Asian Journal of Probability and Statistics

Volume 26, Issue 4, Page 8-21, 2024; Article no.AJPAS.115306
ISSN: 2582-0230

Bayesian Probabilistic Projection of Population Census in the Kingdom of Saudi Arabia

Saheed A. Afolabi [a*]

[a] *Department of Mathematics, College of Computing and Mathematics, King Fahd University of Petroleum and Minerals, Dhahran 31261, Kingdom of Saudi Arabia.*

Author's contribution

The sole author designed, analyzed, interpreted and prepared the manuscript.

Article Information

DOI: 10.9734/AJPAS/2024/v26i4605

Open Peer Review History:
This journal follows the Advanced Open Peer Review policy. Identity of the Reviewers, Editor(s) and additional Reviewers, peer review comments, different versions of the manuscript, comments of the editors, etc are available here: https://www.sdiarticle5.com/review-history/115306

Original Research Article

Received: 01/02/2024
Accepted: 04/04/2024
Published: 13/04/2024

Abstract

Population census supplies a complete and accurate picture of a country's population and its residents' characteristics. Modeling population growth has been worked upon by different scholars before now with more of a classical approach and less of a Bayesian approach. Therefore, an attempt is made in this work to apply Bayesian probabilistic projection on the usual exponential growth rate model in estimating population parameters and predicting population census in the Kingdom of Saudi Arabia (KSA) across thirteen (13) regions. The obtained data from WorldData and United Nations Population were used for the estimation and projection with the application of appropriate prior, likelihood, and posterior selection through Bayesian inference. This approach is reasonably accurate and well-calibrated with a significant precision of 0.01025 approximately 99% model accuracy for the period due to the estimated population parameters that were used: to make a comparison with the 2019 Population Census of Saudi Arabia which was perfectly closed and to forecast for the next 80 years using out-sample cases.

**Corresponding author: Email: afolabisa63@gmail.com;*

Keywords: Population growth rate; conjugate prior; Predictive Inference (PI); Highest Density Interval (HDI); projection.

1 Introduction

A population is simply the entire group of individuals about which we seek information. According to Kenneth [1], the discipline of demography has developed over the past several centuries in an endeavour to make measurements of population and life course in a reliable and relevant manner. As such, it is one of the best examples of how mathematics and statistics are used in the real world.

A population census can be found as the complete process of collection, organization, analysis, and dissemination of demographic and socio-economic related information at a particular period to all the people in a country or in a purely delineated region of a country. Obisesan et al. [2] added that one of the most significant sources of information that serves as the foundation for the country's official statistics is the census. A census is often a periodic activity that counts the population of a country and records some demographic information about the enumerated people.

One of the prominent approaches in demography for population projection is using a population growth model. The model can be achieved differently through arithmetic, geometric, exponential, logistic, Gompertz, and many more in which the population parameters can be estimated using various estimation methods like the ordinary least square, maximum likelihood, or Bayesian. Moreover, all these techniques can be related to linear regression as narrated by Alparslan et al. [3] that the mathematical technique of linear regression is the basis of statistical research.

Kenneth [1] made it clear that the growth rate of a population denoted as 'R' is the slope of the graph of the logarithm of population size over time. It measures the rate of change in population size proportionally. Sibly and Hone [4] added that the parameter which best summarizes trends in population abundance or density is population growth. Additionally, Bayesian statistics offer various approaches to describe information about a prior which can be informative or non-informative in nature, and the leverage of these prior claims can vary significantly within similar system of inferences [5].

However, standard population approaches are deterministic, which implies that they generate a single predicted value for each quantity of relevance. Nonetheless, there is a strong desire for probabilistic forecasts through probability distributions for the respective quantity of interest, and hence express uncertainty over the projections [6].

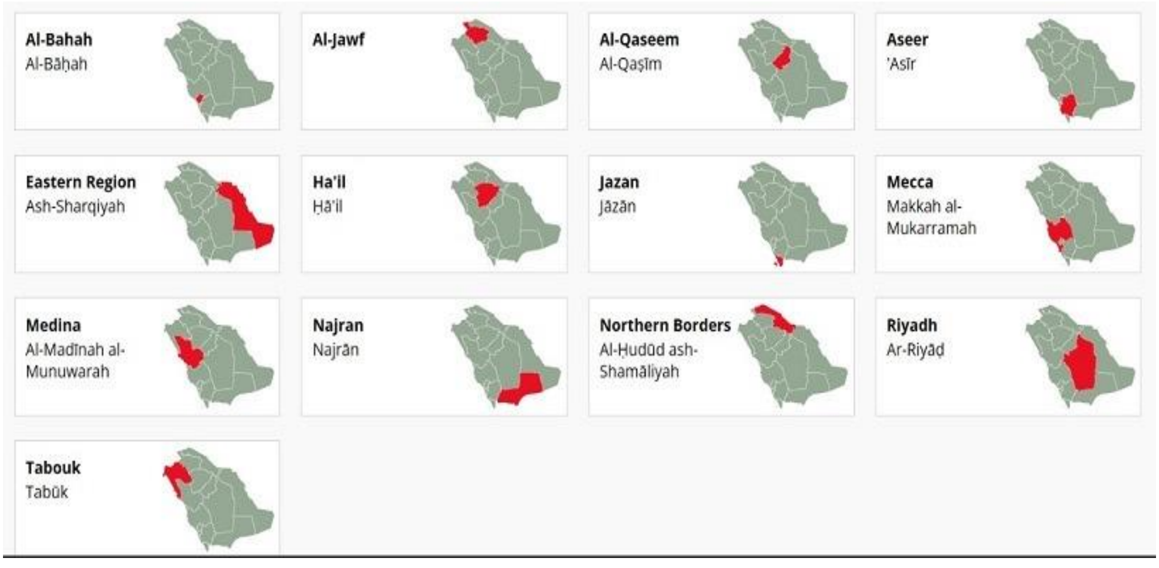

Fig. 1. Thirteen regions of KSA
Source: Saudi Arabia: Regions, Governorates, Cities - Population Statistics in Maps and Charts (citypopulation.de)

