcal{J}$ is referred to as its indeterminate portion on \mathcal{N} , with $g\mathcal{J} + h\mathcal{J} = (g + h)\mathcal{J}$, $0.\mathcal{J} = 0$. Where \mathcal{J} is different from the

*Corresponding author: Email: dhirgam.allawy@qu.edu.iq, dhirgam82@gmail.com;

imaginary $i^2 = -1$, in general, $J^j = J$ if $j > 0$, and is unknown for $j \leq 0$. In 2006, the idea of neutrosophic algebraic structures was also proposed [5].

In [6,7,8,9], the idea of neutrosophic BCI/BCK –algebras, neutrosophic KU-algebras and neutrosophic B-algebras was presented.

Z-algebra is an unique algebraic structure based on logic that was first proposed in 2017 by Chandramouleeswaran et al. [10].

[11] and [12] They provided characteristics and further explanation of Z-algebra.

In this article, we explain the idea of neutrosophic Z-algebra, look at various relevant characteristics, examine a neutrosophic Z-homomorphism, and present some findings.

2 Preliminaries

Definition 2.1: [1] A neutrosophic set $X(J) = \langle X, J \rangle = \{Z + \uparrow J : Z, \uparrow \in X\}$, where $X \neq \emptyset$ and J an indeterminate .

Definition 2.2: [10] let $Z \neq \emptyset$ and $*$ is a binary operation with constant 0 then the algebra $(Z, *, 0)$ named Z-Algebra if satisfying the following axiom:

- $Z_1: Z * 0 = 0$
- $Z_2: 0 * Z = Z$
- $Z_3: Z * Z = Z$
- $Z_4: Z * \uparrow = \uparrow * Z$ When $Z \neq 0$ and $\uparrow \neq 0, \forall Z, \uparrow \in Z$.

Definition 2.3: [10] Let $\delta \neq \emptyset$ and $\delta \subseteq Z$ where $(Z, *, 0)$ is a Z-Algebra, δ is named Z-subalgebra if $Z * \uparrow \in \delta, \forall Z, \uparrow \in \delta$.

Definition 2.4: [10] Let $J \neq \emptyset$ and $J \subseteq Z$, where $(Z, *, 0)$ is a Z-Algebra, J is named

Z-ideal of Z if satisfy (1) $0 \in J$ (2) $Z * \uparrow \in J$, and $\uparrow \in J \Rightarrow Z \in J$.

Definition 2.5: [11] Let $J \neq \emptyset$ and $J \subseteq Z$, where $(Z, *, 0)$ is a Z-Algebra, J is named Z_1 – ideal of Z if satisfy

- (1) $0 \in J$
- (2) $((Z * \lambda) * Z) * \uparrow \in J$, and $\uparrow \in J \Rightarrow Z \in J, \forall Z, \uparrow, \lambda \in Z$.

Definition 2.6: [11] Let $J \neq \emptyset$ and $J \subseteq Z$, where $(Z, *, 0)$ is a Z-Algebra, J is named Z_2 –ideal of Z if satisfy

- (1) $0 \in J$
- (2) $(Z * \lambda) * (Z * \uparrow) \in J$, and $\uparrow \in J \Rightarrow Z \in J, \forall Z, \uparrow, \lambda \in Z$

Definition 2.7: [12] Let $J \neq \emptyset$ and $J \subseteq Z$, where $(Z, *, 0)$ is a Z-Algebra, J is named Z_p –ideal of Z if satisfy

- (1) $0 \in J$
- (2) $(Z * \lambda) * (\uparrow * \lambda) \in J$, and $\uparrow \in J \Rightarrow Z \in J, \forall Z, \uparrow, \lambda \in Z$.

Definition 2.8: [10] let $\mathcal{F} \neq \emptyset$ and $\mathcal{F} \subseteq Z$, where $(Z, *, 0)$ is a Z-Algebra, \mathcal{F} is named Z-filter of Z if $Z \mathcal{X} \uparrow = Z * (Z * \uparrow) \in \mathcal{F}, \forall Z, \uparrow \in \mathcal{F}, (Z \neq \uparrow)$.

Example 2.9: let $Z = \{0, Z, \uparrow, \lambda\}$ be set and $*$ is a binary operation defined on Z by the table:

*	0	Z	↑	λ
0	0	Z	↑	λ
Z	0	Z	↑	↑
↑	0	↑	↑	↑
λ	0	↑	↑	λ

Then $(Z, *, 0)$ is Z -Algebra. $\delta = \{0, \uparrow, \downarrow\}$ is Z -subalgebra and $\mathcal{J} = \{0, \uparrow, \downarrow\}$ is a Z_1 -ideal, $\mathcal{J}^s = \{0, \uparrow, \downarrow\}$ is a (Z_2 -ideal, Z_p -ideal) Z -ideal .and $\mathcal{F} = \{0, \uparrow, \downarrow\}$ is Z -filter .

Note : every (Z_1 -ideal, Z_2 -ideal) is an ideal of Z .

Definition 2.10: [11] Let $Z \neq \emptyset$ with two binary operations $*$, \odot and constant 0 then the algebra $(Z, *, \odot, 0)$ named pseudo Z -Algebra (briefly, PZ) if satisfying the following axiom:

- PZ_1 : $\mathcal{Z} * 0 = \mathcal{Z} \odot 0 = 0$
- PZ_2 : $0 * \mathcal{Z} = 0 \odot \mathcal{Z} = \mathcal{Z}$
- PZ_3 : $\mathcal{Z} * \mathcal{Z} = \mathcal{Z} \odot \mathcal{Z} = \mathcal{Z}$
- PZ_4 : $\mathcal{Z} * \uparrow = \uparrow \odot \mathcal{Z}$ When $\mathcal{Z} \neq 0$ and $\uparrow \neq 0, \forall \mathcal{Z}, \uparrow \in Z$.

