Open Journal of Social Sciences, 2024, 12, 1-10
https://www.scirp.org/journal/jss
ISSN Online: 2327-5960
ISSN Print: 2327-5952

Variations between Elective and Non-Elective Mathematics Students of Colleges of Education in Using Metacognitive Strategies to Solve Geometric Theorem Problems

Abubakar Yakubu[1*], Henry Kwami Apoenchir[2]

[1]Department of Mathematics and ICT Education, Bagabaga College of Education, Tamale, Ghana
[2]Department of Mathematics and ICT Education, SDA College of Education, Asokore-Koforidua, Ghana
Email: *abyakubu.stu@cktutas.edu.gh, hapoenchi.stu@cktutas.edu.gh

How to cite this paper: Yakubu, A., & Apoenchir, H. K. (2024). Variations between Elective and Non-Elective Mathematics Students of Colleges of Education in Using Metacognitive Strategies to Solve Geometric Theorem Problems. *Open Journal of Social Sciences, 12,* 1-10.
https://doi.org/10.4236/jss.2024.124001

Received: November 16, 2023
Accepted: March 31, 2024
Published: April 3, 2024

Abstract

As a result of the shift from Objective Based Curricula (OBC) to Standard Based Curricula (SBC) which emphasizes the attainment of standard levels of competence and skills by learners, there is an emergence of high interest in the use of metacognitive strategies in the learning of mathematics. Working in groups or individually, learners design own strategies to solve problems, reflect on their work for evaluation and adoption of alternative approaches to the problem being solved for better results. These are approaches that fall in line with metacognitive strategies. This research piece sought to determine any differences in metacognitive strategies that may exist between elective mathematics students and non-elective mathematics students in colleges of education. Using a mix of two techniques of sampling, proportionate numbers for the sample were drawn from two colleges of education which were purposively selected. Data were collected to test the null hypothesis of whether elective and non-elective mathematics students differed in their use of metacognitive strategies when solving GT problems. Results of the independent samples t-test showed no significant differences ($t_{(298)} = -0.248$, $p = 0.2805$) between elective mathematics students ($M = 45.83$, $Sd = 8.27$) and the non-elective mathematics students ($M = 45.57$, $Sd = 10.27$) when using metacognitive strategies to solve problems on GT.

Keywords

Metacognition, Elective, Non-Elective, Misconceptions

1. Introduction

Colleges of education in Ghana over the years have served as the main institutions in the country involved in producing teachers for all public basic schools in the country and the initial point of implementation of new teaching strategies developed by the Ghana education service (GES). The colleges have equally been collaborators with the GES in carrying out in-service training (INSET) for practicing teachers at the pre-tertiary level as well as a major stakeholder in education reforms implemented over the years. The colleges of education were deeply involved in the development and implementation of a new curricula in Ghana through the support of a UK funded organization; Transforming Teacher Education and Learning (T-TEL) for Teacher Education in tertiary institutions, Senior high schools and basic schools across the country. This is evident from the numerous tutors from the colleges who were authors in the development of the curricula and course modules for the colleges of education.

Redesigning curricula for both the teacher education institution and pre-tertiary education schools are all pitched towards producing students that measure up to set standards in content knowledge and mathematical skills. At the pre-tertiary level, a lapse in any of the topics or concepts in the mathematics curriculum affects the ability of students to attain the national standards as contained in the standards-based curriculum (SBC) for mathematics. A solid foundation in the mathematics content is equally significant as the pedagogical skills required of the trainee teachers of the Colleges of Education, since they are to cater for the mathematics needs of primary and Junior High Schools students in Ghanaian schools.

All the mathematics topics taught at the basic level constitute part of the foundation topics for core mathematics courses for both elective and non-elective mathematics students in the colleges of education. Geometric Theorem (GT) is a topic of remarkable significance but it has been perceived as a difficult concept by basic, senior high schools and college students (WAEC, 2014, 2015, 2016, 2017; IOE, UCC, 2020, 2021).

