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The Analysis and the Measurement of Poverty: An Interval-Based Composite Indicator Approach

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Abstract: The study of poverty and its quantification is a critical yet unresolved problem in social science. This work seeks to use a new composite indicator to assess poverty as a multidimensional concept. However, subjective decisions, such as various weighting systems on the indicator's creation, may affect its perception. In order to solve this issue, we propose to use random different composite indicators based on simulated weightings and specifications to get a comprehensive interval-based composite indicator. Our method generates robust and trustworthy measurements based on a meaningful conceptual model of poverty. Furthermore, we use some interval parameters such as the upper bound, center, and lower bound to compare the different intervals related to the different statistical units and rankings to aid in analyzing extreme situations and policy scenarios. In Sicily, Calabria, Campania, and Puglia, we identify urgent circumstances. The findings reveal a consistent indicator measurement and the shadow sector's influence on the final measurements.

Keywords: poverty; composite indicators; interval data; interval-based composite indicators

JEL Classification: C02; C15; C43; I3; I32

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1. Introduction: Measuring Poverty

Poverty measurement and, in general, poverty is a fundamental theme in social science literature. Poverty is an actual refutation of human rights because it determines the impossibility of covering relevant expenses. Simultaneously, the relationship with well-being is complicated because of both concepts' multidimensional structures (D'Ambrosio 2018). Therefore, the measurement and monitoring of poverty are nowadays fundamental. At a macro level, poverty and inequality can impact modern societies in the long run. In literature, poverty and well-being are usually associated, and they are fundamental concepts to understand. In particular, poverty gravely affects a person's well-being (household, children, migrants). Structural poverty can lead to an erosion of the basements on which the societies are born. At the same time, poverty can impact people's lives and be a problem for the institutions (it is necessary to think about the effects of poverty on children).

Institutions can increase poverty by creating obstacles to access to income, and poverty can eradicate institutions. In this regard, there exists a specific causal loop that can become deleterious (see Sindzingre 2007). The United 2030 Agenda and the Sustainable Development Goals (SDGs) have created structured monitoring systems adopted in different countries (Mauro et al. 2018). In this context, it is necessary to investigate the phenomenon, its determinants, and the possibilities of policy intervention. Simultaneously, identifying poverty states is very relevant and needs adequate methodologies (Alkire and Foster 2011). In literature, another crucial problem is identifying the areas in which the poor live and design adequate intervention policies.

At this point, the adoption of adequate quantitative methodologies is necessary. Modern data richness calls for approaches that consider integrating a group of indicators for the selected statistical units. Indicators' synthesis takes place at different stages (Maggino 2009; Nardo et al. 2005). One relevant problem is to manage the different numbers of indicators

(Mauro et al. 2018). In that respect, income is considered the most significant predictor of economic status and well-being (Hansen and Kneale 2013). At the same time, income is not a unique indicator to measure poverty. Therefore, there are at the same relevant approaches such as the construction of composite indicators, which can be considered to measure poverty correctly (Marlier and Atkinson 2010; Atkinson and Marlier 2011). In this case, it is necessary to consider different characteristics that can adequately consider other relevant aspects that can measure poverty. In this way, there is also the use of many different approaches to measuring poverty using different methodologies (using population census, administrative data, household surveys see (Baker and Schuler 2004)). A composite indicator arises in this context, which synthesizes and combines different indicators useful to measuring poverty. Composite indicators are generated by constructing a linear weighted function of a combined normalized sub-indicator (Saisana et al. 2005; Aiello and Attanasio 2006). One relevant problem is that the different approaches can often lead to different results, so it is usually necessary to provide sensitivity analysis and robustness checks. These methodologies are addressed to analyze the results' sensitivity of the different methodological choices (Saisana et al. 2005). Our proposal aim is different: the composite indicator's uncertainty is directly measured by considering the critical factors impacting the indicator variability. In this respect, the possible changes to the indicator results are simultaneously considered considering the different identified factors. Therefore, the crucial varying factors (for instance, the composite indicator specification) are identified. By considering a Monte-Carlo simulation, the interval-based composite indicator considering all the different results simultaneously is finally obtained. A Monte-Carlo simulation is necessary, where there are many sources of uncertainty as usual in composite indicator construction to internalize the different effects of the different assumptions (i.e., different weights). In this sense, the interval comprehends all the results of the different composite indicators simulated.

So, in this work is obtained not a unique measure but an interval, and entire intervals, rather than single values, are compared. Then, of course, it is possible to interpret the different intervals adequately. The specific aim of the study is to provide a composite interval indicator that measures both a measure that synthesizes different indicators to measure poverty and takes into account the variability of the different results due to different choices in the construction of the same composite indicator. The use of aggregate measures based on interval data can also be found in the works (Dehnel and Walesiak 2019; Walesiak and Dehnel 2020).

The second section starts by considering the different ways to measure poverty; the composite indicators are approaching one of the most frequent ways to measure poverty. The third section departs from the concept of composite indicators to describe the interval-based composite indicator's approach. The fourth section describes the data we have used. Finally, the last section describes the obtained results in terms of interval-based composite indicators of poverty in Italy.

2. Measuring Multidimensional Poverty: A Literature Review

One relevant approach in measuring poverty is to use and consider groups of indicators to synthesize these indicators. Following (Mauro et al. 2018), this synthesis problem is fundamental to social indicators literature. Therefore, the same authors consider a multivariate approach to measuring multidimensional poverty and well-being analysis. At the same time, they consider the different indicators' synthesis to monitor the different outcomes obtained. From this perspective, the authors' main contribution is that the general level of well-being that the different persons can reach can be linked with the level of substitutability of the different dimensions obtained (Mauro et al. 2018).

