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Research Article

Fractional-Order Adaptive *P*-Laplace Equation-Based Art Image Edge Detection

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In recent years, with the rapid development of image processing research, the study of nonstandard images has gradually become a research hotspot, for example, fabric images, remote sensing images, and gear images. Some of the remote sensing images have a complex background and low illumination compared with standard images and are easy to be mixed with noise during acquisition; some of the fabric images have rich texture information, which adds difficulty to the related processing, and are also easy to be mixed with noise during acquisition. In this paper, we propose a fractional-order adaptive *P*-Laplace equation image edge detection algorithm for the problem of image edge detection in which the edge and texture information of the image is lost. The algorithm can apply for the order adaptively to filter the noise according to the noise distribution of the image, and the adaptive diffusion factor is determined by both the fractional-order curvature and fractional-order gradient of the iso-illumination line and combined with the iterative approach to realize the fine-tuning of the noisy image. The experimental results demonstrate that the algorithm can remove the noise while preserving the texture and details of the image. A fractional-order partial differential equation image edge detection model with a fractional-order fidelity term is proposed for Gaussian noise. The model incorporates a fractional-order fidelity term because this fidelity term smoothes out the rougher parts of the image while preserving the texture in the original image in greater detail and eliminating the step effect produced by other models such as the Perona-Malik (PM) and Rudin-Osher-Fatemi (ROF) models. By comparing with other algorithms, the image edge detection effect is measured with the help of evaluation metrics such as peak signal-to-noise ratio and structural similarity, and the optimal value is selected iteratively so that the image with the best edge detection result is retained. A convolutional mask image edge detection model based on adaptive fractional-order calculus is proposed for the scattered noise in medical images. The adaption is mainly reflected in the model algorithm by constructing an exponential parameter relation that is closely related to the image, which can dynamically adjust the parameter values, thus making the model algorithm more practical. The model achieves the scattering noise removal in four steps.

1. Introduction

Nowadays, access to information is becoming more and more important, and information exists in a variety of forms, not simply in the form of voice transmission, but in a variety of forms, including information technology data, text, images, and video. Nowadays, a lot of information is extracted from images, but in the process of transmitting or photographing images, we often use devices or transmission media limitations, making the acquired images more or less mixed with different shapes and colors of noise, because the existence of noise which leads to the visual effect of the whole image becomes very poor, which will seriously affect people's access to information in the image. For this reason, we regard noise removal as an important task in image processing, because if the noise can be minimized, it will help to obtain more local information in the image, which is also related to the solution of the problems such as the accuracy of image segmentation, the accuracy of target recognition, and the completeness of edge extraction. Therefore, to obtain better image quality, it is essential to perform edge detection operations on images. It is because of this demand that researchers spend efforts to explore better image edge detection algorithms to obtain as much information as possible for subsequent processing and analysis of images. Digital images are often disturbed by noise from various sources during acquisition and capture, which not only deteriorates the visual effect of digital images but also increases the difficulty of subsequent image processing. Therefore, edge detection is an important part of digital image processing. Scholars at home and abroad have conducted a lot of research on image edge detection and proposed edge detection algorithms such as median filtering, Wiener filtering, mean filtering, and low-pass filtering. However, these algorithms also make the edge and texture information of the image lost while edging detection, and the image becomes blurred. Therefore, how to be able to remove the noise while preserving the edge and texture information of the image is the difficulty of image edge detection.

The Fourier transform, a mathematical idea widely used in partial differential equation edge detection algorithms, is a central theory that refracts the frequency domain components of signal information in the entire continuous time range with the help of the interrelationship between the time domain and the frequency domain, most importantly, of course, its spectral information. Although the Fourier transform has a strong local capability in the frequency domain, it still needs to be further strengthened. With further optimization of subsequent studies, researchers found that fractional order can be applied to the image domain, which in turn gave rise to an emerging field, namely, fractional-order adaptive calculus, which can be further optimized based on integer-order differential equations. It is also of interest to study the fractional-order adaptive calculus for processing in the image domain. For scattered noise, the noise mainly exists in medical images, and the image is mainly manifested as black and white chaotic distribution of points; this noise will affect the clarity of the image; the image is very destructive, so it will not be conducive to doctor's access to information in the image; although the traditional method can achieve part of the edge detection effect, the overall performance is still very poor, such as Gaussian filter and wavelet transform, with the development of fractional-order definition; the edge detection model based on the mask template can remove this noise well, so further research on this model is also very meaningful.

2. Related Studies

The current research on image edge detection algorithms has been the focus of the image processing direction; the essence of the algorithm is to remove the noise points in the image, by designing the relevant model algorithm to reconstruct the image. Over the next few decades, researchers have experimented with image edge detection algorithms, which have given birth to many classical edge detection algorithms, such as Gaussian filtering and Wiener filtering, and subsequent developments such as wavelet transform and low-pass filtering. The spatial domain filtering refers to those direct filtering operations on the image, which can not only undeniably eliminate some of the noise in the image but also seriously damage the image edge detail information, causing local distortion of the image, filtering, etc.