Kingdom of Saudi Arabia, the largest nation on the Arabian Penisula, has undergone changes in all dimensions of its population. The population size had increased tremendously due to natural growth and immigration. Age-structure pressures in the Kingdom's population expansion have impacted the education, labor force and women's employment sectors [7]. Fig. 1 above established the thirteen regions of the Kingdom, which are: Riyadh, Mecca (Makkah), Medina (Madinah), Al-Bahah, Al-Qassim, Eastern, Aseer (Asir), Tabuk, Ha'll, Northern Borders, Jazan, Najran and Al-jawf.

2 Literature Review

Sibly and Hone [4] investigated population growth rate and its determinants using one of the major growth rate models named exponential growth rate model. Their study focused on the different facets of population ecology, and it was argued by them that the key role that the population growth rate plays in predicting future population trends contributes to its significance. In fact, if the density dependence's form were constant and well-known, future population dynamics might be somewhat predictable. They later concluded that the best way to define and analyze population management, density dependency, resource and interference competition, environmental stress impacts, and ecological niche shape is in terms of population growth rate.

Obisesan et al. [2] studied the population census in Nigeria with the application of mathematical projections through absolute, relative, arithmetic, geometric and exponential population growth models. They were able to use 1991 as the initial year and 2006 as the final year across 28 states out of 36 due to some missing values to generate the growth rates (R). Then, the rates were used to make prediction for population census in 2020 with the application of 2006 initial population size over the period (t). It was later recommended to the government and all concerned agencies to get ready for the impending population increase and its consequences.

Wakefield et al. [8] carried out research on the population pharmacokinetic with pharmacodynamic modeling using the Bayesian approach by combining three normal distributions. In the same vein, Azose and Raftery (2015) applied the Bayesian hierarchical models to probabilistic projection of international migration of some selected counties of Western Asia including Saudi Arabia using the World Population Prospects (WPP010) data set. Migon and Gamerman [9] proposed growth models in a generalized form via Bayesian approach as an improvement to logistic, Gompertz and modified exponential growth rate models.

Raftery et al. [10] investigated the population projections for all countries using Bayesian approach by breaking down the population into age and sex for proper planning in their research. After converting United Nations population statistics into age-specific rates, the Bayesian hierarchical models' estimates were applied using Markov Chain Monte Carlo (MCMC) and coupled with a projection model with cohort components. They were able to forecast up-to the year 2100 for all the countries and it was indicated from their result that in most countries, the supporting ratio (people aged between 20 to 64 per person aged over 65 i.e. 65+) will decline precipitously over the next few decades.

The methodological concepts of population projections were investigated by Vanella et al. [11] using public health data in Germany with the emphasis of the deterministic and stochastic techniques with Normal distribution for both qualitative and quantitative forecasting using simulation of Monte Carlo. Chao et al. [12] also carried out the research on the skewed levels of ratio of sex at birth (SBR) due to sex-selective abortions in several countries since 1970s using Bayesian probabilistic projections based on 3.26 billion birth records for the forecast between 2021 to 2100 based on various scenarios of sex ratio transition and they were able to provide adequate recommendation.

Alparslan et al. [3] suggested new Bayesian inference paradigm differentially distributed private linear regression using technique of Markov Chain Monte Carlo (MCMC) to get regression coefficients for appropriate model prediction and they were able to provide numerical results. Chao et al. [13] also made use of the Bayesian modelling approach to estimate and project the sex ratio at births' levels and trends in seven provinces of Nepal ranging from the year 1980 to 2050.

Several researchers listed above have previously worked on population growth modeling using more classical and Bayesian methods. But none of them has incorporated Bayesian estimation on exponential growth model for country's population prediction through estimated parameters [growth rate; R and logarithmic initial population size; $\ln K(0)$]. Hence, this paper is being undertaken to estimate the population parameters, make probabilistic projection, compare the predicted values to the actual population census of the year 2019 across the thirteen regions of the Saudi Arabia Kingdom and produce population of cohort and pyramid by age and sex using Bayesian Approach.

3 Methodology

This section gives the probabilistic methods in achieving set objectives of this study through Bayesian inference with the application of Bayes theorem.

3.1 Bayes theorem

This theorem presents the probability of an event (E) based on the prior knowledge of conditions that might be in relation to that event. It can be mathematically expressed as:

$$P(E \mid S) = \frac{P(E).P(S \mid E)}{P(S)} \tag{1}$$

where $P(S) = \sum_{i=1}^{n} P(E_i).P(S \mid E_i)$ for discrete case and $P(S) = \int_{-\infty}^{\infty} P(E_i).P(S \mid E_i)dE_i$ for continuous case.

Relating (1) to the parameter estimation using the approach of Bayesian as this paper concerns, we have the expression:

$$f(\theta_i \mid x) = \frac{f(\theta_i).f(x \mid \theta_i)}{\int_{-\infty}^{\infty} f(\theta_i).f(x \mid \theta_i)d\theta_i} \tag{2}$$

where $f(x \mid \theta_i)$ denotes the probability distribution of the random variables *X*, it can as well be expressed in likelihood form given as $L(\theta_i \mid x)$, $f(\theta_i)$ is the prior information about the parameter θ_i and $f(\theta_i \mid x)$ is the posterior distribution.