It is widely accepted today that poverty, by its nature, is a multidimensional concept (Anand and Sen 1997; Sen 1982, 1988, 1992). For further discussion about the multidimensional measurement of poverty, see (Asselin 2002). The multidimensional nature of poverty is analyzed as a concept and the measurement methodologies by (Alkire et al. 2015). If poverty is a multidimensional concept, it is necessary to analyze it by considering adequate methodologies. Multidimensional poverty calls naturally for the use of good indicators (Asselin 2002). Simultaneously, the problem to consider is how to conciliate between different approaches and different methodologies. Therefore, it is necessary to consider the different subjectivity of the choices (the choices of weightings in composite indicators see Nardo et al. 2005), leading to different results.

In this context, relevant sets of indicators need to be considered. The different indicators, which explicitly characterize poverty as a concept, need to consider the essential dimensions like income needs and capabilities. It is essential, too, at the same time, to use other indicators related to the framework and the living situation (Lok-Dessallien 1999).

Another different approach is the one followed in (Abdu and Delamonica 2018). Here the authors consider a composite indicator to measure multidimensional child poverty. In this case, the multidimensional approach considers the complexity of the poverty phenomenon by considering different aspects that can be combined to provide a unique measure. Thus, this measure can be considered a synthesis. All these measures are significant because they allow us to consider a measurement used to evaluate policies and programs explicitly.

Additionally, Kim (2016) raises the problem of considering a multidimensional and longitudinal perspective on measuring poverty. In this sense, it is the idea to measure the concept of poverty. The novelty introduced by the author is considering the dimension of time. At the same time, the author concludes that the weighting of the "social capital" and the weight for health can have a higher impact over time.

There are also some elements of uncertainty on collecting the correct variables which can be considered. For instance, it is necessary to collect the income as an essential variable on measuring poverty as a specific part of the surveys considered (Hansen and Kneale 2013). Therefore, one of the most relevant approaches to measure poverty is through composite indicators. The composite indicators depart from the use of different indicators to provide a synthesis of the same indicators. The construction of the composite indicators is considered in the following section.

At the same time, uncertainty and vagueness of the concepts could be significant. In that regard, poverty measurement needs to consider fuzzy logic (see in this context Cerioli and Zani 1990). This approach to measuring multidimensional poverty is also considered by (Lemmi and Betti 2006), which uses fuzzy sets simultaneously (Costa and Angelis 2008; Stéphane and Noel 2005). These approaches show many different dimensions exist, which need to be carefully checked and considered in building composite indicators.

Composite indicators are a relevant and consistent way to measure poverty. The usual approach is described in different works. First, the different indicators need to be synthesized, and it is necessary to consider different phases. (see in this respect (Mauro et al. 2018; Maggino 2009; Nardo et al. 2005)). The different method selects the indicators and then synthesizes the underlying concept and measures the latent variable. Following (Mauro et al. 2018), the synthesis of different indicators allows monitoring specific outcomes of the considered statistical units.

In Asselin (2002) the different quantitative methodologies used to construct indicators on the multidimensional poverty context are reviewed. The suggestion in Asselin (2002) is that the use of multivariate methodologies (principal component analysis for example), can be an advantage for the choice of the specific weights used.

Different approaches in measuring the multidimensional poverty measurement are also in (Kakwani and Silber 2008). An alternative approach to analyzing the measurement of multidimensional poverty is by (Bourguignon and Chakravarty 2003). Various approaches were proposed in this context. For instance, De Muro et al. (2011) consider the composite indicator approach to measuring poverty. Their approach is based on the penalty of the geographical areas characterized by single "unbalanced" statistical units. To approximate the different variables in composite poverty indicators, we can usually follow the procedure to construct a composite indicator (Nardo et al. 2005). The different phases can be considered selecting the different indicators used, the aggregation method's choice, and the considered scaling of the different indicators used. In the end, it is possible to obtain the considered latent variable as a component indicator. From the selection of the different indicators, then scale the different indicators using various methodologies. Usually, an aggregation function and a weighting scheme that defines the single indicator's relevance or importance on the composite indicator created are considered. Last, it is possible to compare the distinct values of the composite indicators produced and determine their ranking. Here, the critical point is that the composite indicators' different components can hide relevant policy messages (see Mabughi and Selim 2006). In this respect, many different critical points exist. Several choices, for example, the weighting of the composite indicator, are based on subjective choices. For this reason, robustness analysis and sensitivity analysis are usually followed by various analyses in which different approaches are considered and compared to evaluate the impact of each approach on the final results.

This analysis is usually performed by considering the different impacts on the rankings (Nardo et al. 2005). The approach we will consider is different. It is based on the interval data by taking into account simultaneously many different random measurements in which we try to cover all the meaningful options. In the construction of composite indicators, sensitivity analysis could be fundamental to assessing the different approaches and assumptions (Saisana et al. 2005) the approach we will present in the next section allows us to "endogenize" the sensitivity analysis on the structure of the composite indicator computed.

3. Methodology

3.1. Methodology: Interval-Based Composite Indicators

In that context, composite indicators' construction can be based on subjective choices (Becker et al. 2017; Nardo et al. 2005). However, these subjective choices (for instance, the composite indicator's weighting scheme, see (Nardo et al. 2005) can lead to different results. Therefore, the recent literature aims to construct composite indicators, avoiding the subjectivity of considering an assumption or a different one. Therefore, the target is to measure the different impacts of the suitable choices for constructing the indicator (see Paruolo et al. 2013).

Uncertainty techniques can be considered in this respect because they can measure uncertainty in constructing the composite indicator (for instance, using probabilistic rankings). See for a discussion (Nardo et al. 2005; Saisana et al. 2005). Therefore, the idea is to consider some robustness checks and sensitivity analysis by considering different assumptions to evaluate its robustness. This work aims to internalize this robustness by considering the interval of possible results, which can be obtained by varying the composite indicator's assumptions (see Drago 2017, 2018; Gatto and Drago 2020; Drago and Gatto 2018). In that way, it is essential to define the "model" for the composite indicators initially. Then, it is essential to declare the different factors that lead to the composite indicator variability. From the model, it is possible to identify the different internal sources of variability in the construction of the composite indicator, which leads to the uncertainty of the outcome.