Definition 2.11: [11] Let $\delta \neq \emptyset$ and $\delta \subseteq Z$, where $(Z, *, \odot, 0)$ is PZ then δ is named a pseudo Z -subalgebra if $\mathcal{Z} * \uparrow, \mathcal{Z} \odot \uparrow \in \delta, \forall \mathcal{Z}, \uparrow \in \delta$.

Example 2.12: Let $Z = \{0, \uparrow, \downarrow, \lambda\}$ be set and $*$, \odot are a binary operations defined on Z by the table as follows:

*	0	\uparrow	\downarrow	λ	\odot	0	\uparrow	\downarrow	λ
0	0	\uparrow	\downarrow	λ	0	0	\uparrow	\downarrow	λ
\uparrow	0	\uparrow	\downarrow	λ	\uparrow	0	\uparrow	\downarrow	λ
\downarrow	0	\uparrow	\downarrow	λ	\downarrow	0	\uparrow	\downarrow	λ
λ	0	\uparrow	\downarrow	λ	λ	0	\uparrow	\downarrow	λ

Then $(Z, *, \odot, 0)$ is pseudo Z -algebra , $\delta = \{\uparrow, \downarrow, \lambda\}$ is a pseudo Z -sub algebra.

3 Neutrosophic Z-algebra

Definition 3.1: A neutrosophic Z -algebra is the triple $(Z(\mathcal{J}), *, (0, 0\mathcal{J}))$ (briefly, \mathcal{NZ}) (where $(Z, *, 0)$ be a Z -algebra , $Z(\mathcal{J}) = \langle Z, \mathcal{J} \rangle$ a neutrosophic set)

if $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\uparrow, \mathfrak{q}\mathcal{J})$ are any two elements of $Z(\mathcal{J})$ with $\mathcal{Z}, \mathfrak{h}, \uparrow, \mathfrak{q} \in Z$ satisfies

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\uparrow, \mathfrak{q}\mathcal{J}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \uparrow \wedge \mathfrak{h} * \mathfrak{q})\mathcal{J})$$

An element $\mathcal{Z} \in Z$ is represented by $(\mathcal{Z}, 0\mathcal{J}) \in Z(\mathcal{J})$,

$$(\mathcal{Z}, 0\mathcal{J}) * (\mathfrak{h}, 0\mathcal{J}) = (\mathcal{Z} * \mathfrak{h}, 0\mathcal{J}) = (\mathcal{Z} \wedge \sim \mathfrak{h}, 0) . \text{ where } \sim \mathfrak{h} \text{ is the negation of } \mathfrak{h} \text{ in } Z$$

$$\text{And } (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\uparrow, \mathfrak{q}\mathcal{J}) \Leftrightarrow (\mathcal{Z} = \uparrow \text{ and } \mathfrak{h} = \mathfrak{q})$$

Definition 3.2: A neutrosophic pseudo Z -algebra is $(Z(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ (briefly, \mathcal{NPZ}) (where $(Z, *, \odot, 0)$ be a pseudo Z -algebra

If $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\uparrow, \mathfrak{q}\mathcal{J})$ are any two elements of $Z(\mathcal{J})$ with $\mathfrak{x}, \mathfrak{h}, \uparrow, \mathfrak{q} \in Z$ satisfies

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\uparrow, \mathfrak{q}\mathcal{J}) = (\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \uparrow \wedge \mathfrak{h} * \mathfrak{q})\mathcal{J})$$

$$(\uparrow, \mathfrak{q}\mathcal{J}) \odot (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\uparrow \odot \mathcal{Z}, (\mathcal{Z} \odot \mathfrak{q} \wedge \mathfrak{h} \odot \uparrow \wedge \mathfrak{h} \odot \mathfrak{q})\mathcal{J})$$

Where $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\uparrow, \mathfrak{q}\mathcal{J}) = (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \odot (\uparrow, \mathfrak{q}\mathcal{J})$

When $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \neq (0, 0\mathcal{J})$ and $(\uparrow, \mathfrak{q}\mathcal{J}) \neq (0, 0\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\uparrow, \mathfrak{q}\mathcal{J}) \in Z(\mathcal{J})$

Theorem 3.3: Every $\mathcal{NZ} (Z(\mathcal{J}), *, (0, 0\mathcal{J}))$ with condition $(0, 0\mathcal{J}) * (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\mathcal{Z}, \mathfrak{h}\mathcal{J})$ is a Z -algebra and conversely, not.

Proof: let $(\mathcal{X}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is \mathcal{NZ}

Let $r = (\mathcal{Z}, \mathfrak{h}\mathcal{J})$ and $0 = (0, 0\mathcal{J})$

$$\begin{aligned} Z_1: r * 0 &= (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (0, 0\mathcal{J}) = (\mathcal{Z} * 0, (\mathcal{Z} * 0 \wedge \mathfrak{h} * 0)\mathcal{J}) = (0, (0 \wedge 0)\mathcal{J}) = (0, 0\mathcal{J}) \\ Z_2: 0 * r &= (0, 0\mathcal{J}) * (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (0 * \mathcal{Z}, (0 * \mathfrak{h} \wedge 0 * \mathcal{Z})\mathcal{J}) = (\mathcal{Z}, (\mathfrak{h} \wedge \mathcal{Z})\mathcal{J}) = (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \\ Z_3: r * r &= (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\mathcal{Z} * \mathcal{Z}, (\mathcal{Z} * \mathfrak{h} \wedge \mathfrak{h} * \mathcal{Z} \wedge \mathfrak{h} * \mathfrak{h})\mathcal{J}) \\ &= (\mathcal{Z}, (\mathcal{Z} \wedge \mathfrak{h} \wedge \mathfrak{h} \wedge \mathcal{Z} \wedge \mathfrak{h})\mathcal{J}) \\ &= (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \end{aligned}$$