The depth of content knowledge of elective mathematics students in the colleges of education differ from that of the non-elective mathematics students. Both at Senior High Level (pre college level) and the college of education, elective mathematics students study additional mathematics courses or topics. These differences have the potential of influencing the cognition of these two groups of students differently towards the study of mathematics. College tutors' knowledge of these variations may assist them to adopt strategies that can address potential gaps that may exist between these two groups of students. According to Betty, Elsa, and Gert (2014), Williams and Upchurch (2001), emphasize the importance of metacognitive skills in learning and recommends that metacognitive knowledge, strategies and skills should explicitly be attended to in teaching and learning.

Core Mathematics Topics Perceived as Difficult by Students

Domingo (2016) in a study mentions the topics in which secondary school teachers encounter challenges in teaching to include Geometric Theorem (GT) and applications of trigonometry in elevation, depression and bearings. Continuous teacher lapses in teaching some mathematics topics would invariably result in students perceiving such topics as difficult topics. The challenges and negative perceptions encountered in GT concepts may have its roots from the primary schools. Fielker, Tahta and Brookes (1979) enumerate a number of reasons for these challenges and its resultant negative perceptions. They observed that the average primary school mathematics teacher has fear for the word "Geometry" and an obsession with shapes and concepts, perhaps because of the numerous theorems in geometry the teacher encountered in high school. Fielker et al., notes further that the curriculum devotes more time to arithmetic than geometry.

Fletcher and Anderson (2012) cited in Tay and Mensah-Wonkyi (2018) narrates that it is common practice for candidates to avoid questions on GT when alternatives are available and for the rare occasions when they attempt GT questions, very little knowledge is exhibited. Also, the WAEC (2012), chief examiners report shows that candidates who attempted questions on GT could not recall the right theorem to use and added that candidates lacked in-depth knowledge of geometry. Several other WAEC mathematics chief examiner consistently report of difficulties encountered by candidates in GT (WAEC, 2014, 2015, 2016, 2017, 2018). In one such report the chief examiner for mathematics writes that "for part (b), the task of candidates was to compute the shaded portion of a semi-circle with an inscribed triangle. The question also demanded that candidates should deduce a rule for the area of the semi-circle and the area of the triangle. The chief examiner noted further that majority of candidates could not perceive the shaded portion to be within the semi-circle. Rather, they considered the whole area of the circle as having the shaded area so they totally deviated with the concept for finding the shaded portion (WAEC, 2017). In another report the chief examiner explains that the answers provided by candidates to part (b) of a question revealed lack of knowledge in geometry, as such, most candidates performed poorly in answering the question. Candidate could not apply the geometric concepts of isosceles triangles, a line which is tangent to a circle and the relationship between interior and exterior angles of a triangle. (WAEC, 2016).

Anamuah-Mensah, Mereku and Asabere-Ameyaw (2004) cited in Bismark, Christian, Raymond and Stanley (2019) identified among other topics that Geometry was one of the weak content areas of students. Factors that account for these challenges' students encounter include, unavailability of resources for teaching and learning, instructional strategies used by teachers, learning approaches of students and curricula related variables (Fabiyi, 2017; Telima, 2011).

Fabiyi (2017) conducted a survey on core topics perceived by students as difficult. Of the 23 topics, geometric concepts topped the list of eight topics that were perceived by students as difficult topics to learn. Students' perceptions might be as a result of negative peer discussions, poor teacher instructional strategies and above these factors, lack of self-learning and evaluation strategies of concepts. Reviewing or evaluating the strategies that yielded the desired answers to a mathematics problem is not an approach that is familiar with students. Mostly students are reluctant to go beyond the answers. Once they derive the right answers it becomes the end of the story.

Senyefia (2017) also conducted a study with senior high school students and concluded on a number of topics which the students perceived as difficult topics, most of which were geometry concepts. Topics such as Ratio and Proportion, Circle Theorem, Plane Geometry, Trigonometry & Bearings, Logarithms, Mensuration, Business Mathematics, Sequence and Series and Coordinate Geometry were identified as the concepts in the senior high school core mathematics curriculum that are perceived to be difficult. He went further to establish that female students held a stronger perception about the difficulty of these concepts.