3.2 Bayesian linear regression

From the literature reviews, the exponential growth rate model with final population size *K(t)*, initial population size *K*(0*)*, years *t* and Rate *R* itself is defined as:

$$K(t)=K(0)\exp(Rt) \tag{3}$$

Taking the natural logarithm of the (3) and relating to the simple linear regression, we have:

$$\ln K(t)=\ln K(0) + Rt + \varepsilon_i \tag{4}$$

where $\varepsilon_i \cong N(0,\sigma^2)$ and $\ln K(t) \cong N(\mu,\sigma^2)$

According to Will Koehrsen, (2018) the $\ln K(t) \cong N(\mu,\sigma^2)$ in (4) implies that it follows normal distribution of a continuous probability distribution. Meaning that the mean $\mu = \ln K(0) + Rt$ and hence; $\ln K(t) \cong N[\ln K(0) + Rt, \sigma^2]$. For simplicity, let $y_i = \ln K(t)$, $x_i = t$ such that $\beta_0 + \beta_1 = \ln K(0) + R$ as the population parameters. Hence, the probability distribution function (pdf) of the model is presented as:

$$f(y_i \mid x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{1}{2}\left(\frac{y_i - \beta_0 - \beta_1 x_i}{\sigma} \right)^2 \right]$$

$$f(y_i \mid x_i) = \frac{1}{\sqrt{2\pi\sigma^2}} exp\left[-\frac{(y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^2} \right] \tag{5}$$

where μ and σ^2 are the unknown parameters to be estimated for the population growth. Transforming (5) into the likelihood form; we have the derivation of:

$$L(\mu,\sigma^2 \mid y_i) = (2\pi)^{-n/2}(\sigma^2)^{-n/2} exp\left[-\frac{\sum_{i=1}^{n}(y_i - \beta_0 - \beta_1 x_i)^2}{2\sigma^2} \right]. \tag{6}$$

3.3 Bayesian probabilistic projection

This section deals with the construction of prior, likelihood and posterior distributions in relation to (6) together with means and variances' expressions. Zolaktaf [14] and [15] distinguished among noninformative, informative and conjugate priors. For this study, an assumption of the standard noninformative prior $p(\mu,\sigma^2) \;\alpha\; 1/\sigma^2$ is used to yield the conjugate prior through the posterior distribution.

3.3.1 Likelihood function

By the application of the prior by denoting $\phi = \frac{1}{\sigma^2}$, the likelihood function is expressed as:

$$L(\mu,\sigma^2 \mid y_i) = (\sigma^2)^{-n/2} exp\left[-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(y_i - \mu)^2 \right]. \tag{7}$$

By appropriate simplification and using the decomposition concept of $\sum_{i=1}^{n}(y_i - \bar{y} + \bar{y} - \mu)^2$, we have the form:

$$L(\mu,\phi \mid y_i) = \phi^{n/2} exp\left[-\frac{\phi}{2}\left((n-1)s^2 + n(\mu - \bar{y})^2 \right) \right]. \tag{8}$$

3.3.2 Prior distribution

As established by Clyde et al. [16], the prior distribution of equation (8) follows Normal Gamma mixture distribution with parameters $(\mu_0, n_0, \sigma_0^2, v_0)$ which implies that $\mu,\phi \sim NormGamma(\mu_0, n_0, \sigma_0^2, v_0)$, then the joint probability function is given as;

$$f(\mu,\phi) = \frac{(n_0\phi)^{1/2}}{\sqrt{2\pi}} exp\left[-\frac{n_0\phi(\mu - \mu_0)^2}{2} \right] \times \frac{\phi^{\frac{v_0}{2}-1} exp\left[-\frac{v_0\sigma_0^2\phi}{2} \right]}{\left(\frac{v_0\sigma_0^2}{2} \right)^{v_0/2} \Gamma(v_0/2)} \tag{9}$$

while marginal distributions for the μ and ϕ are expressed as:

$$f(\mu \mid \phi) = \frac{1}{\sqrt{2\pi \frac{\sigma^2}{n_0}}} exp\left[-\frac{(\mu - \mu_0)^2}{2\sigma^2 / n_0} \right] \text{ and}$$

$$f(\phi) = \frac{\phi^{\frac{v_0}{2}-1} exp\left[-\frac{\phi}{2/(v_0\sigma_0^2)} \right]}{\left(\frac{v_0\sigma_0^2}{2} \right)^{v_0/2} \Gamma(v_0/2)}. \tag{10}$$

3.3.3 Posterior distribution

By combining equations (8) and (9), the posterior distribution is derived using the mathematical expression $f(\mu,\phi \mid y_i) = L(\mu,\phi \mid y_i) f(\mu,\phi)$.

$$f(\mu,\phi \mid y_i) = \phi^{n/2} exp\left[-\frac{\phi}{2}\left((n-1)s^2 + n(\mu - \bar{y})^2\right) \right] \times \frac{(n_0\phi)^{1/2}}{\sqrt{2\pi}} exp\left[-\frac{n_0\phi(\mu - \mu_0)^2}{2} \right] \times \frac{\phi^{\frac{v_0}{2}-1} exp\left[-\frac{v_0\sigma_0^2\phi}{2} \right]}{\left(\frac{v_0\sigma_0^2}{2} \right)^{v_0/2} \Gamma(v_0/2)}.$$