Z_4 : if $r * s = s * r$, when $r \neq 0$ & $s \neq 0, \forall r, s \in \mathcal{Z}(\mathcal{J})$

let $r = (\mathcal{Z}, \mathfrak{h}\mathcal{J}), s = (\mathfrak{f}, \mathfrak{q}\mathcal{J}),$

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) = (\mathfrak{f}, \mathfrak{q}\mathcal{J}) * (\mathcal{Z}, \mathfrak{h}\mathcal{J})$$

$$(\mathcal{Z} * \mathfrak{f}, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \mathfrak{f} \wedge \mathfrak{h} * \mathfrak{q})\mathcal{J}) = (\mathfrak{f} * \mathcal{Z}, (\mathfrak{f} * \mathfrak{h} \wedge \mathfrak{q} * \mathcal{Z} \wedge \mathfrak{q} * \mathfrak{h})\mathcal{J})$$

Suppose $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \neq (0, 0\mathcal{J})$ & $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \neq (0, 0\mathcal{J})$ we get

$$0 * \mathfrak{f} = \mathfrak{f} * 0 \Rightarrow \mathfrak{f} = 0$$

$$\text{and } 0 * \mathfrak{q} \wedge 0 * 0 = 0 * 0 \wedge \mathfrak{q} * 0 \Rightarrow \mathfrak{q} = 0$$

We get a contradiction.

Then $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is a \mathcal{Z} -algebra.

Theorem 3.4: Every $\mathcal{NPZ}, (\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ with condition $(0, 0\mathcal{J}) * (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (0, 0\mathcal{J}) \odot (\mathcal{Z}, \mathfrak{h}\mathcal{J}) = (\mathcal{Z}, \mathfrak{h}\mathcal{J})$ is a pseudo \mathcal{Z} -algebra and conversely, not.

Proof: it is easy as above.

Definition 3.5: Let $\mathfrak{S}(\mathcal{J}) \neq \emptyset$ and $\mathfrak{S}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J}), (\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is $\mathcal{NZ}, \mathfrak{S}(\mathcal{J})$ is named a neutrosophic \mathcal{Z} -subalgebra (briefly, \mathcal{NZ}^s) of $\mathcal{Z}(\mathcal{J})$ if

- 1) $(0, 0\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
- 2) $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathfrak{S}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
- 3) $\mathfrak{S}(\mathcal{J})$ Contains a proper sub set which a \mathcal{Z} -algebra.

Definition 3.6: Let $\mathfrak{S}(\mathcal{J}) \neq \emptyset$ and $\mathfrak{S}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J}), (\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ is $\mathcal{NPZ}, \mathfrak{S}(\mathcal{J})$ is called a neutrosophic pseudo \mathcal{Z} -subalgebra (briefly, \mathcal{NPZ}^s) of $\mathcal{Z}(\mathcal{J})$ if

- 1) $(0, 0\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
- 2) $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$ & $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \odot (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathfrak{S}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathfrak{S}(\mathcal{J})$
- 3) $\mathfrak{S}(\mathcal{J})$ Contains a proper sub set which a pseudo \mathcal{Z} -algebra.

Theorem 3.7: If $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \neq \emptyset$ and $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$ for $\omega \neq 0, (\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is \mathcal{NZ} , where $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}(\mathcal{J}): (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J})\}$

Then 1) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ is \mathcal{NZ}^s .

2) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0, 0\mathcal{J})}(\mathcal{J})$.

Proof: 1) clearly $(0, 0\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$

$\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ contain a proper sub set which a \mathcal{Z} -algebra.

Let $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\uparrow, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J})$, $(\uparrow, \mathfrak{q}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \Rightarrow$

$\mathcal{Z} * \omega = \omega$, $\mathcal{Z} * \omega \wedge \mathfrak{h} * \omega = \omega$ & $\uparrow * \omega = \omega$, $\uparrow * \omega \wedge \mathfrak{q} * \omega = \omega$ since $\omega \neq 0 \Rightarrow$
 $\mathcal{Z} = \mathfrak{h} = \uparrow = \mathfrak{q} = \omega$

$$\begin{aligned} [(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\uparrow, \mathfrak{q}\mathcal{J})] * (\omega, \omega\mathcal{J}) &= [\mathcal{Z} * \uparrow, (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \uparrow)\mathcal{J}] * (\omega, \omega\mathcal{J}) \\ &= [(\mathcal{Z} * \uparrow) * \omega, ((\mathcal{Z} * \uparrow) * \omega \wedge (\mathcal{Z} * \mathfrak{q} \wedge \mathfrak{h} * \uparrow) * \omega)\mathcal{J}] \\ &= [\omega * \omega, (\omega * \omega \wedge \omega * \omega)\mathcal{J}] \\ &= (\omega, \omega\mathcal{J}) \end{aligned}$$

This shows that $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\uparrow, \mathfrak{q}\mathcal{J}) \in \mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$

Then $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ is \mathcal{NZ}^s .

(2) it's easy.

Theorem 3.8: If $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \neq \emptyset$ and $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, for $\omega \neq 0$,

$(\mathcal{Z}(\mathcal{J}), *, \odot, (0,0\mathcal{J}))$ is \mathcal{NPZ} , where $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) = \{(\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}(\mathcal{J}): (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\omega, \omega\mathcal{J}) = (\omega, \omega\mathcal{J}) \text{ \& } (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \odot \omega, \omega\mathcal{J} = \omega, \omega\mathcal{J}\}$

Then 1) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J})$ is \mathcal{NPZ}^s .