How Students Learn Geometric Theorem

Zepeda, Elizabeth, Ronevich and Nokes-Malach (2015) reports that research findings have it that, general instructional strategies in science may improve metacognition, which is important in teaching of science. This assertion shows that metacognition in mathematics, which is invariably a science may improve students' achievement. Broening (1939) list some general teaching methods as; lecture, test-determined teaching, recitation, unit plan appreciation, project, problem solving, photoplay, radio etc. these methods of teaching which contain some components for metacognitive process can result in improvement of the approaches to teaching and learning of Geometric Theorem and mathematics generally.

Resourceaholic (2014) elaborates the phases of teaching GT as learning vocabulary, investigations, Proofs and memorizing theorems. The vocabulary stage involves revising with students on the parts of circles using circular cutouts which is properly labeled with all or part of the names missing. Other activities suggested are vocabulary knowledge surveys and literacy walls.

Investigation: students are given the opportunity to investigate the parallel lines in a circle through the use of ICT, geoboards and paper folding. Students use circular paper, folding along straight lines and measuring angles formed, to investigate circle theorems.

Resources: engaging students to explore and use resources such as computer applications that can be downloaded on their android phones.

Memorizing theories: GT is a topic that require recall of the theorems as well as understanding. Some level of self-drill to memorize the theorems is therefore required of students. In solving problems on circle theorems students sometimes need to use a combination of two or more of the theorems.

Akinsola and Olowojaiye (2008) corroborates that enough evidence suggests that the traditional method frequently used in schools is inappropriate to improve students' attitude towards mathematics, and this directly impacts achievement. They argue that there is the need to shift to instructional strategies that would result in students' developing positive attitude towards mathematics. Akinsola and Olowojaiye suggest that in addition to other innovative strategies, Study-Based Instructional strategy (SQBIS) and Behaviuoral-Objective Based Instructional strategy (BOBIS) should be adopted. These among other more innovative strategies should be used to teach GT topics which are perceived by students as difficult concepts (Fabiyi, 2017).

Gunhan (2014) recommended in a study that the school curriculum should place more emphasis on reasoning skills. In teaching geometric concepts students should be presented with problems that would prompt different reasoning skills. Students reasoning skills can be grouped into initiative reasoning skills and creative reasoning skills; the former refers to students who memorize and use algorithms and the latter refers to the group that develop the concept and creates the procedure to solve mathematical problems (Mumu & Tanujaya, 2019).

Rohman and Retnowati (2018) suggested worked examples as an option for solving geometry questions on concepts that are introduced for the first time. The approach they offered was that, the worked examples should be paired with similar problems when the concepts are freshly introduced to novice learners. They add that when using worked examples, the preliminary stage should be the recall of relevant previous knowledge which should be guided or initiated by the teacher. Rohman and Retnowa clarifies that related prior knowledge of concepts leads to enhanced student learning.

Statement of the Problem

Based on misconceptions about memorization that college students would have acquired from high school (Nist, 1993), they mostly resort to rote or routine approaches to learning mathematics which yields short-term quick results. Quick results make rote approaches very attractive for high school and college students and for most of them it is the only option usually considered when tackling questions in mathematics. Özkan and Kesen (2008) found that students use of rote learning strategies was more regular and also noted that the educational system was a factor which urged students to memorize. Though this trend was observed in language learning it runs through as a student habit in other subjects as well (*sic*).

The unattractiveness of GT questions (WAEC, 2014, 2015, 2016, 2017, 2018; IOE UCC, 2020, 2021) makes it appear as though candidates perceive it to be difficult. This perception may be the result of poor teacher instructional strategies, student to student interaction or misinformation on geometry and the inability to use metacognitive approaches when they encounter problems on GT. Problem-solving encouraged as an instructional mode of lesson delivery in the basic school curriculum is considered as a metacognitive strategy (Broening, 1939; Ratner, 1991).

Many other metacognitive strategies remain unexplored as a strategy to lesson delivery and student learning in both pre-tertiary schools and the Colleges of Education in Ghana. Literature on the impact of metacognitive strategies in learning mathematics and poor perceptions held by students about GT, emphasize the need for more research on how students and teachers can effectively exploit metacognitive strategies in teaching and learning GT.