The literature [1] improves the PM model in another way by using a Gaussian kernel to regularize the gradients, which results in a more stable and improved PM model. This model not only identifies the edge regions of the image very well but also deals with the fact that the passing gradients will reduce the sensitivity to noise, which makes the image in general slowdown the step effect. The literature [2] also improves the PM model by proposing a tensor-type diffusion model, who mainly uses the diffusion tensor to replace the diffusion coefficient function in the PM model, although this improved model mainly addresses images with flow-like structures like fingerprints, which can be well enhanced for this class of images. In the literature [3, 4], it was found that the edge and slope regions of an image can be identified by differential curvature, and both constructed two adaptive diffusion coefficients to discriminate the whole image. In the literature [5], a nonlinear anisotropic combined diffusion model was designed; this model is mainly by choosing different diffusion methods when facing different regions of the image, homogeneous diffusion for smooth regions, and mean curvature diffusion for edge regions, and the effectiveness of this model was experimentally demonstrated. The literature [6] combined the concept of fractional order to improve a model of anisotropic diffusion, which also led to the research on the application of fractional order in the field of image edge detection. The literature [7] proposed an improved image edge detection model with anisotropic diffusion by different diffusion coefficient functions. The literature [8] gives an image diffusion model, which reflects the complexity of image texture with the help of local variance of the image, and also introduces fractional order in the model. In the literature [9], an adaptive fractional-order image edge detection model is proposed to improve the PM model, which introduces fractional-order and fidelity terms, both of which will greatly improve the situation of "step effect" in the PM model and also reduce the running time of the algorithm model and remove the noise in the image while preserving. It also reduces the running time of the algorithm model and removes the noise from the image while preserving most of the texture in the image.

In addition, scholars have proposed many improved edge detection algorithms for the PDE model based on the framework of the PM model and ROF model in recent years. The literature [10] was the first to propose a fuzzy algorithm for image enhancement, which is the first algorithm to apply fuzzy sets to the field of image enhancement, and many researchers have joined the study because of the novelty of the algorithm. The literature [11] proposed an edge enhancement algorithm with the help of fuzzy theory to obtain the image feature values, and the method is good for enhancing the image detail information. The literature [12] proposed a fuzzy contrast enhancement algorithm that adapts the local contrast for image enhancement. The literature [13] proposes an algorithm that first selects weights with the help of fuzzy rules to minimize the mean squared error of the image output, and experiments show that this rule-controlled median filter enhancement algorithm is effective. The literature [14] designs three different filters with the help of the set fuzzy rules and enhances the three target regions in the image with the help of these three filters. The overall image enhancement effect is significant because the appropriate filters are used to process the corresponding image regions. In the literature [15], a fuzzy rule based on the fire operator was established to enhance different regions of the image. In [16], a fuzzy rule is established to divide the image regions, and then, the fuzzy rule is combined with wavelet transform to reconstruct the image. In the paper [17], an image enhancement algorithm was designed to determine the limit size by fuzzy rules, which combines the laws of fuzzy logic and histogram to process mammogram images.

3. Art Image Edge Detection Model Based on Fractional-Order Adaptive *P*-Laplace Equation

3.1. Fractional-Order Adaptive P-Laplace Equation. When the concept of fractal theory was introduced, the theory of fractional-order calculus appeared in the vision of researchers, and through continuous research and development, the theory of fractional-order calculus was proved to be feasible for image enhancement. At present, there are two widely accepted definitions, one is the Grümwald-Letnikov definition and the other is the Riemann-Liouville definition, and the most widely used definitions in the frequency domain are two kinds of transformations, namely, the Fourier transform and the *P*-Laplace transform.

The origin of the G-L definition was first derived from the integer order and then slowly spread to the fractional-order domain, as follows.

First, understand the first-order derivative, which is mathematically defined as shown below.

$$g'(x) = \lim_{\Delta x \longrightarrow 0} \frac{g(x+\Delta x) - g(x)}{\Delta x}. \quad (1)$$

Then, the transition to the second-order derivative is shown as follows.

$$g''(x) = \lim_{\Delta x \longrightarrow 0} \frac{g(x+2\Delta x) - 2g(x+\Delta x) + g(x)}{\Delta x^2}. \quad (2)$$

Finally, extending to the nth integer-order derivative definition,

$$g^n(x) = \lim_{a \longrightarrow 0} \frac{1}{a^n} \sum g(x - ak) \begin{pmatrix} n \\ k \end{pmatrix} (-1)^k. \quad (3)$$

The central idea of the definition of R-L is to summarize a law after integrating the function several times and then extend this law to all real numbers, which is the idea of the definition of R-L we mentioned. For a productive function $f(x)$, integrating the function n times gives

$$R^n f(x) = \int\int g(t)dt = \lim_{n \longrightarrow \infty} \frac{1}{n} \sum_{i=1}^{n} \left(x^i - t^{i-1}\right) f(t). \quad (4)$$