2) $\mathcal{A}_{(\omega, \omega\mathcal{J})}(\mathcal{J}) \subseteq \mathcal{A}_{(0,0\mathcal{J})}(\mathcal{J})$.

Proof: it is easy as above.

Theorem 3.9: If $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0,0\mathcal{J}))$ is \mathcal{NZ} , where
 $\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{J}): \mathcal{Z} \in \mathcal{Z}\}$ Then $\mathcal{Z}_\xi(\mathcal{J})$ is a \mathcal{NZ}^s of $\mathcal{Z}(\mathcal{J})$.

Proof: clearly $(0,0\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$ and the third condition is satisfied for $\mathcal{Z}_\xi(\mathcal{J})$

Let $(\uparrow, \uparrow\mathcal{J}), (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$, $\uparrow, \mathfrak{h} \in \mathcal{Z} \Rightarrow$

$$(\uparrow, \uparrow\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) = (\uparrow * \mathfrak{h}, (\uparrow * \mathfrak{h})\mathcal{J})$$

This shows that $(\uparrow, \uparrow\mathcal{J}) * (\mathfrak{h}, \mathfrak{h}\mathcal{J}) \in \mathcal{Z}_\xi(\mathcal{J})$

Then $\mathcal{Z}_\xi(\mathcal{J})$ is a \mathcal{NZ}^s of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.10: If $\mathcal{Z}_\xi(\mathcal{J}) \neq \emptyset$ and $\mathcal{Z}_\xi(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \odot, (0,0\mathcal{J}))$ is \mathcal{NPZ} , where

$\mathcal{Z}_\xi(\mathcal{J}) = \{(\mathcal{Z}, \mathcal{J}): \mathcal{Z} \in \mathcal{Z}\}$ Then $\mathcal{Z}_\xi(\mathcal{J})$ is a \mathcal{NPZ}^s of $\mathcal{Z}(\mathcal{J})$.

Proof: it is easy as above.

Example 3.11: Let $*$ is a binary operation defined on $\mathcal{Z}_\xi(\mathcal{J}) = \{(0,0\mathcal{J}), (\mathcal{Z}, \mathcal{J}), (\uparrow, \uparrow\mathcal{J}), (\lambda, \lambda\mathcal{J})\}$ as follows:

*	$(0,0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\uparrow, \uparrow\mathcal{J})$	$(\lambda, \lambda\mathcal{J})$
$(0,0\mathcal{J})$	$(0,0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\uparrow, \uparrow\mathcal{J})$	$(\lambda, \lambda\mathcal{J})$
$(\mathcal{Z}, \mathcal{J})$	$(0,0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(0,0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$
$(\uparrow, \uparrow\mathcal{J})$	$(0,0\mathcal{J})$	$(0,0\mathcal{J})$	$(\uparrow, \uparrow\mathcal{J})$	$(\uparrow, \uparrow\mathcal{J})$
$(\lambda, \lambda\mathcal{J})$	$(0,0\mathcal{J})$	$(\mathcal{Z}, \mathcal{J})$	$(\uparrow, \uparrow\mathcal{J})$	$(\lambda, \lambda\mathcal{J})$

Then $(\mathcal{Z}_\xi(\mathcal{J}), *, (0,0\mathcal{J}))$ is a \mathcal{NZ}^s of $\mathcal{Z}(\mathcal{J})$

Theorem 3.12: Let $\{\mathcal{A}(\mathcal{J})_\gamma: \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_\gamma \neq \emptyset$ be a collection of \mathcal{NZ}^s of $\mathcal{Z}(\mathcal{J})$ if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \neq \{(0,0\mathcal{J})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \text{ is a } \mathcal{NZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{J}).$$

Proof: since $(0,0\mathcal{J}) \in \mathcal{A}(\mathcal{J})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

$$(0,0\mathcal{J}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \neq \emptyset$$

And the third condition was achieved for $\mathcal{A}(\mathcal{J})_{\gamma}, \forall \gamma \in \mathcal{S} \Rightarrow$

The third condition was achieved for $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \neq \{(0,0\mathcal{J})\} \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{J}) \neq (0,0\mathcal{J}) \Rightarrow$$

$\{(0,0\mathcal{J})\} \subseteq \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$, which is a \mathcal{Z} – algebra

$$\text{Let } (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}(\mathcal{J})_{\gamma}, \forall \gamma \in \mathcal{S}$$

Since $\mathcal{A}(\mathcal{J})_{\gamma}$ is a $\mathcal{NZ}^{\mathcal{S}}$, $\forall \gamma \in \mathcal{S}$ of $\mathcal{Z}(\mathcal{J})$ then

$$(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}(\mathcal{J})_{\gamma}, \forall \gamma \in \mathcal{S}, \Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$$

hence $\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$ is a $\mathcal{NZ}^{\mathcal{S}}$ of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.13: Let $\{\mathcal{A}(\mathcal{J})_{\gamma}; \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_{\gamma} \neq \emptyset$ be a collection of $\mathcal{NPZ}^{\mathcal{S}}$ of $(\mathcal{Z}(\mathcal{J}), *, \oplus)$, $(0,0\mathcal{J})$ is \mathcal{NPZ} if

$$\bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \neq \{(0,0\mathcal{J})\} \Rightarrow \bigcap_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \text{ is a } \mathcal{NPZ}^{\mathcal{S}} \text{ of } \mathcal{Z}(\mathcal{J}).$$

Proof: it is easy as above.

