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Grasping a Concept as an Image or as a Word – A Categorical Formulation of Visual and Verbal Thinking Processes

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Authors' contributions

This work was carried out in collaboration between two authors. Both authors read and approved the final manuscript.

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ABSTRACT

Aim: Categorical formulations for thought processes and communications are provided.
Study Design: Category theory and Sheaf theory.
Place and Duration of Study: June 2010 to April 2013 in the Department of Mathematics, California Polytechnic State University and the Institute of Economic Research, Kyoto University.
Methodology: In order to capture the changing state of an individual over a time period, we introduce a state controlling "variable" in terms of objects (called generalized time) in a site (i.e., a category with a Grothendieck topology). We also capture communications (information flows) as observation morphisms between individuals. The interplay among the change of the state of an individual, the communication between individuals, and the thought processes of those individuals is unified as the commutativity of the fundamental diagram in Section 3 of [1].
Results: We provide formulations for communication diagrams (information flows) between the language dependency type and the image dependency type which are denoted as Type (L) and Type (I), respectively.
Conclusion: We have used categorical notions to formulate to express Type (I) and Type (L) of thought patterns. Then we have formulated the information flows in terms of the induced morphisms between Type (I) and Type (L).

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1. INTRODUCTION

As in the past, human attitude has long been discussed in relationship to personalities, the integral part of which seems to be acquired through influences from childhood, e.g., family background. Whether individuals have brothers or sisters, whether they are youngest or eldest, or how they are raised, all affect the development of personality. Sometimes an extroverted person may become more inward after having frustrated or defeated in ambition. In order to study objectively such highly complex entities as personalities, we need to study something inherent to human beings not subject to be influenced by environmental factors.

For that purpose, we focus on a "thinking pattern" as a basic structure of personalities, which with some minor exceptions, can be identified even in adulthood. We, in this study, focus on the spontaneous thought process and divide it into two components: visual and verbal thinking.

Then we classify the subjects into two major groups: 1) image-thinking group, and 2) verbal-thinking group, and 3) image/verbal-thinking group. For motivations of this classification, consult [2-4].

For the image-thinking group, a recalling process is prompted by picturing images: for example, the images of numbers for recalling a telephone number. We name this image-oriented group Type (*I*). For Type (*I*), free thought is a series of scenes coming in and out of the mind like successive frames of a film.

The verbal-thinking group is a group of people conducting self-talk, rather than picturing images or characters when recalling something. This group, Type (*L*), relies on a flow of words in recalling and conducting thoughts.

Type (*I*) is capable of exerting high concentration when the mind is set on one strong image. Images promote the generation of physical sensation, thus directing full strength toward the goal. The type (*I*) individuals are generally emotional and active, never be convinced without the associated images, and thus are less likely to be influenced by others. These tendencies, however, may appear capricious, absent minded or forgetful to some people. For example, they may listen to somebody speak, but if the speech does not inspire an image, their minds can not be turned on. Their thinking patterns will be changed concurrently with the images coming to mind.

Type (*L*) has a tendency to be unable to understand things unless he/she is instructed in an organized manner. Type (*L*) individuals generally take time to learn, but once they learn, they are sure to make steady progress. This type of people is capable of processing many thoughts at the same time and considerate enough to feel for others around them. Their thoughts themselves are also conducted in an orderly way. What would happen when Type (*I*) is communicating with Type (*L*)? How about the converse: is it a smooth (canonically induced morphism) case for Type (*L*) to communicate with Type (*I*)? Type (*I*) people understand or interpret their thoughts as images by finding common concepts (images) among individual thoughts (words). Technically speaking, we need to define a morphism (mapping) assigning a class to a class in the case of communication between Type (*I*) and Type (*I*). We will give categorical formulations for all four ordered combinations between Type (*I*) and Type (*L*).

2. METHODOLOGY

The notion of a topos has recently been applied to theoretical physics, especially to quantumgravity by [5-8]. For generalactivity reports in theoretical physics, see [9-11].

In terms of categorical concepts, e.g., functors, natural transformations, direct and inverse limits,brain activities, e.g., understanding and thinking have been formulated. See [12-14] for consciousness study in terms of category theory.

By definition, a presheaf is a contravariantfunctor from a category S to another category, e.g., the category ((sets)) of sets. We take such a domain category as a site S, i.e., a category with a Grothendieck topology. See [15-18] for the notion of a site. Let $\hat{S}$ be the category of presheaves (contravariantfunctors) from the site S to the category ((sets)) of sets. For objects m and P in $\hat{S}$, we define an observation morphism (or a measurement morphism) from m to P over an object V in the site S as a set theoretic map (i.e.,a morphism of ((sets)))

$$s_V : m(V) \longrightarrow P(V), \quad (1_m)$$

where$m$ need not be the presheaf associated with a human entity, i.e., m can be a presheaf associated with a particle.

When m happens to be an individual Q (a human being), then such an observation morphismshould be interpreted as an information flow (i.e., communication) from individual Q to individual Pover V.