At this point, it is possible to consider several replications of the composite indicator considered by taking into account different combinations of the assumptions given. At every stage, a different combination of assumptions is sampled, and a different outcome is computed. Then, they explicitly consider an interval of all the possible obtained composite indicators by considering different combinations of assumptions on the composite indicator. Finally, the different results are collected, and they can be represented utilizing interval-valued data (Billard and Diday 2003; Billard 2008). Thus, these data can be used to represent uncertainty and also inaccuracy (Qi et al. 2020; Barclay et al. 2019) and, in general, composite phenomena (for instance, in (Mlodak 2014; Fura et al. 2017; Schang et al. 2016), statistical units characterized by different statistical features are discussed).

We propose internalizing the uncertainty analysis using the interval-valued data, which is relevant in constructing a composite indicator (Saisana et al. 2005). Our approach allows us to directly measure, represent, and compare the variability of the different assumptions used to construct the composite indicators using random weights and simulating different indicator structures. In this respect, the subjectivity of the weightings can be solved. It is also possible to obtain more consistent public policies that also consider the different composite indicator choices.

Furthermore, using the intervals, it is possible to better design policies because it can better measure the uncertainty related to a different situation, for instance, measured by a composite indicator. The results of this work are beneficial for all who are interested in the construction and the use of composite indicators, including analysts, policy analysts, economic and social researchers, and of course, policymakers. The entire approach is described and visualized in Table 1 and Figure 1.

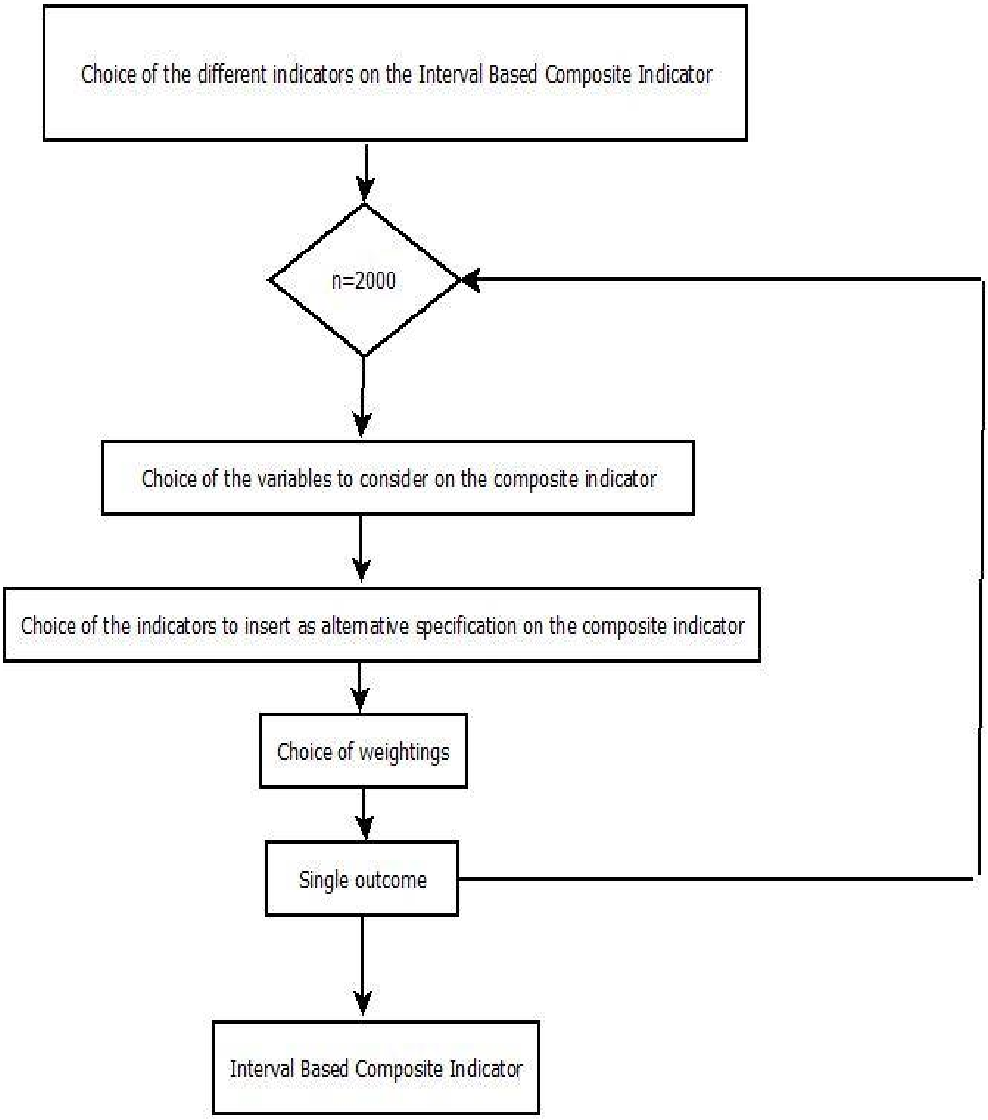

Figure 1. Flow chart of the procedure.

It is essential to note that our final results are interval data and not scalar data. In this logic, the interval allows us to measure the uncertainty explicitly and permits us to obtain a unique measure of the composite indicator (Sunaga 1958). Moreover, the interval data own a specific algebra that allows different computations between intervals (Moore 1979; Sunaga 1958) and statistical analyses (Gioia and Lauro 2005; Lauro and Palumbo 2000).

Therefore, in this respect, the process is started by considering n number of different composite indicators with $n = 1, \ldots N$ (they contribute to creating the interval), computed by random combinations of factors (Saltelli 2016; Saltelli et al. 2008). Then each interval based composite indicator is built by having:

$$I[X]^c = \left[\underline{X^c}, \overline{X^c}\right] \quad (1)$$

where c is the considered, measured phenomenon to measure with the indicator X for $c = 1 \ldots C$ (Palumbo and Lauro 2003).