Theorem 3.14: Let $\{\mathcal{A}(\mathcal{J})_{\gamma}; \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_{\gamma} \neq \emptyset$ be a collection of $\mathcal{NZ}^{\mathcal{S}}$ of $\mathcal{Z}(\mathcal{J})$ if $\mathcal{A}(\mathcal{J})_1 \subseteq \mathcal{A}(\mathcal{J})_2 \subseteq \dots$ then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$ is a $\mathcal{NZ}^{\mathcal{S}}$ of $\mathcal{Z}(\mathcal{J})$.

$$\text{Proof: obviously } (0,0\mathcal{J}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma} \neq \emptyset \Rightarrow \exists (\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$$

\Rightarrow For some $\gamma \in \mathcal{S}$ $(\mathcal{Z}, \mathfrak{h}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}(\mathcal{J})_{\gamma}$ and $(\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{A}(\mathcal{J})_{\gamma} \subseteq \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$

$$\Rightarrow (\mathcal{Z}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma}$$

Let $\mathfrak{S}(\mathcal{J})_\gamma$ be a proper sub set of $\mathcal{A}(\mathcal{J})_\gamma$, for some $\gamma \in \mathcal{S}$ which a Z - algebra,

then for any $\gamma \in \mathcal{S}, \mathfrak{S}(\mathcal{J})_\gamma \in \bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma$ then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_{\gamma \in \mathcal{S}}$ is \mathcal{NZ}^s of $Z(\mathcal{J})$.

Theorem 3.15: Let $\{\mathcal{A}(\mathcal{J})_\gamma: \gamma \in \mathcal{S}\}$ and $\mathcal{A}(\mathcal{J})_\gamma \neq \phi$ be a collection of \mathcal{NPZ}^s of $(Z(\mathcal{J}), *, \odot, (0,0\mathcal{J}))$ is \mathcal{NPZ} if $\mathcal{A}(\mathcal{J})_1 \subseteq \mathcal{A}(\mathcal{J})_2 \subseteq \dots$ then

$\bigcup_{\gamma \in \mathcal{S}} \mathcal{A}(\mathcal{J})_\gamma$ is \mathcal{NPZ}^s of $Z(\mathcal{J})$.

Proof: it is easy as above.

Definition 3.16: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq Z(\mathcal{J}), (Z(\mathcal{J}), *, (0,0\mathcal{J}))$ is \mathcal{NZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic Z -ideal (briefly, \mathcal{NZ}^i) of $Z(\mathcal{J})$ if :

- 1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$

Remark 3.17: Let $\mathcal{D}(\mathcal{J})$ is a \mathcal{NZ}^i of $Z(\mathcal{J})$ if

$(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$ and $(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) = (0,0\mathcal{J})$ then $(\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Proof: let $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$ and $(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) = (0,0\mathcal{J}) \Rightarrow$

$(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$ and $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J}), (\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$

Since $\mathcal{D}(\mathcal{J})$ is a $\mathcal{NZ}^i \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Definition 3.18: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq Z(\mathcal{J}), (Z(\mathcal{J}), *, \odot, (0,0\mathcal{J}))$ is \mathcal{NPZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudo Z -ideal (briefly, \mathcal{NPZ}^i) of $Z(\mathcal{J})$ if :

- 1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.
- 2) If $(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$

And $(\mathcal{L}, \mathfrak{h}\mathcal{J}) \odot (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$.

Definition 3.19: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq Z(\mathcal{J}), (Z(\mathcal{J}), *, (0,0\mathcal{J}))$ is \mathcal{NZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic Z_1 -ideal (briefly, \mathcal{NZ}^{i1}) of $Z(\mathcal{J})$ if :

- 1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $[(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{r}, \mathfrak{w}\mathcal{J})] * (\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{L}, \mathfrak{h}\mathcal{J}), (\mathfrak{r}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in Z(\mathcal{J})$

Definition 3.20: Let $\mathcal{D}(\mathcal{J}) \neq \phi$ and $\mathcal{D}(\mathcal{J}) \subseteq Z(\mathcal{J}), (Z(\mathcal{J}), *, \odot, (0,0\mathcal{J}))$ is \mathcal{NPZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudo Z_1 -ideal (briefly, \mathcal{NPZ}^{i1}) of $Z(\mathcal{J})$ if :

- 1) $(0,0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) $[(\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{r}, \mathfrak{w}\mathcal{J})] * (\mathcal{L}, \mathfrak{h}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{L}, \mathfrak{h}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{L}, \mathfrak{h}\mathcal{J}), (\mathfrak{r}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in Z(\mathcal{J})$

And $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \odot (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \odot (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

Definition 3.21: Let $\mathcal{D}(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is \mathcal{NZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic Z_2 -ideal (briefly, \mathcal{NZ}^{i2}) of $\mathcal{Z}(\mathcal{J})$ if :

- 1) $(0, 0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] * [(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$.

Definition 3.22: Let $\mathcal{D}(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ is \mathcal{NPZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudo Z_2 -ideal (briefly, \mathcal{NPZ}^{i2}) of $\mathcal{Z}(\mathcal{J})$ if :

- 1) $(0, 0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] * [(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{f}, \mathfrak{q}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

And $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \odot (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \odot [(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \odot (\mathfrak{f}, \mathfrak{q}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$.