Then, generalizing this to all real numbers, the definition of the α-order calculus of R-L can be expressed as

$$L_{R-D}^n f(x) = \frac{1}{\eta(\alpha)} \int\int (x-\eta)^\alpha g(\eta) d\eta. \quad (5)$$

3.2. Fractional-Order P-Laplace Equation for Signals. When it comes to fractional-order differential operations, here, we mainly talk about differential operations under Fourier transform. Based on the property that the Fourier transform can better handle discrete data information, the time domain signal can be transformed into frequency domain signal in image processing and then discrete processing, so that we can better process the image information. For the implementation of the partial differential equation-based model code presented later, the process is also obtained by interconverting the time and frequency domains. For a productive real function $2f(x) \in L(R)$, its Fourier transform is defined as

$$F(\rho) = \int k^{-i2\pi\omega x} f(x) dx. \quad (6)$$

The corresponding inversions are

$$f(\rho) = \frac{1}{\sigma} \int F(\rho) e^{-i\omega x} dx. \quad (7)$$

It is known that the general real $\alpha \in R+$ differentiation of $2f(x) \in L(R)$ can be expressed as

$$K^m f(x) = \frac{1}{m} \frac{\partial^2 F(x)}{\partial \rho \partial x}. \quad (8)$$

The form of $Df(x)\alpha$ in the Fourier transform domain can be obtained from the above equation.

$$L^\kappa f(x) = \frac{1}{\sqrt{m}} \int (i2\pi\omega)^\alpha e^{i2\pi\omega\theta} d\omega. \quad (9)$$

In Figure 1, we divide the whole image into two parts to see, at $0 < \omega < 1$, the curve mainly reflects the smooth region of the image, and according to the transformation form of the curve, we can see that in the smooth region, the weakening ability of the fractional order to this region is weaker than the integer order, so it has better ability to preserve the details of the smooth image; for the region of $\omega > 1$, it mainly reflects the texture and edge regions of the image, which belongs to the location of high-frequency information, and from the figure, we can see that as the fractional order increases, it has a stronger ability to enhance the whole image, both of which are significantly better than the integer order. In general, the fractional order is significantly better than the integer order for each region of the image, so it is

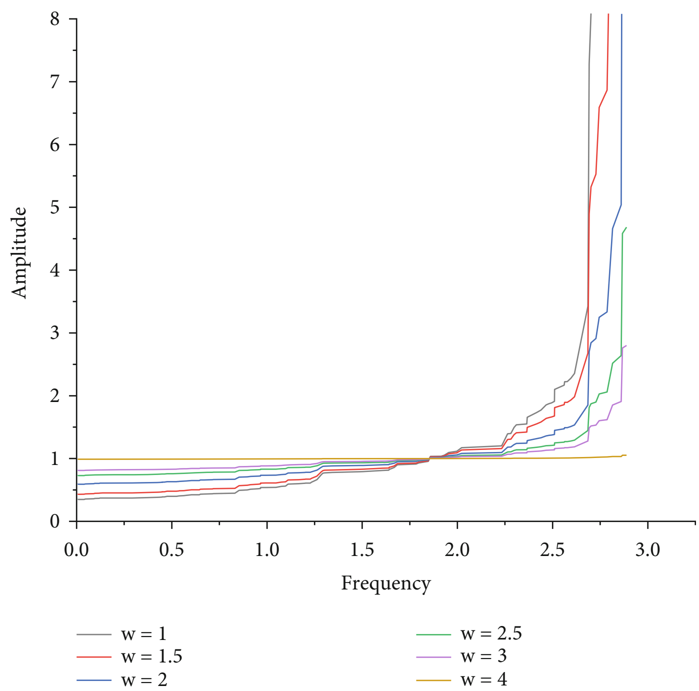

Figure 1: Amplitude-frequency characteristic curve of fractional-order differentiation.

practical to use the fractional order in the field of image enhancement.

In the frequency domain, the fractional-order integral operation is an operation when the fractional-order differential order takes a negative value because the integral operation and the differential operation are inverse operations, so through the differential operation expression, we can invert the frequency domain expression of the fractional-order integral, that is, when taking $\beta=-\alpha, I=D^{-1}$, then the mathematical expression is as follows.

$$I^{\alpha} f(\eta)=(i\omega)^{\alpha} f(\omega)=|\omega|^{\alpha} e^{i2\pi\alpha} f(\omega), \tag{10}$$

where I_{β} is the integral operator of order β. Similarly, according to Equation (10), we can draw its corresponding frequency-domain relationship curve, in which we mainly show the curve when α takes negative values, i.e., $\alpha=-0.01$, $-0.1-$,-0.2,-0.4,-0.7,-1 (as shown in Figure 2).