Ignoring the terms without the parameters μ and ϕ with the application of the fact $n_1 = n_0 + n$ (sample size) and $v_1 = v_0 + n$ (degree of freedom),

$$f(\mu,\phi \mid y_i) = exp\left[-\frac{n_1\phi(\mu - \mu_1)}{2} \right].exp\left[-\frac{\phi}{2}(-n_1\mu_1^2 + n\bar{y}^2 + n_0\mu_0^2) \right]. \tag{11}$$

From (11), posterior mean and variance are presented as:

$$E(\mu,\phi \mid y_i) \equiv \mu_1 = \frac{n_0\mu_0 + n\bar{y}}{n_0 + n} \text{ and}$$

$$V(\mu,\phi \mid y_i) \equiv \sigma_1^2 = \frac{1}{v_1}\left[(n-1)s^2 + v_0\sigma_0^2 + \frac{nn_0}{n_1}(\bar{y} - \mu_0)^2 \right]. \tag{12}$$

The credible interval (C.I.) of the posterior distribution for the parameter estimation, which is coined from the Highest Density Interval (HDI) can be expressed as:

$$C.I. = E(\mu,\phi \mid y_i) \pm t_{\alpha/2,v_1}\, \mathrm{SD}(\mu,\phi \mid y_i). \tag{13}$$

4 Results and Discussion

This section gives the study's analysis using the technique of Bayesian estimation for the set objectives through discussion based on the acquired data.

4.1 Exploratory data analysis

This section expresses the preliminary study about the dataset from 1961 to 2020 of the population figures in a graphical form.

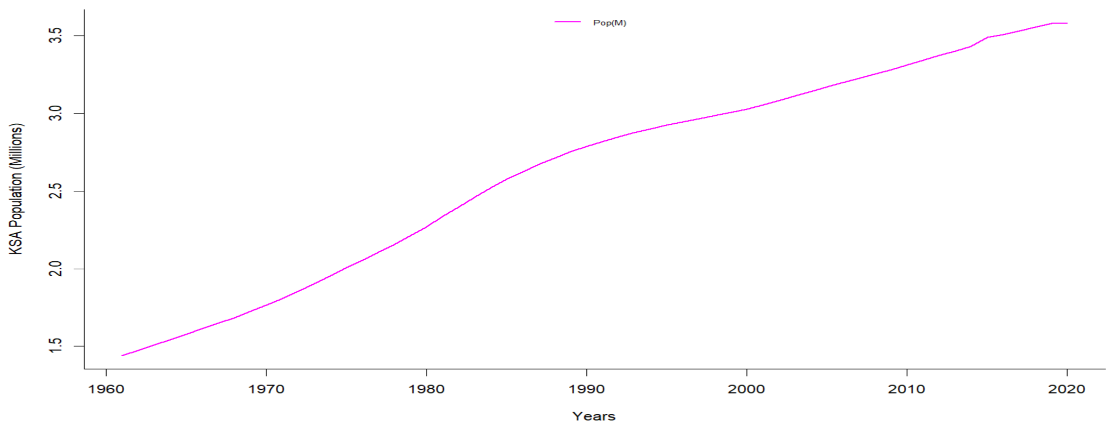

Fig. 2. Time Plot of Population in Kingdom of Saudi Arabia

The population of Saudi Arabia increased steadily from 1961 to 2019 before somewhat declining in the year 2020 due to Covid-19 pandemic, as seen in Fig. 2 above.

4.2 Population parameters' estimation

The approach of Bayesian was applied on the obtained data to estimate the population parameters for adequate population growth modelling as displayed in Table 1 below.

Table 1. Estimation of parameters

Parameters	Mean	SD	SE	HDI	P-Value
β_0	1.48355	0.0265799	3.188e-04	1.43100 ± 1.53576	0.000
β_1	0.03773	0.0007572	9.097e-06	0.03501 ± 0.03922	0.000
σ	0.10126	0.0097111	5.980e-05	0.08433 ± 0.12233	0.000

Key: HDI-Highest Density Interval

It is indicated from the results that the growth rate (R or β_1) is statistically significant, likewise the natural logarithm of the initial population [ln$K$(0) or β_0] and the standard deviation of the model (σ). However, the model precision was as well established with the variance value of 0.01025.

Precisely, 3.8 percent increase in the coefficient of the 60 years which establishes the growth raises the final population to be estimated at 5% level of significance with the population parameter model of $\ln K(\text{t}) = 1.48355 + 0.03773t$.

The above Fig. 3 revealed the contribution of the population parameters in the model estimation. It is cleared that the population growth rate, the initial population and the parameter of precision positively contributed to the population estimation with the model employed.

4.3 Population prediction

This section gives the population estimates based on the probabilistic projection for the next 80 years, i.e., from 2021 to 2100.