Definition 3.23: Let $\mathcal{D}(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is \mathcal{NZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic Z_q -ideal (briefly, \mathcal{NZ}^{iq}) of $\mathcal{Z}(\mathcal{J})$ if :

- 1) $(0, 0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] * [(\mathfrak{f}, \mathfrak{q}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

Definition 3.24: Let $\mathcal{D}(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, \odot, (0, 0\mathcal{J}))$ is \mathcal{NPZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic pseudo Z_q ideal (briefly, \mathcal{NPZ}^{iq}) of $\mathcal{Z}(\mathcal{J})$ if :

- 1) $(0, 0\mathcal{J}) \in \mathcal{D}(\mathcal{J})$
- 2) If $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] * [(\mathfrak{f}, \mathfrak{q}\mathcal{J}) * (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

And $[(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \odot (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \odot [(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \odot (\mathfrak{a}, \mathfrak{w}\mathcal{J})] \in \mathcal{D}(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{D}(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in \mathcal{D}(\mathcal{J}), \forall (\mathcal{Z}, \mathfrak{b}\mathcal{J}), (\mathfrak{a}, \mathfrak{w}\mathcal{J}), (\mathfrak{f}, \mathfrak{q}\mathcal{J}) \in \mathcal{Z}(\mathcal{J})$

Definition 3.25: Let $\mathcal{D}_\xi(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}_\xi(\mathcal{J}) \subseteq \mathcal{Z}_\xi(\mathcal{J})$, $\mathcal{D}_\xi(\mathcal{J})$ is named a neutrosophic Z -ideal (briefly, $\mathcal{NZ}^{\xi i}$) of $\mathcal{Z}_\xi(\mathcal{J})$ if :

- 1) $(0, 0\mathcal{J}) \in \mathcal{D}_\xi(\mathcal{J})$
- 2) If $(\mathcal{Z}, \mathcal{Z}\mathcal{J}) * [(\mathfrak{f}, \mathfrak{f}\mathcal{J}) * (\mathfrak{a}, \mathfrak{a}\mathcal{J})] \in \mathcal{D}_\xi(\mathcal{J})$, and $(\mathfrak{f}, \mathfrak{f}\mathcal{J}) \in \mathcal{D}_\xi(\mathcal{J}) \Rightarrow (\mathcal{Z}, \mathcal{Z}\mathcal{J}) * (\mathfrak{a}, \mathfrak{a}\mathcal{J}) \in \mathcal{D}_\xi(\mathcal{J}), \forall (\mathcal{Z}, \mathcal{Z}\mathcal{J}), (\mathfrak{f}, \mathfrak{f}\mathcal{J}), (\mathfrak{a}, \mathfrak{a}\mathcal{J}) \in \mathcal{D}_\xi(\mathcal{J})$

Theorem 3.26: Every $\mathcal{NZ}^{\xi i}$ of $\mathcal{X}_\xi(\mathcal{J})$ is a \mathcal{NZ}^i of $\mathcal{X}_\xi(\mathcal{J})$.

Proof: suppose that $(\mathcal{Z}, \mathcal{Z}\mathcal{J}) = (0, 0\mathcal{J})$ in 2 \Rightarrow it's proofed.

Definition 3.27: Let $\mathcal{D}(\mathcal{J}) \neq \emptyset$ and $\mathcal{D}(\mathcal{J}) \subseteq \mathcal{Z}(\mathcal{J})$, $(\mathcal{Z}(\mathcal{J}), *, (0, 0\mathcal{J}))$ is \mathcal{NZ} , $\mathcal{D}(\mathcal{J})$ is named a neutrosophic Z -filter (briefly, \mathcal{NZ}^f) of $\mathcal{Z}(\mathcal{J})$ if :

- 1) $(0,0J) \notin \mathcal{D}(J)$
- 2) $\forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{D}(J)$ and $(\mathcal{Z}, \mathfrak{h}J) \neq (\uparrow, \mathfrak{q}J) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}J)\mathfrak{X}(\uparrow, \mathfrak{q}J) = (\mathcal{Z}, \mathfrak{h}J) * [(\mathcal{Z}, \mathfrak{h}J) * (\uparrow, \mathfrak{q}J)] \in \mathcal{D}(J)$

Definition 3.28: Let $\mathcal{D}(J) \neq \emptyset$ and $\mathcal{D}(J) \subseteq \mathcal{Z}(J)$, $(\mathcal{Z}(J), *, \odot, (0,0J))$ is \mathcal{NPZ} , $\mathcal{D}(J)$ is named a neutrosophic pseudo Z-filter (briefly, \mathcal{NPZ}^f) of $\mathcal{Z}(J)$ if :

- 1) $(0,0J) \notin \mathcal{D}(J)$
- 2) $\forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{D}(J)$ and $(\mathcal{Z}, \mathfrak{h}J) \neq (\uparrow, \mathfrak{q}J) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}J)\mathfrak{X}(\uparrow, \mathfrak{q}J) = (\mathcal{Z}, \mathfrak{h}J) * [(\mathcal{Z}, \mathfrak{h}J) * (\uparrow, \mathfrak{q}J)] \in \mathcal{D}(J)$

And $\forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{D}(J)$ and $(\mathcal{Z}, \mathfrak{h}J) \neq (\uparrow, \mathfrak{q}J) \Rightarrow$
 $(\mathcal{Z}, \mathfrak{h}J)\mathfrak{X}(\uparrow, \mathfrak{q}J) = (\mathcal{Z}, \mathfrak{h}J) \odot [(\mathcal{Z}, \mathfrak{h}J) \odot (\uparrow, \mathfrak{q}J)] \in \mathcal{D}(J)$

Definition 3.29: If $(\mathcal{Z}(J), *, \odot, (0,0J))$ & $(\mathcal{Z}'(J), \dot{*}, \dot{\odot}, (\dot{0}, \dot{0}J))$ be two \mathcal{NZ} , a mapping $f: \mathcal{Z}(J) \rightarrow \mathcal{Z}'(J)$ is named a neutrosophic Z- homomorphism (briefly, \mathcal{NZ}^h) if satisfied

- 1) $f[(\mathcal{Z}, \mathfrak{h}J) * (\uparrow, \mathfrak{q}J)] = f(\mathcal{Z}, \mathfrak{h}J) \dot{*} f(\uparrow, \mathfrak{q}J), \forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{Z}(J)$
- 2) $f(0,0J) = (\dot{0}, \dot{0}J)$
- 3) If f is 1-1 $\Rightarrow f$ is named a neutrosophic Z- monomorphism.
- 4) If f is onto $\Rightarrow f$ is named a neutrosophic Z- epimorphism.
- 5) If f is 1-1 and onto $\Rightarrow f$ is named a neutrosophic Z-isomorphism.