From the composite interval indicator obtained it is possible to compute the center:

$$X^c_{center} = \frac{1}{2}\left(\underline{X^c} + \overline{X^c}\right). \quad (2)$$

Furthermore, the range or the width obtained:

$$X^c_{range} = \overline{X^c} - \underline{X^c}, \quad (3)$$

and finally, the radius:

$$X^c_{radius} = \frac{1}{2}\left(\overline{X^c} - \underline{X^c}\right) \quad (4)$$

The range and the width represent the variability of the considered interval composite indicators (Gioia and Lauro 2005). The parameters on which the ranking analysis is computed for the different intervals are the center, the minimum, the maximum, and the range (Mballo and Diday 2005; Song et al. 2012). In order to measure the uncertainty, it is possible to consider the difference between the upper and the lower bound of the computed interval (see also Grzegorzewski 2018).

Finally, it is possible to analyze at the same time the prototype (an average interval) using interval arithmetic. The interval arithmetic and the capacity to handle these composite indicators as intervals allow different advantages. First, they represent a more robust version of a classical composite indicator (based on a single value) and consider the internal variability. This is determined by the various composite indicators' different performances on the same conceptual "model" (Nardo et al. 2005). Finally, they can be used and considered in comparisons as a scalar (it is possible to use, for instance, the center) or genuinely as intervals (considering center, minima, and maxima). In this case, it is possible to use analytical approaches such as interval arithmetic to evaluate, for instance, a prototype (the statistical average of the different interval-based composite indicators). Furthermore, these interval-based composite indicators can contain a higher quantity of information so that the decision could be based on a more precise evaluation.

3.2. Methodology and Data

In the first step, we have to define the composite indicator model. The model is given by taking into account the following choices:

(1) The essential variables to be considered on the composite indicator;
(2) The significant number on the total to be considered;
(3) The relevant aggregation function;
(4) The weights applied on the composite indicator.

All these data come from the ASVIS database, which is considered a unique source. The date for each variable is 31 December 2016. The different indicators of their original name and their name are defined in Table 1. Each indicator is considered a statistical unit in the Italian regions (for the year 2016).

Table 1. Indicators considered.

Indicators Considered and Their Reference Date on the ASVIS Database
Percentage of families living below the threshold of absolute poverty (31 December 2016)—Sotpovas (families who live under a level of absolute poverty)
Index of great economic difficulty (31 December 2016)—Diffeco (an index of economic difficulty)
Percentage of population living in poverty or social exclusion (31 December 2016)—Poves (social exclusion)
Index of severe material deprivation (31 December 2016)—Depriv (material deprivation)
Percentage of individuals in low working-intensity households (31 December 2016)—Basintlav (low labor intensity)
Percentage of people who live in households with an equivalent disposable income, less than 60% of median income (31 December 2016)—Reddmed (income)

In Table 2, we compute the descriptives for each variable to evaluate some outliers of the data.

Table 2. Descriptive statistics of the indicators considered.

	Sotpovas	Diffeco	Poves	Depriv	Basintlav	Reddmed
min	3.58	3.9	16.1	5	6.1	8.9
1st qu.	5.755	7.25	20.1	6.75	8.35	13.75
median	10.37	8.8	24.4	9.4	9.9	16
mean	12.434	11.29	30.28	11.39	12.74	20.84
3rd qu.	16.32	14.8	39	14.55	16.7	27.55
max.	34.94	21.6	55.6	26.1	26.7	41.8

We chose the dataset related to 2016 to ensure the most recent set of data jointly available together, at higher reliability of the observations considered. Data reliability is a significant issue (see, for instance, Kamanou et al. 2005). In this sense, these six variables are the most relevant we can consider for our model. In that way, these variables are considered the most significant in the framework we are explicitly considering. In this sense, it is possible to proceed with the data analysis to evaluate our initial indicators and the structure of the indicators used as components or factors of the interval-based composite indicator. All variables are destimulants in the study, but this is not usually the case in other studies. The stimulants and destimulants (Kuc-Czarnecka et al. 2020) as factors that positively or negatively affect the considered phenomenon were introduced in (Hellwig 1972). These definitions can also be found in (Walesiak 2018). Other authors (Mazziotta and Pareto 2016, 2018) use the terms 'positive polarity' and 'negative polarity' instead of the concept of stimulant and destimulant.

Some descriptive analyses of our data are considered. In this respect, we explore our variables by observing if some situations require special attention (for instance, significant outliers). In this vein, it is possible to compute the descriptive statistics for the variables and examine the critical structure of the data we can observe. Then it is possible to consider the correlation matrices of the variables. In particular, the correlation matrix can be usefully considered and visualized as a network with a specific threshold. These are relevant in practice because we can think of specific weighting schemes that show a high correlation. In extreme cases, the choice can be made not to use these indicators.

In this respect, it is necessary to evaluate our choices primarily. We considered the correlation matrix of the different variables to avoid select variables that eventually showed more relevant correlation problems.

Therefore, it is possible at this point to define our model of composite interval indicator by considering these specific factors (for the terminology in the composite indicators, see Nardo et al. 2005):

1. The indicator choice;
2. The number of the indicator choice on the total number of indicators considered (in this respect, we can explore alternative configurations of the composite indicator);
3. The different weightings.

At the same time, we normalize each indicator by providing standardization for each of them. Following (Nardo et al. 2005), we used a simple standardization for each considered component in the simulations:

$$I_{qr}^{t} = \frac{x_{qr}^{t} - x_{qr=\bar{r}}^{t}}{\sigma_{qr=\bar{r}}^{t}} \tag{5}$$

Given $\bar{r}$ as the reference region, the component (or the indicator) q for region r is x_{qr}^{t}, where the mean is $x_{qr=\bar{r}}^{t}$ for the component and the $\sigma_{qr=\bar{r}}^{t}$ is the standard deviation (Nardo et al. 2005).