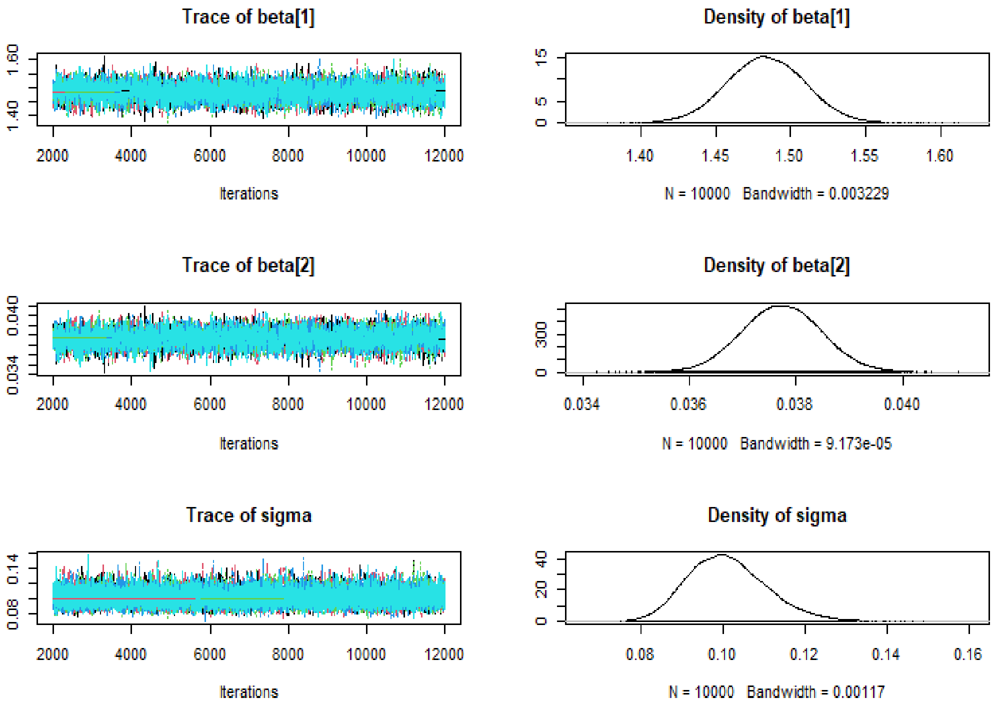

Fig. 3. Diagnostic Plots of the Population Parameters
Keys: - β0: beta[1] and β1: beta[2]

It can be affirmed from the above Table 2 that the predicted population of the Kingdom of Saudi Arabia for the next 80 years (2021 to 2100) based on the data available ranges from 44 million plus to around 867million. Perhaps, the 2.5% HDI establishes the range of around 38 million to almost 700 million while the 97.5% HDI gives the estimated population of around 51 million to almost 1126 million.

It is clearly shown from the Fig. 4 above that the aggregate probabilistic projections through the age intervals of the Kingdom of Saudi Arabia defined the 95% median of all the sampled future trajectories. A projection with increasing uncertainty bands over the forecast horizon indicates the age gaps steadily rise from 0-4 years to 45-49 years before declining from 50-54 years up-to 130+ years [17].

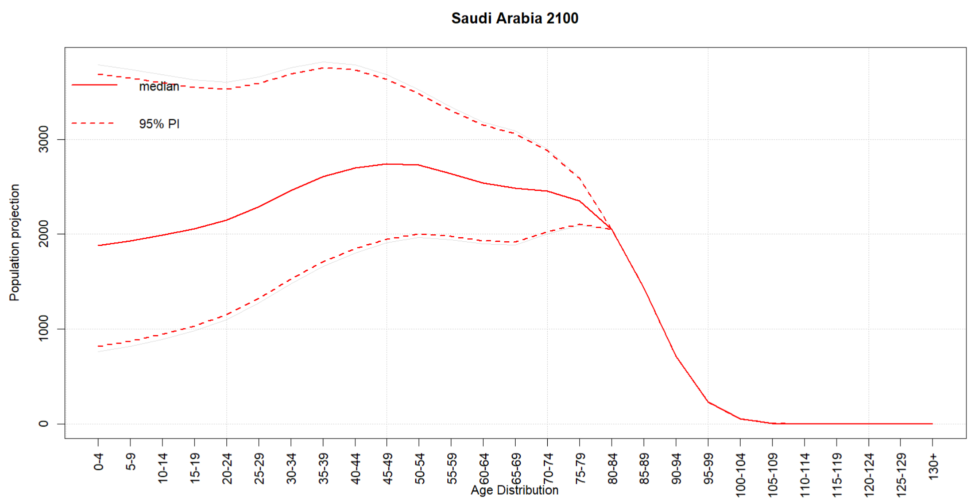

Fig. 4. Future Trajectory Projection of the Kingdom of Saudi Arabia

Table 2. Estimates of population prediction

		Projections (Millions)				**Projections (Millions)**		
Years	**True Value**	**Projected Value**	**Lower Limit (2.5%)**	**Upper Limit (97.5%)**	**Years**	**Projected Value**	**Lower Limit (2.5%)**	**Upper Limit (97.5%)**
2021	35.95	44.03919355	35.39639575	50.81441123	2061	199.19415453	143.59689014	243.95407735
2022	36.408	45.73253636	36.65757163	52.84694999	2062	206.85333178	148.71325668	253.71205951
2023	36.947	47.49098958	37.96368330	54.96078880	2063	214.80701062	154.01191969	263.86035372
2024		49.31705675	39.31633180	57.15917958	2064	223.06651488	159.49937441	274.41457217
2025		51.21333770	40.71717526	59.44550437	2065	231.64360380	165.18234749	285.39095152
2026		53.18253218	42.16793086	61.82328046	2066	240.55048876	171.06780528	296.80637792
2027		55.22744380	43.67037698	64.29616583	2067	249.79985070	177.16296231	308.67841290
2028		57.35098393	45.22635536	66.86796477	2068	259.40485812	183.47529018	321.02532047
2029		59.55617591	46.83777334	69.54263377	2069	269.37918589	190.01252670	333.86609520
2030		61.84615931	48.50660625	72.32428754	2070	279.73703467	196.78268539	347.22049138
2031		64.22419443	50.23489979	75.21720540	2071	290.49315117	203.79406527	361.10905351
2032		66.69366694	52.02477253	78.22583783	2072	301.66284909	211.05526107	375.55314782
2033		69.25809268	53.87841856	81.35481332	2073	313.26203098	218.57517375	390.57499520
2034		71.92112267	55.79811012	84.60894552	2074	325.30721085	226.36302141	406.19770532
2035		74.68654833	57.78620041	87.99324059	2075	337.81553768	234.42835059	422.44531227
2036		77.55830685	59.84512649	91.51290495	2076	350.80481985	242.78104797	439.34281144
2037		80.54048683	61.97741224	95.17335327	2077	364.29355048	251.43135250	456.91619805
2038		83.63733405	64.18567147	98.98021681	2078	378.30093378	260.38986795	475.19250708
2039		86.85325757	66.47261112	102.93935207	2079	392.84691236	269.66757588	494.19985493
2040		90.19283596	68.84103458	107.05684981	2080	407.95219564	279.27584915	513.96748258
2041		93.66082386	71.29384512	111.33904440	2081	423.63828936	289.22646582	534.52580069
2042		97.26215871	73.83404946	115.79252360	2082	439.92752613	299.53162361	555.90643627
2043		101.00196782	76.46476144	120.42413866	2083	456.84309730	310.20395484	578.14228142
2044		104.88557563	79.18920587	125.24101489	2084	474.40908595	321.25654193	601.26754388
2045		108.91851131	82.01072243	130.25056260	2085	492.65050114	332.70293342	625.31779968
2046		113.10651666	84.93276980	135.46048850	2086	511.59331358	344.55716058	650.33004789
2047		117.45555421	87.95892991	140.87880757	2087	531.26449257	356.83375462	676.34276747
2048		121.97181580	91.09291231	146.51385539	2088	551.69204439	369.54776451	703.39597654
2049		126.66173132	94.33855870	152.37430094	2089	572.90505219	382.71477540	731.53129390
2050		131.53197790	97.69984768	158.46915996	2090	594.93371739	396.35092774	760.79200310