Objectives of the Study

The objective of the study was to examine; whether College of Education elective mathematics students differ from non-elective mathematics students in the use of metacognitive strategies to solve GT problems.

Research Hypothesis

There are no significant differences between elective and non-elective mathematics students in the use of metacognitive strategies when solving GT problems.

Relevance of the Study

The task of any mathematics teacher in delivering a lesson is to ensure that his or her learners acquire knowledge and skills per the objectives that are set for the lesson. It is anticipated that the learning strategies outlined in college of education mathematics curriculum would be considered by the developers and college tutors as an approach for lesson delivery that would benefit college mathematics students, as well as secondary and basic school students.

A clear knowledge of how students learn is very crucial for a teacher to be able to facilitate the learning process in class. A teacher's knowledge of perceptions and learning style of students would enable her or him to package the most suitable teaching and learning strategies for the students. This study would serve as reference or supporting document that can be used by College of Education tutors in preparing trainee teachers for their pedagogical competence and enable them to learn better, using metacognitive learning strategies. This study would provide information for teachers and trainee teachers on suitable pedagogical strategies in handling students with different entry behaviours in their mathematics classrooms.

2. Methodology

Comparative case study which is one form of multiple case study designs was employed as an effective design to suit the purpose of the research which tries to compare the differences that may exist between elective and non-elective mathematics college of education students when tackling task on GT. A combination of purposive and stratified sampling techniques was used to select two colleges and the sample respectively from the colleges of education in the northern sector of the country. Bagabaga college of education and Tamale college of education were purposively selected since this combination offered opportunity for selecting a sample that ensured the inclusion of students from all the variety of programmes run by any of the colleges in the northern sector of the country. Employing Krejcie and Morgan (1970) table for sample size determination, a sample of 300 students was determined from a total population of 1398 students. Using stratified sampling techniques, the total sample size of 300 was obtained by selecting 150 students each from the two Strata (elective and non-elective mathematics students) to respond to questionnaires that were administered.

Piloting of the instrument was carried out in Evangelical Presbyterian College of Education, Bimbilla, which is one of the fourteen colleges within the zone of the study. The reliability coefficient of the questionnaire administered for the piloting was computed for Cronbach's Alpha at 0.79. The null hypothesis was tested using Statistical package for the social sciences (SPSS) software to run an independent samples t-test on the data.

Testing the Research Hypothesis

There is no significant difference between elective mathematics (mathematics major) and non-elective mathematics (mathematics minor) students in the use of metacognitive strategies when solving GT problems.

To test the stated hypothesis an independent samples t-test was conducted to compare the difference between elective mathematics students and non-elective mathematics college students in their use of metacognitive strategies when solving problems on GT.

The mean score as captured in Table 1 for major (Elective mathematics students) and minor (non-elective mathematics students) were 45.83 and 45.56 respectively. This suggests some similarity in means for the elective mathematics students (M = 45.83, Sd = 8.27) and non-elective mathematics students (M = 45.57, Sd = 10.27).

The results in Table 2 shows that the means of both the elective mathematics (M = 45.83, Sd = 8.27) and the non-elective mathematics (M = 45.57, Sd = 10.27) are similar. The t value of -0.248 confirms an insignificant difference between the means of the two groups. Additionally, the p value of 0.805 does not provide enough evidence to reject the null hypothesis. These results confirm the null hypothesis that; mathematics major students do not differ from non-mathematics major students in the adoption of metacognitive strategies to learn or solve GT problems. Whilst this finding is corroborated by Pintrich, Smith, Garcia and McKeachie (1991) and Khan (2020) in a similar study, it contradicts the

Table 1. Group means for the elective and non-elective mathematics students.

	Group	N	Mean	Std. Deviation
Metacognitive strategies	Minor	150	45.5667	8.27336
	Major	150	45.8333	10.27175

Source: Field Survey (2021).

Table 2. Independent samples t-test for differences between elective mathematics and non-elective mathematics students in the use of metacognitive strategies.