Definition 3.30: If $(\mathcal{Z}(J), *, \odot, (0,0J))$ & $(\mathcal{Z}'(J), \dot{*}, \dot{\odot}, (\dot{0}, \dot{0}J))$ be two \mathcal{NPZ} , a mapping $f: \mathcal{Z}(J) \rightarrow \mathcal{Z}'(J)$ is named a neutrosophic pseudo Z- homomorphism (briefly, \mathcal{NPZ}^h) if satisfied

- 1) $f[(\mathcal{Z}, \mathfrak{h}J) * (\uparrow, \mathfrak{q}J)] = f(\mathcal{Z}, \mathfrak{h}J) \dot{*} f(\uparrow, \mathfrak{q}J), \forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{Z}(J)$
- 2) $f[(\mathcal{Z}, \mathfrak{h}J) \odot (\uparrow, \mathfrak{q}J)] = f(\mathcal{Z}, \mathfrak{h}J) \dot{\odot} f(\uparrow, \mathfrak{q}J), \forall (\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in \mathcal{Z}(J)$
- 3) $f(0,0J) = (\dot{0}, \dot{0}J)$
- 4) If f is 1-1 $\Rightarrow f$ is named "a neutrosophic pseudo Z- monomorphism".
- 5) If f is onto $\Rightarrow f$ is named "a neutrosophic pseudo Z- epimorphism".
- 6) If f is 1-1 and onto $\Rightarrow f$ is named a neutrosophic pseudo Z-isomorphism.

Theorem 3.31: Let $\mathcal{Z}(J)$ & $\mathcal{Z}'(J)$ be two \mathcal{NZ} , $f: \mathcal{Z}(J) \rightarrow \mathcal{Z}'(J)$ be a neutrosophic Z- epimorphism .If $\mathcal{D}(J)$ is a \mathcal{NPZ}^f of $\mathcal{Z}(J) \Rightarrow f(\mathcal{D}(J))$ is a \mathcal{NPZ}^f of $\mathcal{Z}'(J)$.

Proof: let $(\mathcal{Z}, \mathfrak{h}J), (\uparrow, \mathfrak{q}J) \in f(\mathcal{D}(J)) \Rightarrow$

$$(\mathcal{Z}, \mathfrak{h}J) = f(\mathfrak{a}, \mathfrak{w}J) \quad , \quad (\uparrow, \mathfrak{q}J) = f(\mathfrak{Q}, \mathfrak{E}J) \quad \text{where } (\mathfrak{a}, \mathfrak{w}J), (\mathfrak{Q}, \mathfrak{E}J) \in \mathcal{D}(J)$$

Since $\mathcal{D}(J)$ is a \mathcal{NPZ}^f of $\mathcal{Z}(J)$, \Rightarrow

$$(\mathfrak{a}, \mathfrak{w}J)\mathfrak{X}(\mathfrak{Q}, \mathfrak{E}J) = (\mathfrak{a}, \mathfrak{w}J) * [(\mathfrak{a}, \mathfrak{w}J) * (\mathfrak{Q}, \mathfrak{E}J)] \in \mathcal{D}(J)$$

Also $f((\mathfrak{a}, \mathfrak{w}J)\mathfrak{X}(\mathfrak{Q}, \mathfrak{E}J)) \in f(\mathcal{D}(J))$

$$\begin{aligned} (\mathcal{Z}, \mathfrak{h}J)\mathfrak{X}(\uparrow, \mathfrak{q}J) &= (\mathcal{Z}, \mathfrak{h}J) * ((\mathcal{Z}, \mathfrak{h}J) * (\uparrow, \mathfrak{q}J)) \\ &= f(\mathfrak{a}, \mathfrak{w}J) \dot{*} (f(\mathfrak{a}, \mathfrak{w}J) \dot{*} f(\mathfrak{Q}, \mathfrak{E}J)) \\ &= f [(\mathfrak{a}, \mathfrak{w}J) * ((\mathfrak{a}, \mathfrak{w}J) * (\mathfrak{Q}, \mathfrak{E}J))] \\ &= f [(\mathfrak{a}, \mathfrak{w}J)\mathfrak{X}(\mathfrak{Q}, \mathfrak{E}J)] \end{aligned}$$

$(\mathcal{Z}, \mathfrak{h}J)\mathfrak{X}(\uparrow, \mathfrak{q}J) \in f(\mathcal{D}(J)) \Rightarrow$
 $f(\mathcal{D}(J))$ is a \mathcal{NPZ}^f of $\mathcal{Z}'(J)$.

Theorem 3.32: Let $Z(\mathcal{J})$ & $Z(\hat{\mathcal{J}})$ be two NPZ, $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ be a neutrosophic pseudo Z- epimorphism .If $\mathcal{D}(\mathcal{J})$ is a NPZ^f of $Z(\mathcal{J}) \Rightarrow f(\mathcal{D}(\mathcal{J}))$ is a NPZ^f of $Z(\hat{\mathcal{J}})$.

Proof: it is easy as above.