		Projections (Millions)					Projections (Millions)	
Years	**True Value**	**Projected Value**	**Lower Limit (2.5%)**	**Upper Limit (97.5%)**	**Years**	**Projected Value**	**Lower Limit (2.5%)**	**Upper Limit (97.5%)**
2051		136.58948943	101.18089961	164.80780883	2091	617.80940269	410.47293708	791.22311896
2052		141.84146638	104.78598164	171.39999894	2092	641.56467669	425.09811453	822.87145688
2053		147.29538613	108.51951299	178.25587178	2093	666.23336029	440.24438800	855.78570485
2054		152.95901352	112.38607029	185.38597444	2094	691.85057485	455.93032419	890.01649833
2055		158.84041199	116.39039329	192.80127592	2095	718.45279214	472.17515129	925.61649815
2056		164.94795502	120.53739057	200.51318396	2096	746.07788631	488.99878265	962.64047157
2057		171.29033804	124.83214564	208.53356260	2097	774.76518783	506.42184108	1001.14537646
2058		177.87659085	129.27992311	216.87475044	2098	804.55553942	524.46568422	1041.19044899
2059		184.71609043	133.88617518	225.54957961	2099	835.49135426	543.15243067	1082.83729472
2060		191.81857434	138.65654831	234.57139551	2100	867.61667634	562.50498710	1126.14998339

4.4 Comparison of estimated population prediction with population census

Here, an attempt is made to compare the predicted values to the actual population census of the year 2019 across the thirteen regions of the Kingdom of Saudi Arabia to affirm the estimated model performance together with the construction of population cohort and pyramid by age and sex [18].

Table 3 above established that 2019 population figures of twelve out of the thirteen regions of the Kingdom are approximately the same with the lower limit of the 2019 predicted values and lie between the interval estimation. This implies that only the population census of Al-Qassim region is somehow different from the estimated population and does not fall within the range while other regions (Riyadh, Makkah, Madinah, Al-Bahah, Eastern, Asir, Tabuk, Hail, Nothern borders, Jazan, Najran, Al-jawf) are approximately the same.

Both Fig. 5 and 6 can also be referred to as the age-specific results. In Fig. 5 above, A projection with increasing uncertainty bands over the forecast horizon using median and 95% median established that: age gaps steadily rise from 0-4 years to 45-49 years before declining around year 2030, age gaps steadily increase from 50-54 years to 70-74 years before declining around year 2050 while age gaps of 75-79 years to 95-99 years maintain constancy at the initial population before steadily rising around 2010 and sometimes decline towards the final population.