		Levene's Test for Equality of Variance		t-test for Equality of Means			
		F	Sig.	t	df	Sig (2-tailed)	Mean Diff
Strategies	Equal variances assumed	1.720	0.191	−0.248	298	0.805	−0.26667
	Equal variances not assumed			−0.248	285.07	0.805	−0.26667

Source: Field Survey (2021).

findings of Misu, Budayasa, Lukito and Rosdiana (2019), who reported significant differences between mathematics education students and higher mathematics students.

3. Conclusion and Recommendations

The null hypothesis stated that, there was no significant differences between elective and non-elective college of education mathematics students in their use of metacognitive strategies when they encountered problems on GT. To this effect, the instruments employed in soliciting data sought to determine the metacognitive strategies that both categories of students resort to, when they encounter task on GT.

Both elective mathematics and non-elective mathematics students appeared to be similar in their use of metacognitive strategies to solve problems on GT. These are findings which are corroborated by Khan (2020) in a similar study, however it contradicts the findings of Misu, Budayasa, Lukito and Rosdiana (2019), who reported significant differences between mathematics education students and higher mathematics students.

Since the results showed no differences between the two groups it suggests that similar pedagogical strategies could be adopted for both elective and non-elective college of education mathematics students, however individual differences among students in either group may need different attention.

Since no significant differences were noticed between mathematics elective mathematics and non-elective mathematics students, teachers in mathematics classrooms should concentrate more on addressing individual mathematics needs rather than packaging different strategies for presenting mathematics lessons for elective and non-elective mathematics students, especially when they are in separate classrooms.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

Akinsola, M. K., & Olowojaiye, F. B. (2008). Teacher Instructional Methods and Student Attitudes towards Mathematics. *International Electronic Journal of Mathematics Education, 3,* 60-73. https://doi.org/10.29333/iejme/218

Anamuah-Mensah, J., Mereku, D. K., & Asabere-Ameyaw, A. (2004*). Ghanaian Junior Secondary School Students' Achievement in Mathematics and Science: Results from Ghana's Participation in the 2003 Trends in International Mathematics and Science Study*. Ministry of Education Youth and Sports.

Betty, B., Elsa, M., & Gert, W. (2014). A Metacognitive Approach to Pair Programming: Influence on Metacognitive Awareness. *Electronic Journal of Research in Educational Psychology, 12,* 33-60. https://doi.org/10.14204/ejrep.32.13104

Bismark, M., Christian, K. K., Raymond, B. K. E., & Stanley, A. (2019). *Geometry Topics* *in Mathematics Perceived Difficult to Study by Senior High School Students in the Cape Coast Metropolis.* University of Cape Coast. https://www.academia.edu/39740783

Broening, A. M. (1939). Chapter IV: General Methods of Teaching. *Review of Educational Research, 9,* 295-302. https://doi.org/10.3102/00346543009003295

Domingo, A. (2016). *Analysis of Mathematical Topics Perceived Difficult to Teach by High School Teachers in Belize*. https://www.researchgate.net/publication/306291682

Fabiyi, T. R. (2017). Geometry Concepts in Mathematics Perceived Difficult to Learn by Senior High School Students. *IOSR Journal of Research & Method in Education, 7,* 83-90. https://doi.org/10.9790/7388-0701018390

Fielker, D., Tahta, D., & Brookes, W. (1979). Strategies for Teaching Geometry to Younger Children. *Educational Studies in Mathematics, 10,* 85-133. https://doi.org/10.1007/BF00311177

Fletcher, J. A., & Anderson, S. (2012). Improving Students' Performance in Mensuration at the Senior High School Level Using the Geometer's Sketchpad. *Journal of Science and Mathematics Education, 6,* 63-79.

Gunhan, B. C. (2014). A Case Study of the Investigation of Reasoning Skills in Geometry. *South African Journal of Education, 34,* 268-306. https://doi.org/10.15700/201412071156

Institute of Education (IOE), University of Cape Coast (UCC) (2020, 2021). *Mathematics Chief Examiner's Report*.

Khan, B. R. (2020). Metacognitive Skills of Students in a Mathematics Class with Supplemental Instruction and Online Homework. *Journal of Mathematics Education at Teachers College, 11,* 33-41.