Similarly, it is divided into two parts for analysis, when $0<\omega<1$, it shows the low-frequency region, and the effect shown by the fractional-order integral operator in this region is better than the integer order, which can protect the low-frequency information of the image; when $\omega>1$, it corresponds to the high-frequency information region, and the trend graph of the curve indicates the effectiveness of the fractional-order integral operator; although no matter integer order or fractional order, it can protect the high-frequency information of the image well; the superiority of the fractional-order operator is shown with the increase of the fractional-order operator. Therefore, the fractional-order integral operator can protect the originality and accuracy of the image on the whole, which is the advantage of the fractional-order integral operator compared with the integer-order differential operator.

3.3. Artistic Image Edge Detection Method Based on Fractional-Order P-Laplace Equation. In the process of image acquisition, due to the influence of environmental factors, noise factors, light, and other factors, the quality of the acquired images change to a certain extent, and there will be adverse effects such as image blurring and grayscale offsetting. In this paper, we mainly take the various types of insulator images of composite insulators as an example for processing. Firstly, we need to preprocess the digital images simply by using digital image processing technology and then determine the hydrophobic grade, and this topic adopts the image processing technology based on fractional-order P-Laplace equation to process the composite insulator images and get the processed images after The processed image is analyzed by using the maximum area ratio method and the improved shape factor method to obtain the composite insulator. The processed image is analyzed using the maximum area ratio method and the improved shape factor method to obtain the art image edge detection method, and the image analysis flow chart is shown in Figure 3.

Since the collected composite insulator image is a color image, it should be converted into a grayscale image first in the processing process, and the grayscale image changes from 0 to 255, where 0 indicates the darkest black image and 255 indicates the brightest white image. To improve the operation speed of the subsequent algorithm and the accuracy of the hydrophobic image recognition, it is necessary to grayscale the image in this process, and the process of histogram equalization of grayscale images is mainly the process of transforming the original image into its uniform

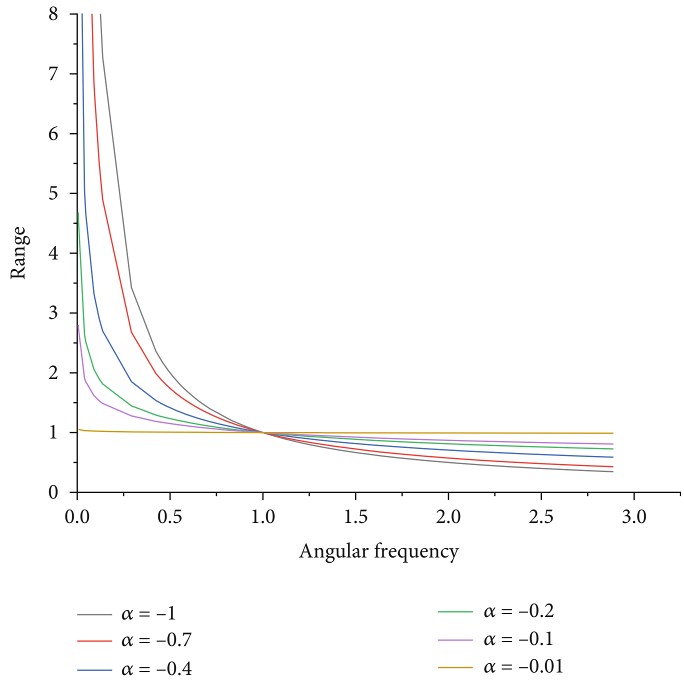

FIGURE 2: Magnitude-frequency characteristic curve of fractional-order integration.

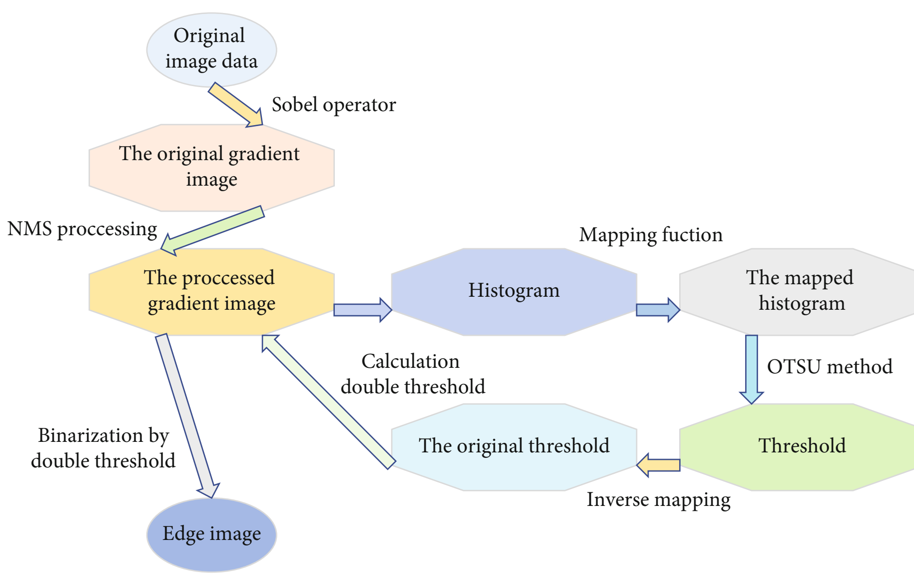

FIGURE 3: Image edge detection analysis flow.