Then, we aggregated the different indicators by obtaining the outcome. The algorithm is described in Figure 1. In the figure, the construction of the interval-based composite indicator is described. First, it is necessary to choose the variables to be considered entirely for constructing the composite indicator (a set of feasible indicators to consider for the construction of the indicator). Then, to consider the uncertainty related to the construction of the composite indicator, a set of possible different random specifications is considered. In this sense, they simulated 2000 different composite indicators by choosing a different combination of the variables considered and weights. So, a set of different composite indicators is obtained, followed by the final interval. In the end, the intervals are estimated using 2000 simulations defined a priori as sufficient to estimate the intervals for each region. Thus, different interval-based composite indicators have been calculated, with each interval including a different number of simulations than the previous interval.

Additionally, the tables provide further evidence that supports this assertion. Two thousand runs seem to be sufficient for producing results that are consistent and stable. Appendix A presents the results of the interval-based composite indicator from 1000 and 20,000 simulations (Tables A1 and A2). It is possible to see that the findings are not significantly different from the results obtained after running 2000 simulations (see Figure 1). Furthermore, the ranks are pretty robust.

It is also considered a choice of the relevant number of variables on the composite indicator. They are used for different indicators in the total consideration to evaluate different measurement approaches in constructing the poverty measure. In this respect, different results are obtained due to the variability of the different measures. There can be a weak association between the different indicators so that some regions can perform better in some indicators than others.

These characteristics can vary during the process of construction of the interval-based composite indicator. However, other elements on the construction of the composite indicator do not vary. For instance, the standardization of the different variables does not vary. At the same time, any outlier detection and missing imputation are not considered (in our case, there are no missing data detected on the analysis).

The computation of different parameters for the interval composite indicators is considered: we obtain four measures: the minimum, a measure for the maximum, and center and radius. It is possible to note that the composite indicator's outcome comprehends a ranking for the minimum, the maximum, the center, and the radius. Therefore, the composite indicator can be interpreted as continuous. Furthermore, interval arithmetic makes it possible to compute the different prototypes (the interval average, which can be helpful as a benchmark).

At this point, it is possible to compute a different composite indicator by considering the random selection of a particular combination from the feasible initial indicators chosen. In this sense, our Monte Carlo simulation considers a maximum of four factors out of six with a random weight (we obtained different composite indicators by considering both the components and their weights). It was possible to sample the simulated weights using random number generation; then, we combined the results. In order to arrive at the ultimate value, we divided each preliminary weight for the total. Given the simulated composite indicator structure, we constrained the total of the final weights to be 1.

In total, 2000 unique composite indicators were obtained, based on the method above, and, finally, an interval was quantified. We computed the interval data representing the different poverty measurements using these results by considering the defined model. We considered the quantile 0.10 to be the minimum and the quantile 0.90 to be the maximum in order to avoid outliers and provide a robust version of the interval.

The different rankings were obtained by taking into account the different characteristics of the interval data: the minimum, the center, the maximum, and also the range. In the end, a different ranking that can take into account the alternative scenarios was obtained. Thus, the findings seem to be robust when considering various quantiles in the study. However, the scenarios with quantiles 0.05 and 0.95 and 0.01 and 0.99 produced the most significant shift in the ranking, with Campania taking first place rather than Calabria (Tables A3 and A4). This finding shows that the situation in Campania, depending on the factors, may be critical.

The interpretation of the center (or mid-point) and the range (or width) is essential. In this respect, it is possible to interpret the center as the "result" of the composite interval indicator, which is comparable to the most probable scenario (in this way, compared to the classical composite indicator analysis, the center could be used). In order to compare the center, the same composite indicator is computed using the equal weights scenario (Table 3). The interval range simultaneously is essential because it shows a critical difference in the results between different composite indicators. It is also possible to observe some scenarios producing relevant results when significant differences exist between the different indicators used to construct the composite indicator.

Table 3. Interval Based Composite Indicators: minimum, center, and maximum (ranked for center), center and range (ranked for range), a classical composite indicator using the equal weights scenario. Reg means region, Lb lower bound, Ce center, Ub upper bound, Rk rank, Ra range, Ew equal-weighted scenario.

Reg	Lb	Ce	Ub	Rk	Reg	Ce	Ra	Rk	Reg	Ew	Rk
Sicilia	1.29	1.72	2.16	1	Calabria	1.34	1.11	1	Sicilia	1.76	1
Campania	1.29	1.60	1.90	2	Sardegna	0.75	1.08	2	Campania	1.60	2
Calabria	0.78	1.34	1.89	3	Sicilia	1.72	0.86	3	Calabria	1.31	3
Puglia	0.61	0.87	1.12	4	Molise	0.35	0.76	4	Puglia	0.87	4
Sardegna	0.21	0.75	1.29	5	Basilicata	0.71	0.69	5	Basilicata	0.72	5
Basilicata	0.36	0.71	1.06	6	Campania	1.6	0.61	6	Sardegna	0.72	6
Molise	−0.03	0.35	0.73	7	Abruzzo	0.09	0.58	7	Molise	0.38	7
Abruzzo	−0.20	0.09	0.38	8	Piemonte	−0.39	0.52	8	Abruzzo	0.08	8
Lazio	−0.37	−0.21	−0.04	9	Puglia	0.87	0.51	9	Lazio	−0.22	9
Piemonte	−0.65	−0.39	−0.13	10	Friuli-Venezia Giulia	−0.84	0.49	10	Piemonte	−0.42	10
Liguria	−0.56	−0.46	−0.36	11	Lazio	−0.21	0.33	11	Liguria	−0.48	11
Umbria	−0.60	−0.47	−0.34	12	Valle d'Aosta	−0.66	0.28	12	Umbria	−0.48	12
Marche	−0.69	−0.55	−0.42	13	Marche	−0.55	0.27	13	Marche	−0.54	13
Valle d'Aosta	−0.80	−0.66	−0.52	14	Umbria	−0.47	0.26	14	Valle d'Aosta	−0.66	14
Lombardia	−0.87	−0.76	−0.66	15	Veneto	−1.03	0.26	15	Lombardia	−0.78	15
Friuli-Venezia Giulia	−1.09	−0.84	−0.60	16	Toscana	−0.98	0.23	16	Friuli-VG	−0.84	16
Toscana	−1.09	−0.98	−0.87	17	Lombardia	−0.76	0.21	17	Toscana	−0.98	17
Emilia-Romagna	−1.13	−1.03	−0.93	18	Emilia-Romagna	−1.03	0.2	18	Veneto	−1.01	18
Veneto	−1.16	−1.03	−0.90	19	Liguria	−0.46	0.2	19	Emilia-Romagna	−1.03	19

4. Results

In order to analyze the results accurately, it is essential to interpret the different composite interval indicators computed. We can see the results in Table 3, including calculating the center and radius of the first interval determined with minimum and maximum values.