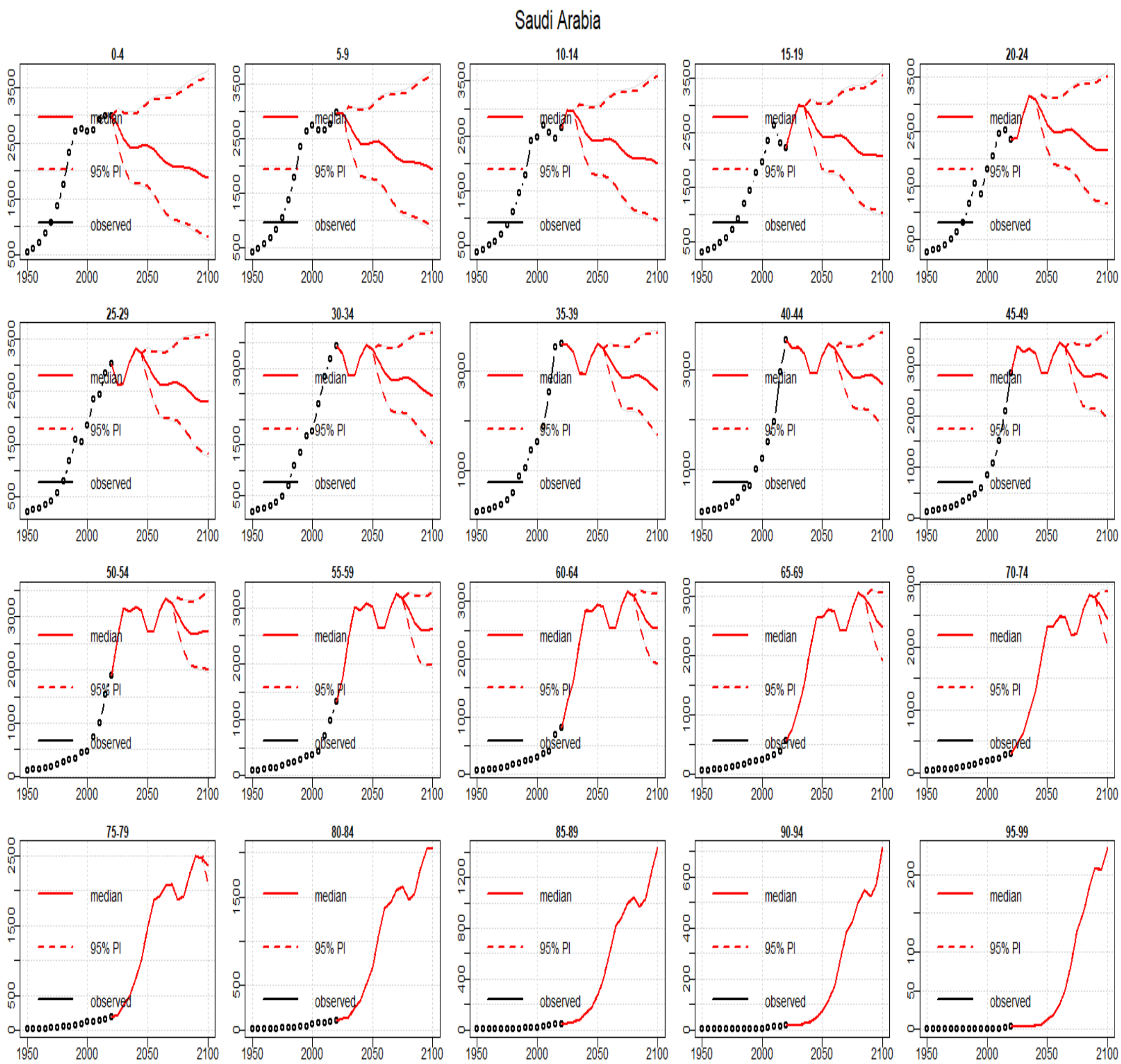

Fig. 5. Probabilistic Population Cohort Projection by Age in the Kingdom of Saudi Arabia

Table 3. Population census and prediction's comparison in millions

Regions	2019 Census	2019 Pred_L	2019 Pred	2019 Pred_U
Riyadh	8.661	8.353	10.337	11.891
Makkah	9.033	8.713	10.782	12.403
Madinah	2.240	2.160	2.673	3.075
Al-Qassim	1.488	1.435	1.776	2.043
Eastern Region	5.149	4.966	6.145	7.069
Asir	2.308	2.226	2.755	3.169
Tabuk	0.950	0.916	1.133	1.304
Hail	0.731	0.705	0.873	1.004
Nothern Borders	0.383	0.369	0.457	0.526
Jazan	1.637	1.579	1.954	2.248
Najran	0.608	0.587	0.726	0.835
Al-Bahah	0.497	0.479	0.593	0.682
Al-jawf	0.532	0.513	0.635	0.730
KSA	34.218	33.003	40.840	46.980

Fig. 6 depicts the age gaps with various uncertainties about absolute numbers of probabilistic population pyramid projection by age and sex at both left and right using median and 95% median predictive interval. Between 2020 to 2100, uncertainty occurs from age 0 to 49 years while no uncertainty for other age groups. However, the population will be on average more than the way it was in 2020 with a significantly higher percentage of people under 49 years of age and a lower percentage of those in their 70s and above.