distribution. To expand the grayscale range of the image, it is more evenly distributed and does not deviate too much from the normal range, making the image look sharper. The idea of histogram equalization is to turn the gray histogram of the original image from a relatively concentrated gray interval into a uniform distribution over the entire gray range, using the cumulative distribution function as the transformation function.

$$\gamma_K = \sum \rho(\sigma) 0 \le \sigma \le 1. \tag{11}$$

Since there is a large amount of random noise in the detected images, to remove the effects of these noises, we design a median filter to remove the effects of these noises. Median filtering is a nonlinear processing technique based on the traditional idea of statistical sorting that can provide accurate suppression of noise [18]. And the noise is filtered while the image detail information (such as sharpening corners and edges) is better protected, thus eliminating isolated noise points, and the two-dimensional median filtering output is

$$f(x, y) = \mathrm{sigma}\{f(x-k, y-l), k, l \in R\}. \tag{12}$$

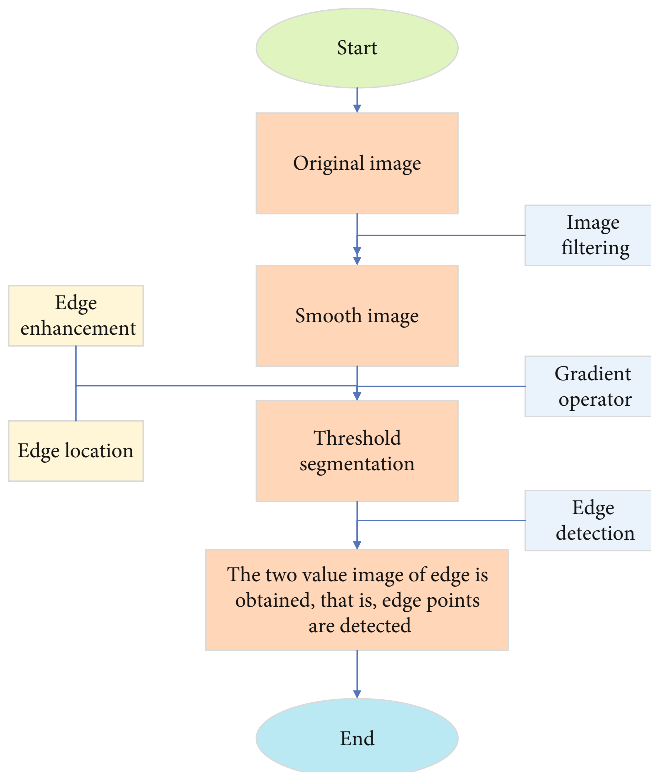

FIGURE 4: Art image edge detection noise processing flow.

To obtain better noise immunity, we need to optimize and improve the traditional detection method with the basic operational steps: first, perform image preprocessing; then, perform edge detection; finally, enhance the obtained results and wait for the processing to obtain the desired results, but image preprocessing often blurs the image, which leads to loss of details [19]. The structural elements can be selected according to the essential features of the image and selected and adjusted for different situations to facilitate their application. The following operations can be constructed by improving them based on the basic operator.

$$E_{\xi}(x, y) = f \otimes \xi - f \odot \xi. \tag{13}$$

However, if we want to effectively suppress both the bright and dark noise of the image, we need to synthesize that, taking into account the smoothing theory of the image, the following operator is constructed.

$$E_{\tau}(x, y) = (f \otimes \tau) \times \tau - (f \odot \tau)^{\tau}, \tag{14}$$

$$E_{\rho}(x, y) = (f \odot \rho)^{\rho} + (f \oplus \rho). \tag{15}$$

For the processing of art image edge detection noise, the process shown in Figure 4 can be referred to. Compared with the classical signal singularity detection algorithm, the detection algorithm based on the fractional-order adaptive P-Laplace equation can directly obtain the initial wavelet coefficients without inner product calculation and then decompose the wavelet coefficients by interpolating the pairwise filter to achieve the detection of singular points. This algorithm can improve the accuracy of edge point coefficients based on reducing the computation of classical new sign singularity coefficients and then obtain more accurate detection results [20].

To sum up, the edge detection algorithm of this topic can integrate the fractional order of the P-Laplace equation and decompose directly from the image pixel matrix to get the requested edge image. Compared with the classical detection algorithm, the algorithm in this paper can get the initial image edge coefficients quickly, and the tower decomposition for image edge coefficients avoids the integration process in the classical detection algorithm, which can effectively avoid the noise of integration and get more accurate image edge coefficients.