Definition 3.33: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ be a \mathcal{NZ}^h then $\ker(f) = \{(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J})\}$ is named the kernel of f .

Definition 3.34: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ be a NPZ^h then

$\ker(f) = \{(\mathcal{Z}, \mathfrak{b}\mathcal{J}) \in Z(\mathcal{J}): f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J})\}$ is named the kernel of f .

Remark 3.35: (1) Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ is a \mathcal{NZ}^h , then $\ker(f)$ is not a \mathcal{NZ}^f of $Z(\mathcal{J})$.
 (2) \mathcal{NZ}^f is not \mathcal{NZ}^i and conversely .
 (3) \mathcal{NZ}^f is not \mathcal{NZ}^s and conversely .

Remark 3.36: (1) Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ is a NPZ^h, then $\ker(f)$ is not a NPZ^f of $Z(\mathcal{J})$.
 (2) NPZ^f is not NPZⁱ and conversely .
 (3) NPZ^f is not NPZ^s and conversely .

Theorem 3.37: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ be a \mathcal{NZ}^h then

- 1) If the identity of $Z(\mathcal{J})$ is $(0,0\mathcal{J}) \Rightarrow$ the identity of $Z(\hat{\mathcal{J}})$ is $f(0,0\mathcal{J})$.
- 2) If \mathcal{U} is a \mathcal{NZ}^s of $Z(\mathcal{J})$, then $f(\mathcal{U})$ is a \mathcal{NZ}^s of $Z(\hat{\mathcal{J}})$.
- 3) If \mathcal{U} is a \mathcal{NZ}^s of $Z(\hat{\mathcal{J}})$, then $f^{-1}(\mathcal{U})$ is a \mathcal{NZ}^s of $Z(\mathcal{J})$.

Proof: it's clear.

Theorem 3.38: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ be a NPZ^h then

- 1) If the identity of $Z(\mathcal{J})$ is $(0,0\mathcal{J}) \Rightarrow$ the identity of $Z(\hat{\mathcal{J}})$ is $f(0,0\mathcal{J})$.
- 2) If \mathcal{U} is a NPZ^s of $Z(\mathcal{J})$, then $f(\mathcal{U})$ is a NPZ^s of $Z(\hat{\mathcal{J}})$.
- 3) If \mathcal{U} is a NPZ^s of $Z(\hat{\mathcal{J}})$, then $f^{-1}(\mathcal{U})$ is a NPZ^s of $Z(\mathcal{J})$.

Proof: it's clear.

Theorem 3.39: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ is a \mathcal{NZ}^h then f is a neutrosophic Z- monomorphism $\Leftrightarrow \ker(f) = \{(0,0\mathcal{J})\}$

Proof: it's clear.

Theorem 3.40: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ is a NPZ^h then f is a neutrosophic Z- monomorphism $\Leftrightarrow \ker(f) = \{(0,0\mathcal{J})\}$

Proof: it's clear.

Theorem 3.41: Let $f: Z(\mathcal{J}) \rightarrow Z(\hat{\mathcal{J}})$ is a \mathcal{NZ}^h then $\ker(f)$ is a \mathcal{NZ}^i of $Z(\mathcal{J})$.

Proof: $f(0,0\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J}) \Rightarrow (0,0\mathcal{J}) \in \ker(f)$

Let $(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * [(\uparrow, \mathfrak{q}\mathcal{J}) * (\lambda, \omega\mathcal{J})] \in \ker(f)$ and $(\uparrow, \mathfrak{q}\mathcal{J}) \in \ker(f) \Rightarrow$

$$\begin{aligned} f((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * [(\uparrow, \mathfrak{q}\mathcal{J}) * (\lambda, \omega\mathcal{J})]) &= (\hat{0}, \hat{0}\mathcal{J}) \text{ and } f(\uparrow, \mathfrak{q}\mathcal{J}) = (\hat{0}, \hat{0}\mathcal{J}) \\ (\hat{0}, \hat{0}\mathcal{J}) &= f((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * [(\uparrow, \mathfrak{q}\mathcal{J}) * (\lambda, \omega\mathcal{J})]) \\ &= f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * [f(\uparrow, \mathfrak{q}\mathcal{J}) * f(\lambda, \omega\mathcal{J})] \\ &= f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * [(\hat{0}, \hat{0}\mathcal{J}) * f(\lambda, \omega\mathcal{J})] \end{aligned}$$

$$\begin{aligned} &= f(\mathcal{Z}, \mathfrak{b}\mathcal{J}) * f(\mathfrak{a}, \mathfrak{u}\mathcal{J}) \\ &= f((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \end{aligned}$$

We get $((\mathcal{Z}, \mathfrak{b}\mathcal{J}) * (\mathfrak{a}, \mathfrak{u}\mathcal{J})) \in \ker(f)$. then $\ker(f)$ is a \mathcal{NZ}^i of $\mathcal{Z}(\mathcal{J})$.

Theorem 3.42: Let $f: \mathcal{Z}(\mathcal{J}) \rightarrow \mathcal{Z}(\hat{\mathcal{J}})$ is a \mathcal{NPZ}^h then $\ker(f)$ is a \mathcal{NPZ}^i of $\mathcal{Z}(\mathcal{J})$.

Proof: it is easy as above.

4 Conclusion

We discussed the idea of a neutrosophic \mathcal{Z} -algebra and neutrosophic pseudo \mathcal{Z} – algebra looked into some of its properties, and the concept of neutrosophic \mathcal{Z} -ideal, neutrosophic \mathcal{Z} -sub algebra, neutrosophic \mathcal{Z} -filter and neutrosophic \mathcal{Z} - homomorphism are studied and a few properties are obtained.

Competing Interests

Authors have declared that no competing interests exist.

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