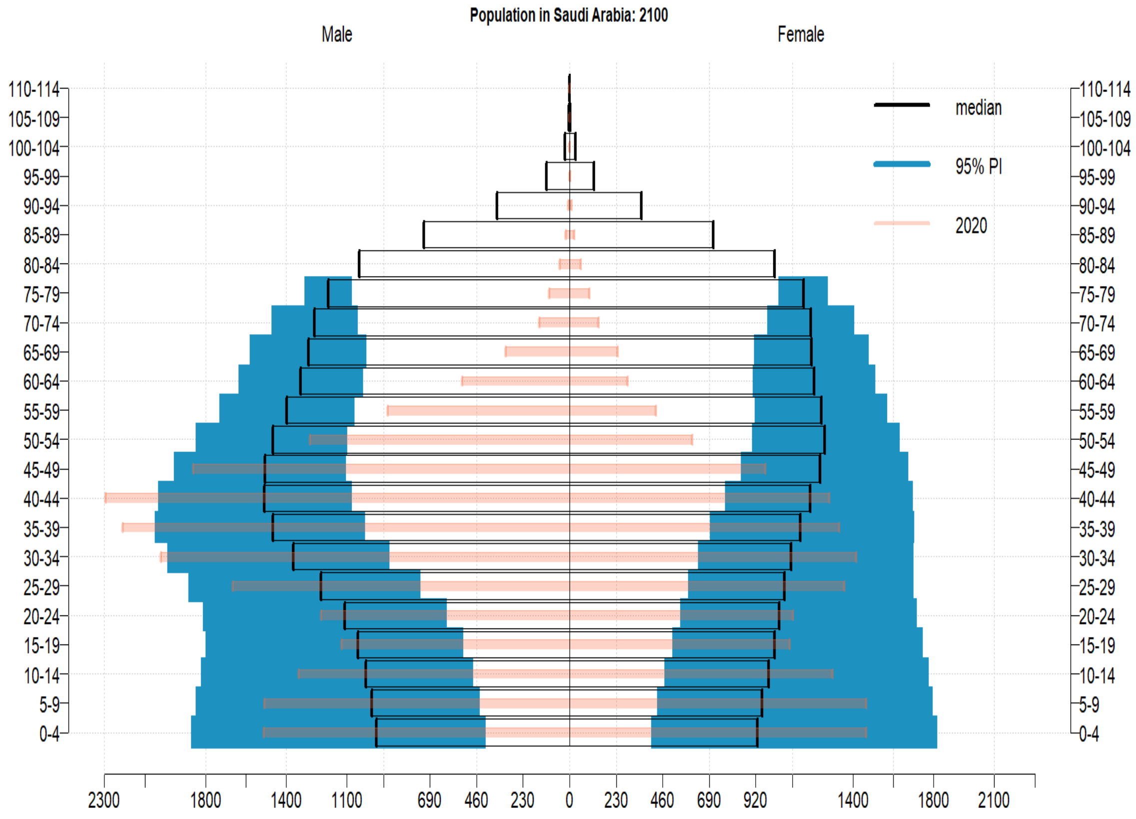

Fig. 6. Probabilistic Population Pyramid Projection by Age and Sex in KSA

5 Conclusion

This study employed Bayesian probabilistic projection on the exponential growth rate model in estimating and predicting population in the Kingdom of Saudi Arabia (KSA) across the thirteen regions with the obtained data. With precision of 0.01025 and $p<0.05$ over the used period, the model $\ln K(t) = 1.48355 + 0.03773t$ provided a good calibration of the estimated population parameters. The prediction generated using 2.5% and 97.5% highest density interval (HDI) from 2021 to 2100 maintained closed range. These population forecast values were found to be closer to the population census while comparing the year 2019 of the regions, also a projection with increasing uncertainty bands over the forecast horizon was discovered as the age gaps steadily rise from 0-4 years to 45-49 years before declining from 50-54 years up-to 130+ years. It is now recommended that Bayesian approach is better for future projection based on the HDI.

Competing Interests

Author has declared that no competing interests exist.

References

[1] Kenneth WW. Essential Demographic Methods; 2013.

[2] Obisesan KO, Oladoja OM, Lawal AO. Mathematical Projection of Population Census in Nigeria; 2020.

[3] Alparslan B, Yıldırım S, Birbil Şİ. Differentially Private Distributed Bayesian Linear Regression with MCMC. 2023;1–20.

[4] Sibly RM, Hone J. Population Growth Rate and its Determinants: An Overview, Philos. Trans. R. Soc. B Biol. Sci. 2022;357:1153–1170.

[5] Akanbi OB, Afolabi SA. Application of a Modified g -Parameter Prior in Bayesian Model Averaging to Water Pollution in Ibadan, J. Environ. Earth Sci. 2019;9:51–63.

[6] A.E. Raftery, N. Li, H. Ševčíková, P. Gerland, G.K. Heilig, Bayesian Probabilistic Population Projections for all Countries, Proc. Natl. Acad. Sci. U. S. A. 109 (2012) 13915–13921.

[7] AA. Salam, Population and Household Census , Kingdom of Saudi Arabia 2010 : Facts and Figures, Int. J. Humanit. Soc. Sci. 2013;3:258–263.

[8] Wakefield J, Aarons L, Racine-Poon A. The Bayesian Approach to Population Pharmacokinetic/Pharmacodynamic Modeling; 2016.

[9] Migon HS. Gamerman D. Generalized exponential growth models a bayesian approach, J. Forecast. 1993;12:573–584.

[10] Raftery AE, Li N, Ševčíková H, Gerland P, Heilig GK. Bayesian probabilistic population projections for all countries, Proc. Natl. Acad. Sci. U. S. A. 2012;109:13915–13921.

[11] Vanella P, Deschermeier P, Wilke CB, An Overview of Population Projections—Methodological Concepts, International Data Availability, and Use Cases, Forecasting. 2020;2:346–363.

[12] Chao F, Gerland P, Cook AR, Guilmoto CZ, Alkema L. Projecting sex imbalances at birth at global, regional and national levels from 2021 to 2100: Scenario-based Bayesian probabilistic projections of the sex ratio at birth and missing female births based on 3.26 billion birth records, BMJ Glob. Heal. 2021;6.

[13] Chao F, Kc S, Ombao H. Estimation and probabilistic projection of levels and trends in the sex ratio at birth in seven provinces of Nepal from 1980 to 2050: a Bayesian modeling approach, BMC Public Health. 2022;22:1–15.

[14] Zolaktaf N. Conjugate Priors, Uninformative Priors; 2016.

[15] Olubusoye OE, Akanbi OB. STA 774 Lecture Six - Normal Distribution with Unknown Mean and Unknown Variance; 2016.

[16] Clyde M, Çetinkaya-Rundel M, Rundel C, Banks D, Chai C, Huang L. Inference and Decision-Making with Multiple Parameters | An Introduction to Bayesian Thinking; 2019.

[17] Azose JJ, Raftery AE. Bayesian Probabilistic Projection of International Migration, Demography. 2015; 52:1627–1650.

[18] Will Koehrsen, Introduction to Bayesian Linear Regression , Towar. Data Sci. 2018;2017.

Peer-review history:
The peer review history for this paper can be accessed here (Please copy paste the total link in your browser address bar)
https://www.sdiarticle5.com/review-history/115306