4. Experimental Results and Analysis

Firstly, edge extraction is performed for the same model using different algorithms, and the detection results obtained are shown in Figure 5. From Figure 5, we can see that Wiener, AFFT, PM, Wavelet, NL-Means, GAFIA, FOIA, and the algorithm in this paper have certain edge detection abilities for noisy images with noise intensity of 1 and 0.5, respectively, but the traditional FOIA model with fractional order $\alpha = -0.9$ has poor edge detection. The model in this paper can preserve the texture better to a certain extent, which can be seen in Figure 5; the residual map of this model retains better image details, and it can be seen that most of the detailed information of the image is preserved, and the

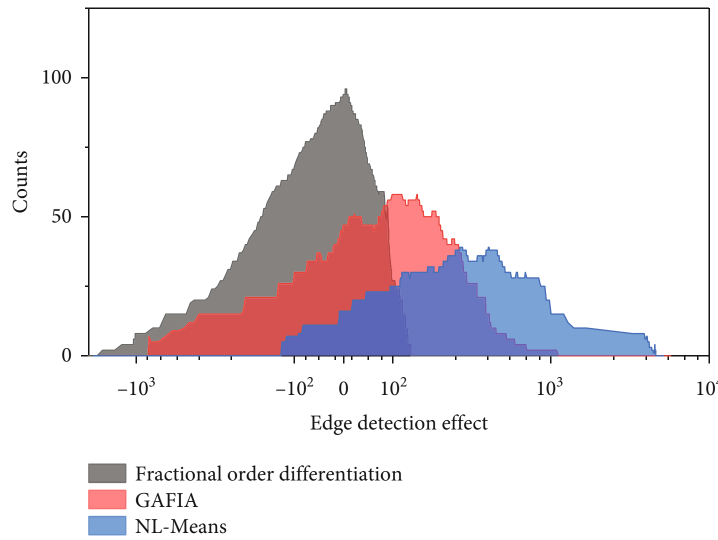

Figure 5: Comparison of the edge detection effect of several classical edge detection algorithms.

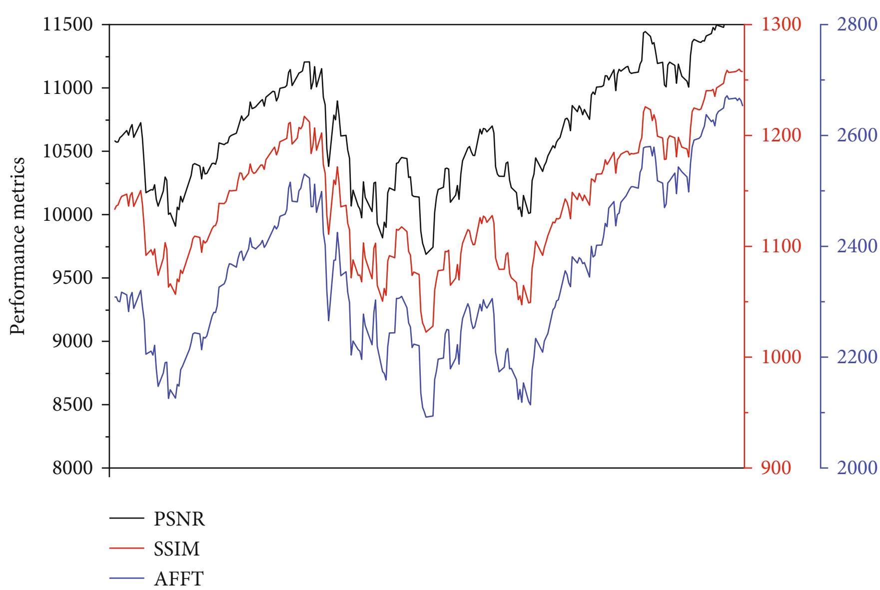

Figure 6: Performance metrics of different model algorithms for images with different noise PSNR and SSIM.

performance index also objectively reflects the effectiveness of this model, within a certain noise intensity range, the index of this model. The numerical values of this model are significantly better than several other comparison models, which can also confirm the effectiveness of this model.

Of course, it is not comprehensive to evaluate the merits of the edge detection effect of each algorithm model by visual images alone. Therefore, to better evaluate the edge detection ability of each model, we evaluate the PSNR and SSIM by the objective performance metrics of each model. Figure 6 shows the PSNR and SSIM of each model for different ultrasonic images at different noise intensities. The data in Figure 6 show that the index value of each model decreases faster with the increase of noise intensity, but the performance index of this model decreases slowly, which also reflects that this model has a better edge detection effect on ultrasonic images with high noise intensity. However, as the noise intensity increases, its index decreases faster, which is also reflected in the images, i.e., the overall brightness of the images is darker and blurred, and the performance indexes of the model are better for both the kidney ultrasound images and the other two ultrasound images, which also objectively reflects the effectiveness of the model. To demonstrate the effectiveness of the model more visually, the line graphs of the performance indexes in Figure 6 are used to show the performance of the model in comparison with other models at various noise intensities. The SSIM index of this model is much better than other models regardless of the noise intensity, and considering these two performance indexes, this model has better results.