Such an object V in S is called a generalized time (period). Then the image Im$s_V$of $s_V$is asubset of $P(V)$ which is the information (the measurements) which the observer P receives from theobserved m over the generalized time period V.

In (1_m), let $m = P$. Namely, we get a self-observation map

$$p_V : P(V) \longrightarrow P(V). \quad (1_p)$$

See [12-14] for consciousness related topics. The elements in the set $P(V)$ areregarded as "thoughts" over a generalized time V, and a map (endomorphism) from $P(V)$ to $P(V)$ isregarded as "thinking," i.e., a thought process over V.

Next, consider a covering in the site S

$$\{V \longleftarrow V_i\}_{i \in I}, \quad (2)$$

which would be $V = \cup_{i \in I} V_i$if category S were a topological space. Next, we will introduce a special presheaf. Apresheaf P is said to be a sheaf if the following glueing condition holds. For $s_i \in P(V_i)$ and $s_j \in P(V_j)$, $i, j \in I$, satisfying

$$s_i|_{V_i \times V_j} = s_j|_{V_i \times V_j}, \quad (3)$$

there exists a unique $s \in P(V)$ to satisfy

$$s|_{V_i} = s_i, \quad \text{for all } i \in I. \tag{4}$$

Note that a presheaf associated with a (functioning) brain is a sheaf. See also our paper [1]. This is because a (functioning) brain has an ability to form a global object from a given local data by glueing local informations. See also [12-14].

We can also formulate the notion of understanding (recognizing) in terms of categorical conceptsas follows. For $s \in P(V)$, if there exist W in S and a morphism $V \xrightarrow{\varphi} W$ to satisfy

$$s = P(\varphi)(s') \tag{5}$$

for$s' \in P(W)$, then $s' \in P(W)$, is said to be an understanding of $s \in P(V)$. (Note that the map $P(\varphi)$ would become the restriction map if S were a topological space.) Or, $s \in P(V)$ is recognized as $s' \in P(W)$.

As examples of understanding (or recognizing), we give the following Example 1 and 2.

Example 1: Each time one is allowed to see a only $10cm \times 10cm$ area of an entire large painting. By glueing these local data, i.e., Eq.(3), we obtain global information $s \in P(V)$. Then each local datum $s_i \in P(V_i)$ is the restriction of $s \in P(V)$ to V_i, i.e., Eq.(4). When such a large painting is well known, partial glueing as in Eq.(3) is enough due to memory to obtain the global $s \in P(V)$.

Example 2: When one is shown (or one hears) only a few notes of a well known music piece, one may not be able to recognize (understand) the music piece (the global information). However, when enough notes are shown, one can recognize the music piece. Then the previously shown few notes are the image of the understanding map, i.e., Eq.(5), $s = P(\varphi)(s')$.

3. RESULTS AND DISCUSSION

In this section, we treat entities as human beings.

Hence anobservation (measurement) morphism in Eq. (1_m) may be interpreted as an informationflow by communication between two individuals P and Q.

In Eq. (1_m) in the previous section, we let $m = Q$. That is, Eq. (1_m) becomes

$$t_V : Q(V) \longrightarrow P(V). \tag{1'}$$

For a morphism in S

$$\psi : V \longrightarrow V' \tag{6}$$

we have the functionally induced morphism

$$P(\psi): P(V') \longrightarrow P(V). \quad (7)$$

The morphism in Eq.(7) has canonical nature. Namely, the morphism $\psi : V \longrightarrow V'$ induces the functorial (natural) change of the states of P from the period V to the period V'. Mathematically speaking, the contravariantfunctor$P$ takes $\psi : V \longrightarrow V'$ in S into $P(\psi) : P(V') \longrightarrow P(V)$ in ((sets)).

Note that if individual P communicates with individual Q over V, as before, we have:

$$Q(V) \xrightarrow{t_V} P(V)$$

i.e., an information flow from $Q(V)$ to $P(V)$. And, the image of this information flow Im$t_V$is the information that $P(V)$ received from $Q(V)$.

Similarly, for Q in $\hat{S}$, we have

$$Q(\psi): Q(V') \longrightarrow Q(V). \quad (8)$$

Let the self observation map for Q be

$$q_V : Q(V) \longrightarrow Q(V). \quad (1_Q)$$

An interpretation of the composition $p_V \circ t_V$, of

$$Q(V) \xrightarrow{t_V} P(V)$$

with the thought process (thinking)

$$P(V) \xrightarrow{p_V} P(V)$$

is the thought process influenced by the communication t_V.

Hence, the commutativity

$$t_V \circ q_V = p_V \circ t_V$$

indicates that, e.g., individual $P(V)$ listens as, for example, $Q(V)$ speaks; the communication $t_V$influences the thinking $p_V$of $P(V)$ and the thought process $q_V$of $Q(V)$ influences the communication t_V. That is, the communication between $P(V)$ and $Q(V)$, (in this case, $P(V)$ is the listener) is well engaged, i.e., a good communication between P and Q over the state-determining variable V.