Krejcie, R. V., & Morgan, D. W. (1970) Determining Sample Size for Research Activities. *Educational and Psychological Measurement, 30,* 607-610. https://doi.org/10.1177/001316447003000308

Misu, L., Budayasa, I. K., Lukito, A., & Rosdiana (2019). Comparison of Metacognition Awareness of Mathematics and Mathematics Education Students Based on the Ability of Mathematics. *International Journal of Trends in Mathematics Education Research, 2,* 124-127. https://doi.org/10.33122/ijtmer.v2i3.118

Mumu, J., & Tanujaya, B. (2019). Measure Reasoning Skills of Mathematics Students. *International Journal of Higher Education, 8,* 85-91. http://ijhe.sciedupress.com https://doi.org/10.5430/ijhe.v8n6p85

Nist, S. (1993). What the Literature Says About Academic Literacy? *Georgia Journal of Reading, 9,* 11-18.

Özkan, Y., & Kesen, A. (2008). Memorization in EFL Learning. *Academia, 35,* 58-71.

Pintrich, P. R., Smith, D., Garcia, T., & McKeachie, W. J. (1991*). A Manual for the Use of the Motivated Strategies for Learning Questionnaire.* University of Michigan.

Ratner, C. (1991). *Vygotsky's Sociohistorical Psychology and Its Contemporary Applications*. Plenum Press. https://doi.org/10.1007/978-1-4899-2614-2

Resourceaholic (2014). *Ideas and Resources for Teaching Secondary School Mathematics.* https://www.resourceaholic.com/circletheorems.html

Rohman, H. M. H., & Retnowati, E. (2018). How to Teach Geometry Theorems Using Worked Examples: A Cognitive Load Theory Perspective. *Journal of Physics: Conference Series, 1097,* Article 12104. https://doi.org/10.1088/1742-6596/1097/1/012104

Senyefia, B. (2017). Remedial Students' Perception of Difficult Concepts in Senior High School Core Mathematics Curriculum in Ghana. *Asian Research Journal of Mathematics, 3,* 1-13. https://doi.org/10.9734/ARJOM/2017/32330

Tay, M. K., & Mensah-Wonkyi, T. (2018). Effect of Using Geogebra on Senior High School Students' Performance in Circle Theorems. *African Journal of Educational Studies in Mathematics and Sciences, 14, 1-18.*

Telima, A. (2011). Problems of Teaching and Learning of Geometry in Secondary Schools in Rivers State, Nigeria. *International Journal Emerging Science, 1,* 143-152.

West African Examination Council (WAEC) (2012). *Mathematics Chief Examiners Report*. https://kuulchat.com/wassce/chief_examiners_report/2012%20Mathematics.pdf

West African Examination Council (WAEC) (2014). *Mathematics Chief Examiners Report*. https://www.waecgh.org/uploads/examinersreport/2014/mathematicsb14m.pdf

West African Examination Council (WAEC) (2015). *Mathematics Chief Examiners Report*. https://www.waecgh.org/uploads/examinersreport/2015/mathematicsb15m.pdf

West African Examination Council (WAEC) (2016). *Mathematics Chief Examiners Report*. https://www.waecgh.org/uploads/examinersreport/2016/mathematicsb16m.pdf

West African Examination Council (WAEC) (2017). *Mathematics Chief Examiners Report*. https://www.waecgh.org/uploads/examinersreport/2017/mathematicsb17m.pdf

West African Examination Council (WAEC) (2018). *Chief Examiners Report Resume*. https://www.waecgh.org/uploads/examinersreport/2020/generalresumew20.pdf

Williams, L., & Upchurch, R. (2001). In Support of Student Pair-Programming. *ACM SIGCSE Bulletin, 33,* 327-331. https://doi.org/10.1145/366413.364614

Zepeda, C. D., Elizabeth, R. J., Ronevich, P., & Nokes-Malach, T. J. (2015). Direct Instruction of Metacognition Benefits Adolescent Science Learning, Transfer, and Motivation: An *in Vivo* Study. *Journal of Educational Psychology, 107,* 954-970. https://doi.org/10.1037/edu0000022