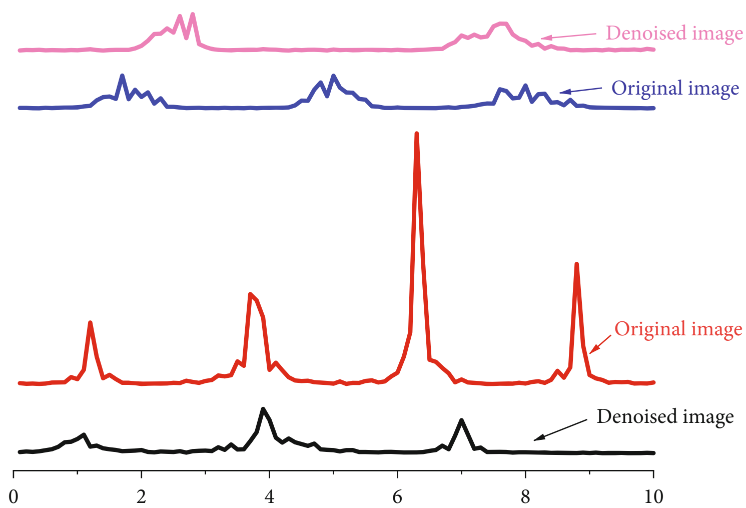

Figure 7: Comparison of grayscale values of ultrasonic images.

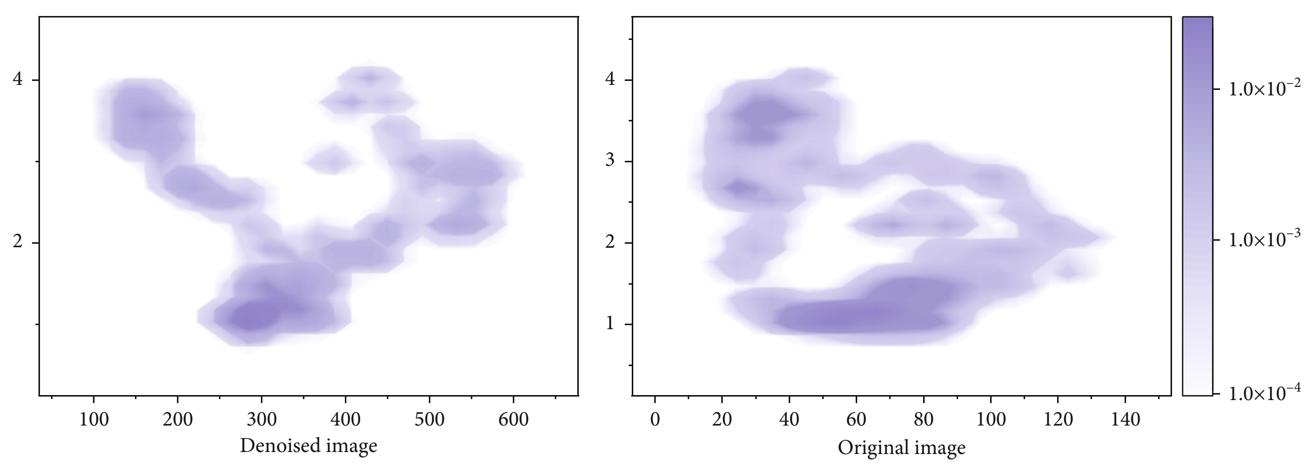

Figure 8: Detailed edge images in different directions.

In Figure 7, the gray value comparison between the edge detection image, the noise image, and the original image is mainly reflected in Figure 7, in which the blue color represents part of the gray value curve of the original image, the light red color represents part of the gray value of the edge detection image, and the dark red color curve represents part of the gray of the noise image. From the figure, we can see that the grayscale values of the images under the noise intensity have changed substantially, and the grayscale values are larger or smaller, while the image after edge detection fits the original image to a certain extent, and the noise in the noisy image is removed.

The final complete edge can be obtained from the detailed edge images in three directions, and the edge detection results of each layer are shown in Figure 8. It can be seen that the edge details detected by the first layer of decomposition are lost more, and there are many discontinuous edges. The clearer and more complete edge lines can be obtained after the second layer decomposition, but some details such as the nose and the contour of the background wall are not detected. And these details can be extracted accurately by the third layer decomposition. The edge image of the third layer has a better detection effect on the detailed-rich hair, hats, and features with a more detailed portrayal. The edge lines in the fourth layer are thicker, and the portrayal of edge contours is more obvious. Similar to the detection results of simple images, the lower the decomposition level of interpolated wavelet tower decomposition for complex portraits, the finer the edge lines obtained, and the weaker the detail detection ability. As the decomposition level increases, the edge lines of the image are also thicker, but the more accurate the edge localization is, the more detailed edges can be detected. Here, some other images are selected again for edge extraction using the edge detection algorithm based on the P-Laplace equation, and the corresponding images with complete edge detection are shown. The detection results of different images show that the edge detection algorithm of this topic has a strong detail extraction ability and can obtain good edge extraction results as the edges with weak contrast are also carved more carefully.