After conducting the different comparisons between the regions, it is possible to observe that the data indicate that the obtained center's interval rankings give similar results regardless of the equal weightings scenario (Table 3). In particular, we can see that the first ranks tend to be similar. This result means that the results tend to be robust. Therefore, to evaluate and confirm the robustness of the results, we compared the ranks obtained by the parameters of the interval-based composite indicator and the equal weightings scenario; we computed a correlation matrix based on Kendall's tau correlation coefficient (Table 4).

Table 4. Correlation matrix based on Kendall's tau correlation considering the different ranks.

	Rank Lower Bound	**Rank Center**	**Rank Upper Bound**	**Rank Range**	**Rank Equal Weights**
rank lower bound	1	0.96	0.92	0.58	0.96
rank center	0.96	1	0.95	0.61	0.98
rank upper bound	0.92	0.95	1	0.66	0.95
rank range	0.58	0.61	0.66	1	0.61
rank equal weights	0.96	0.98	0.95	0.61	1

An important observation could be that we can observe differences between the ranks computed by Sardegna and Basilicata (in this sense, the ranks are inverted). At the same time, it can be noted that the results for the range allow essential reflections on the variability of the results. In that respect, the interval range is substantial because it shows how the results vary considering different weightings or assumptions in the composite indicator's construction. In particular, it can be observed that there is an evident variation between the results, due, for example, to the presence of the shadow sector on the different first ranked regions. Thus, for example, Calabria, Sardinia, and Sicily show a higher range than other regions, which the shadow sector's presence can explain (see Smith 2005; Bovi and Castellucci 1999).

By analyzing the center's table of values, the minimum, and the maximum, we can observe that Sicily has the center's highest value, followed by Campania and Calabria. It can then be possible to observe a specific separation of the following regions: Calabria, Puglia, Sardegna, and Basilicata. The regions that perform well are Italy's northern regions, such as Veneto, Emilia-Romagna, Toscana, Friuli Venezia Giulia, and Lombardia. Assuming that the minimum for each region is considered, the conclusion does not change. It is possible to find the exact ranking between the interval center and the other descriptors of the interval as the minimum and the maximum. There are some relevant changes on Basilicata's ranking, which perform at a minimum better than Sardegna. The lowest observations' rankings tend to be the same for the center, considering the worst regions. Based on the results, in this respect, the conclusions can be considered robust. Robust means that we can observe jointly that the first interval tends to have higher values than the other one for the first ranks.

Calabria and Sardegna are ranked first and second, respectively, which indicates that they both perform slightly differently on the maximum ranking. Emilia-Romagna loses a position to Veneto by considering the lowest-performing poverty regions, but the situation remains stable overall considering the highest performing regions in poverty.

The results are consistent with the range of the intervals observed. The interval range is computed considering the difference between the maximum and the minimum and measuring the variation level. Interestingly, Calabria, Sardegna, Sicilia, and Molise show the highest range between the minimum and the maximum computed. On the other hand, Toscana, Lombardia, Liguria, and Emilia-Romagna show the lowest results obtained. The key findings are that the results depend on the shadow sector's presence; some variables are better or worse depending on whether the shadow sector is present.

The variance considers all the different components of the composite indicator and shows important values for regions with a high range. In particular, when there is a higher variance between the original variables, in this case, it is possible to obtain the meaningful radii, which in this case can be interpreted with a different performance on the indicator by using specific groups of variables rather than other groups of variables. In this regard, the shadow market is an essential factor. The index of great economic difficulty is slightly higher than the other variables.

The results are essential in that they allow for the identification and measurement of poverty in Italy. Simultaneously, some regions with very high interval values are observed to have a very high center, which can be interpreted as paying particular attention to these situations (Sicilia, Campania, and Calabria and Puglia perform better). On the one hand, however, some different statistical variables make it possible to obtain significant differences with the single composite indicators' results between the different regions. So, the interval variability can be determined by considering the different variables that characterize the indicators, allowing different performances of the underlying composite indicators.

In this respect, it is possible to determine the range of the interval-based composite indicator by the variance of the different factors that constructed it.

5. Conclusions

This work aimed to measure poverty in Italy consistently using interval data that allows using a Monte-Carlo simulation on the different assumptions on which the composite indicator is constructed to explore the different results. The interval-based composite indicators show that the highest values of the social phenomenon studied, poverty, are obtained in Sicily, Campania, Calabria, and Puglia. At the same time, Calabria and Sardegna have a high value for the computed range (the difference between maxima and minima).

These results improve the existing knowledge (see Stranges 2007; Giuliano et al. 2020) by considering the equal weighting scenario, allowing us to evaluate the result for a single scenario and study how the results vary considering different scenarios using different assumptions. In this sense, we have evaluated quantitatively the sensitivity of the different results considering different scenarios. Instantaneously, it is possible to observe the different impacts of the variables on the final composite indicator, which can be observed in the range of interval-based indicators. So It is possible to measure poverty using an interval-based composite indicator, which combines and considers several different assumptions. This is a relevant innovation; using a Monte-Carlo simulation, we can construct composite indicators considering various assumptions, such as weightings. In this case, a significant finding is also that the range of the composite indicator (determined by the different performances on the variables considered for each region on the indicator) can allow the discovery of critical underlying and latent phenomena which can be discovered using this approach. In general, considering the methodological findings and conclusions, these composite indicators obtain consistent results and consider many different assumptions less sensitive to subjective assessments. In particular, they can consider many different factors of variation of a composite indicator (for example, weightings or different structures of the composite indicator) and consider the different combinations of factors identified in constructing the composite indicator. In the end, the uncertainty of the composite indicator can be endogenized and usefully compared.