The evaluation criteria of the edge detection algorithm are divided into two aspects: qualitative analysis and quantitative analysis. Qualitative analysis is a comparative analysis of the visual effect of the algorithm detection results, based on which the edge detection effect can be classified into five levels: poor, poor, average, good, and good. And quantitative analysis is to compare the good and bad detection results after calculating the specific index parameters for the evaluation of the edge detection algorithm. Noise and edges are usually high-frequency components, and the above differential operator adds high-frequency components by taking derivatives of pixels, so it is difficult to accurately distinguish between noise and edges in images, leading to misclassification of edge regions and noise in many images. The *P*-Laplace algorithm can solve the contradiction between the accuracy of edge detection and the resistance to noise interference in the differential operator. However, the classical edge detection algorithm also has some problems. There is a contradiction between the image resolution and the accuracy of wavelet coefficient solution in the classical detection algorithm, and it is difficult to obtain accurate edge detection coefficients, so it is easy to have problems such as inaccurate positioning during the detection. From the experimental results, it can be seen that the edges extracted by the classical detection algorithm are not continuous and complete enough, and the edge lines are blurred, and many detailed parts are not detected. The edge images obtained by using the *P*-Laplace decomposition algorithm in this paper not only have accurate localization and continuous and complete edges but also do not detect too many pseudoedges as the traditional operator; thus, the detection effect is better than other methods mentioned above. For the detailed part in the background, only the canny operator and the algorithm in this paper can detect it more accurately, and the other algorithms cannot extract the details of the leaf vein part completely. For the detailed part of the pattern on the ball, it can be seen that the detected edges of this algorithm are clear and complete without breakpoints, and the detection effect is the best. Canny operator detects many pseudoedges on both the surface of the ball and the background part, so the detection effect is not accurate enough. The detection results of other algorithms have many discontinuous edges, which are not well-positioned and have poor detection results.

A qualitative and quantitative comparison of the simulation results obtained with various differential operators and classical wavelet algorithms shows that the advantages of the *P*-Laplace algorithm are reflected in the following aspects.

(1) Traditional differential operators such as the Canny algorithm are affected by noise, and the extracted edge images have many isolated noise points, which increase the false rate of edges. In contrast, the *P*-Laplace algorithm obtains few isolated noise points in the edges, and this advantage is more prominent as the number of decomposition layers increases

(2) The edges extracted by the classical detection algorithm are discontinuous, with many interrupted points in the middle, resulting in a poorly defined edge line. In contrast, the edge lines in the edge image obtained by the *P*-Laplace algorithm are clear and smooth, and there are no interrupting points

(3) The classical detection algorithm is not accurate enough for edge localization. For example, the detection of the background leaf vein part in the football image has an obvious deviation compared with the original image. In contrast, the extraction results of the *P*-Laplace algorithm do not have this phenomenon, and the extracted edge positions are consistent with the original image

(4) The classical detection algorithm is more complex and computationally intensive. While the algorithm of this topic implements a simpler process, the computational volume is smaller, which can effectively improve the efficiency of edge detection

Therefore, it can be concluded that the *P*-Laplace algorithm used in this topic has better edge detection capability compared with the traditional differential operator and classical detection algorithm. The algorithm of this topic can quickly perform edge extraction of images and achieve excellent detection results.

5. Conclusion

This paper mainly introduces the application of fractional-order calculus theory based on the image edge detection model; for different types of noise, it is applied to different scenarios; for the common Gaussian noise in life, the theory is used in the partial differential equation, to remove the noise while also eliminating the step effect and then achieve better preservation of the image texture; for the medical scattered noise, the fractional-order calculus function is combined with a model of fuzzy theory to process medical images. The combination of these models has to some extent promoted the development of the image edge detection field. The main work accomplished in this paper includes the following aspects: the two noise models dealt with in this paper are introduced, followed by several edge detection algorithms related to the models proposed in this paper and the related theory of fractional-order calculus in this paper, to have a certain understanding of the theoretical basis of this paper, and finally, point out the image. Finally, we point out several performance evaluation indexes of images, by which the effect of image edge detection is objectively evaluated; we introduce the application of fractional-order calculus in the direction of the partial differential equation and propose a fractional-order calculus-based partial differential equation image edge detection model for common Gaussian noise. The fractional-order fidelity term can make the image after edge detection closer to the original image, that is, more details can be protected from being discarded, while the gray value fidelity term is mainly used to measure the similarity of the gray value variation between images, which will promote the model to obtain better edge detection. In the latter study, it is found that the integer-order calculus is much less effective than the fractional-order calculus in preserving image information, processing high-frequency information, and smoothing low-frequency information. A detailed analysis is also given about the comparison between the two, and finally, the experimental results also show that the image edge detection model based on the partial differential equation of the fractional-order calculus can not only remove the Gaussian noise in the image but also preserve the image detail The implementation principle of singularity detection using classical detection methods is described in detail. The defects of the classical detection algorithm are analyzed from the perspectives of experimental results and mathematical proofs, respectively, namely, there is a contradiction between the image resolution and the accuracy of wavelet coefficient solution in the classical mocha measurement method. On this basis, the advantages of edge image solving based on the P-Laplace equation are analyzed.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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