The interval-based composite center indicates the composite indicator's final value, which may differ from the value identified on a single scalar composite indicator. The lower bound (the minimum) and the upper bound (the maximum) can also be considered critical indicators of extreme scenarios that can be usefully compared. At the same time, this is an important finding and result for policymakers: minima and maxima allow the design of economic policies because there can be uncertainty in the results obtained. Policies can consider these extreme scenarios as policy targets. In this sense, the policy aims to improve the minima representing some relevant territorial weaknesses.

Simultaneously, the range has a vital interpretation: it can identify significant differences in the different single indicators, further explored through a multivariate analysis. In the theoretical field of the subject, a significant result is that, by using these interval-based composite indicators, we can observe that the result of the indicator which uses poverty is also associated with the presence and size of the shadow sector. In this sense, we can observe a relevant association between a poverty indicator (synthesizing different variables related to poverty) and the size of the shadow sector.

Additionally, in this sense, the final results are more reliable than other composite indicators and can be used for policy purposes. In this respect, as theoretical and management implications, there is the possibility of using composite indicators based on intervals that allow the consideration of the variability of the information of a composite indicator. However, the results of the composite indicator are less reliable than intervals based on a lower range because of higher variability (more extensive range) because scenarios or simulations are hugely different. Then, from the ranking analysis based on the lower and upper bound, it is possible to investigate viable policies to improve the ranking based on weaknesses and strong points.

On the other hand, limitations of this work are related to the fact that the number of the variables can be increased, and it is possible to consider a more complex structure of composite indicators, which considers many different blocks of variables. In this sense, a possible future development can be the construction of interval-based composite indicators based on a different structure. Additionally, approaches to define and increase the number of simulations to perform will be considered in the future.

It is possible to address other two other connected limitations. First, there is a higher difficulty explaining and communicating interval-based composite indicators (centers, upper and lower bounds) than composite indicators based on single values (or "tiny interval", see Palumbo and Lauro 2003). In addition, it is challenging to communicate rankings on radii from a single interval-based composite indicator.

It is possible to eliminate or mitigate the subjectivity associated with applying certain assumptions (such as a structural definition of the composite indicator or a weighting system). Using this approach, we endogenize the robustness and the uncertainty analysis based on the uncertain factors in the construction of the composite indicators (on the role of the robustness and uncertainty analysis, see Saisana et al. 2005). There is a clear advantage in interpreting the composite indicator. First, the "center" represents the expected value; the range can allow comparison and reliability of the valuable central value in economic policy. Second, a considerable interval means that the different assumptions can lead to a very different scenario. Small intervals can show a homogeneous and stable situation, leading to an intervention. Finally, a lower and upper bound can lead to policy evaluations because they can show relevant scenarios that can improve or be very important. After all, they can show solid and weak points.

Another relevant point is the exploration of different ways to measure the intervals. In this respect, an attractive possible future development is to consider the intervals for computation robust central tendency (trimmed mean, median, Winsorized mean, Tukey's bi-weight mean) instead of simple interval measures. The approaches to computing the intervals allow for dealing with extreme results for composite indicators. It also proposed several novel approaches (Gatto and Drago 2021) for robustifying the results. For example, they used an interval of ranks of the original values of the composite indicator instead. The extreme scenarios' analysis and the decomposition of the intervals using bi-clustering procedures are proposed (Drago 2019).

Finally, by considering the theoretical point of view, it could be essential to explore the relationships between these indicators' results and the shadow sector.

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Appendix A

Appendix A.1. Interval-Based Composite Indicator: 1000 Simulations (Ranked by Center)

Table A1. Interval based composite indicator computed using 1000 simulations. Reg means region, Lb lower bound, Ce center, Ub upper bound, Ra range, Ew equal-weighted scenario.

Reg	Lb	Ce	Ub	Re	Ce	Ra
Sicilia	1.32	1.73	2.15	Calabria	1.34	1.09
Campania	1.3	1.6	1.89	Sardegna	0.74	1.05
Calabria	0.79	1.34	1.88	Sicilia	1.73	0.83
Puglia	0.61	0.87	1.12	Molise	0.35	0.76
Sardegna	0.21	0.74	1.27	Basilicata	0.7	0.67
Basilicata	0.36	0.7	1.03	Abruzzo	0.09	0.59
Molise	−0.03	0.35	0.73	Campania	1.6	0.59
Abruzzo	−0.2	0.09	0.39	Piemonte	−0.4	0.52
Lazio	−0.37	−0.2	−0.04	Puglia	0.87	0.51
Piemonte	−0.66	−0.4	−0.14	Friuli-Venezia Giulia	−0.85	0.5
Liguria	−0.56	−0.46	−0.36	Lazio	−0.2	0.34
Umbria	−0.6	−0.47	−0.34	Valle d’Aosta	−0.65	0.28
Marche	−0.68	−0.55	−0.42	Umbria	−0.47	0.26
Valle d’Aosta	−0.8	−0.65	−0.51	Marche	−0.55	0.26
Lombardia	−0.87	−0.76	−0.66	Veneto	−1.02	0.25
Friuli-Venezia Giulia	−1.09	−0.85	−0.6	Toscana	−0.98	0.23
Toscana	−1.1	−0.98	−0.87	Lombardia	−0.76	0.2
Veneto	−1.15	−1.02	−0.9	Emilia-Romagna	−1.03	0.2
Emilia-Romagna	−1.13	−1.03	−0.93	Liguria	−0.46	0.2

Appendix A.2. Interval Based Composite Indicator: 20,000 Simulations (Ranked by Range)

Table A2. Interval-based composite indicator computed using 20,000 simulations. Reg means region, Lb lower bound, Ce center, Ub upper bound, Ra range, Ew equal-weighted scenario.

Reg	Lb	Ce	Ub	Re	Ce	Ra
Sicilia	1.3	1.73	2.15	Calabria	1.34	1.1
Campania	1.29	1.59	1.89	Sardegna	0.74	1.08
Calabria	0.79	1.34	1.89	Sicilia	1.73	0.84
Puglia	0.61	0.86	1.12	Molise	0.34	0.78
Sardegna	0.2	0.74	1.28	Basilicata	0.71	0.68
Basilicata	0.37	0.71	1.05	Campania	1.59	0.6
Molise	−0.06	0.34	0.73	Abruzzo	0.09	0.59
Abruzzo	−0.2	0.09	0.38	Piemonte	−0.39	0.52
Lazio	−0.37	−0.21	−0.05	Puglia	0.86	0.52
Piemonte	−0.66	−0.39	−0.13	Friuli-Venezia Giulia	−0.84	0.5
Liguria	−0.56	−0.46	−0.36	Lazio	−0.21	0.33
Umbria	−0.6	−0.46	−0.33	Valle d’Aosta	−0.66	0.28
Marche	−0.68	−0.55	−0.42	Umbria	−0.46	0.27
Valle d’Aosta	−0.8	−0.66	−0.52	Marche	−0.55	0.26
Lombardia	−0.87	−0.76	−0.66	Veneto	−1.03	0.26
Friuli-Venezia Giulia	−1.09	−0.84	−0.59	Toscana	−0.97	0.23
Toscana	−1.09	−0.97	−0.86	Lombardia	−0.76	0.21
Emilia-Romagna	−1.12	−1.02	−0.92	Emilia-Romagna	−1.02	0.2
Veneto	−1.15	−1.03	−0.9	Liguria	−0.46	0.2

Appendix B

Appendix B.1. Interval Based Composite Indicator: Quantile 0.01/0.99 (Ranked by Center and Range)

Table A3. Interval-based composite indicator computed using quantile 0.01/0.99. Reg means region, Lb lower bound, Ce center, Ub upper bound, Ra range, Ew equal-weighted scenario.

Reg	Lb	Ce	Ub	Re	Ce	Ra
Sicilia	1.38	1.78	2.19	Calabria	1.47	1.2
Campania	1.36	1.67	1.98	Sardegna	0.88	1.16
Calabria	0.87	1.47	2.07	Sicilia	1.78	0.8
Puglia	0.66	0.91	1.17	Molise	0.42	0.73
Sardegna	0.3	0.88	1.46	Basilicata	0.79	0.71
Basilicata	0.43	0.79	1.14	Campania	1.67	0.62
Molise	0.05	0.42	0.78	Abruzzo	0.16	0.6
Abruzzo	−0.14	0.16	0.46	Piemonte	−0.33	0.58
Lazio	−0.35	−0.17	0.01	Friuli-Venezia Giulia	−0.79	0.53
Piemonte	−0.63	−0.33	−0.04	Puglia	0.91	0.51
Liguria	−0.55	−0.44	−0.32	Lazio	−0.17	0.36
Umbria	−0.59	−0.44	−0.28	Valle d'Aosta	−0.62	0.3
Marche	−0.66	−0.53	−0.4	Umbria	−0.44	0.3
Valle d'Aosta	−0.77	−0.62	−0.47	Marche	−0.53	0.26
Lombardia	−0.86	−0.74	−0.62	Toscana	−0.96	0.25
Friuli-Venezia Giulia	−1.05	−0.79	−0.52	Veneto	−1	0.24
Toscana	−1.08	−0.96	−0.83	Lombardia	−0.74	0.24
Emilia-Romagna	−1.11	−1	−0.9	Liguria	−0.44	0.23
Veneto	−1.12	−1	−0.88	Emilia-Romagna	−1	0.21

Appendix B.2. Interval Based Composite Indicator: Quantile 0.05/0.95 (Ranked by Center and Range)

Table A4. Interval-based composite indicator computed using quantile 0.05/0.95. Reg means region, Lb lower bound, Ce center, Ub upper bound, Ra range, Ew equal-weighted scenario.

Reg	Lb	Ce	Ub	Reg	Ce	Ra
Sicilia	1.14	1.66	2.19	Calabria	1.37	1.39
Campania	1.19	1.58	1.98	Sardegna	0.78	1.36
Calabria	0.67	1.37	2.07	Sicilia	1.66	1.05
Puglia	0.53	0.85	1.17	Molise	0.29	0.98
Sardegna	0.1	0.78	1.46	Basilicata	0.7	0.89
Basilicata	0.25	0.7	1.14	Campania	1.58	0.79
Molise	−0.2	0.29	0.78	Abruzzo	0.09	0.73
Abruzzo	−0.28	0.09	0.46	Piemonte	−0.37	0.65
Lazio	−0.41	−0.2	0.01	Puglia	0.85	0.64
Piemonte	−0.69	−0.37	−0.04	Friuli-Venezia Giulia	−0.83	0.61
Liguria	−0.57	−0.45	−0.32	Lazio	−0.2	0.42
Umbria	−0.62	−0.45	−0.28	Valle d'Aosta	−0.65	0.36
Marche	−0.72	−0.56	−0.4	Umbria	−0.45	0.33
Valle d'Aosta	−0.83	−0.65	−0.47	Veneto	−1.05	0.33
Lombardia	−0.88	−0.75	−0.62	Marche	−0.56	0.33
Friuli-Venezia Giulia	−1.14	−0.83	−0.52	Toscana	−0.97	0.27
Toscana	−1.1	−0.97	−0.83	Lombardia	−0.75	0.26
Emilia-Romagna	−1.15	−1.02	−0.9	Emilia-Romagna	−1.02	0.25
Veneto	−1.21	−1.05	−0.88	Liguria	−0.45	0.25

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