There are basically two types of thinking processes. The first type depends on language oriented thinking processes. This language oriented type corresponds to a set theoretic map (1_P) in Section 2. Namely, the map $p_V$in (1_P) is defined on elements (thoughts) in $P(V)$. The second type depends on image oriented thinking processes. This image oriented type corresponds to a map $\tilde{p}_V$ defined on classes of elements. That is, $\tilde{p}_V$ is the induced map from p_V:

$$\tilde{p}_V : P(V)/\sim \longrightarrow P(V)/\sim$$

where ~ indicates a (an equivalence) relation on the set $P(V)$ as follows.

We have formulated thinking or thought processes as a morphism in the category of sets, i.e.,

$$p_V : P(V) \longrightarrow P(V), \quad (9)$$

where P is a presheaf associated with an entity like a human, and V is an object of a *t*-site S. Namely, an element (a thought) of $P(V)$ is mapped by p_V to an element of $P(V)$. Such a morphism from $P(V)$ to $P(V)$ itself is said to be an endomorphism. As indicated earlier in paper [1], if $p_V(x) = x$ for all x in $P(V)$, i.e., $p_V = Id_{p_V}$, such an entity P thinks nothing over the generalized time period V of the *t*-site S.

Type (L) people whose thinking processes depending upon language have an element-dependent morphism. On the other hand, Type (I) people who grasp the image (the common concept) among individual thoughts, i.e., elements of $P(V)$, process their thinking as a class-dependent morphism. The classification by finding the common concept among thoughts has the following properties. When a thought x is similar (or related) to y in $P(V)$, we write it by $x \sim y$. Then we have,

$$\begin{cases} x \sim x, \text{ and} \\ \text{if } x \sim y \text{ then } y \sim x, \text{ and} \\ \text{if } x \sim y \text{ then } y \sim z, \text{ then } x \sim z. \end{cases}$$

Namely, ~ is an equivalence relation on $P(V)$.

Let the partition of $P(V)$ by the equivalence classes be $\tilde{P}(V) = P(V)/\sim$. Let us consider the induced morphism by p_V in (9):

$$\tilde{p}_V : \tilde{P}(V) \longrightarrow \tilde{P}(V), \quad (10)$$

where $\tilde{p}_V$ is defined by $\tilde{p}_V(\overline{x}) = \overline{p}_V(x) \in \tilde{P}(V)$.

The induced morphism $\tilde{p}_V$ has the following interpretation. For a thought x representing the class $\overline{x}$ (the common concept of all thoughts in this equivalence class), the thought-process morphism p_V takes x to the element $p_V(x)$. Then the assigned class by $\tilde{p}_V(\overline{x})$ is the class $\overline{p}_V(x)$ to which $p_V(x)$ belongs.

Namely, for an individual thought x in $P(V)$, the equivalence class $\overline{x}$ represents the image-interpretation P has (over V) about x. In the case of thought processes of the entity P of Type (I), a thought process takes images to images, i.e., $\tilde{P}(V) \longrightarrow \tilde{P}(V)$, rather than element-wise individual thoughts to thoughts, i.e., $P(V) \longrightarrow P(V)$. Eq.(10) indicates that the image x associated with the thought x is thought-processed (thinking) to the image $\overline{p}_V(x)$ associated with the thought $p_V(x)$.

The communication combinations between Type (L) and Type (I) can be expressed symbolically as follows:

$$\begin{cases} (i) & \text{Type } (I) \longrightarrow \text{Type } (I) \\ (ii) & \text{Type } (L) \longrightarrow \text{Type } (I) \\ (iii) & \text{Type } (I) \longrightarrow \text{Type } (L) \\ (iv) & \text{Type } (L) \longrightarrow \text{Type } (L) \end{cases}$$

(i) Communication between the same Type (*I*)

Recall the communication (information flow) between individuals P and Q over a generalized time period V can be interpreted as an observation morphism.

$$t_V : Q(V) \longrightarrow P(V). \tag{11}$$

(See [1, section 3]). When P and Q are of Type (I), we need to construct (define) the induced morphism $\tilde{t}_V$ from $\tilde{Q}(V)$ to $\tilde{P}(V)$. Then we must obtain commutativity of the fundamental diagram in section 3 of [1].

Recall that the commutativity indicates the compatibility conditions associated with the states corresponding to a morphism $V \longrightarrow V'$ in S.

The construction of $\tilde{t}_V$ can be done canonically as follows. For $\overline{x} \in \tilde{Q}(V)$, represented by a thought x in $Q(V)$, define

$$\tilde{t}_V(\overline{x}) = \bar{t}_V(x). \tag{12}$$

Summarizing this construction, we get the commutative diagram:

$$\begin{array}{ccc} Q(V) & \xrightarrow{t_V} & P(V) \\ \downarrow \pi_Q & & \downarrow \pi_P \\ \tilde{Q}(V) & \xrightarrow{\tilde{t}_V} & \tilde{P}(V) \end{array} \tag{13}$$

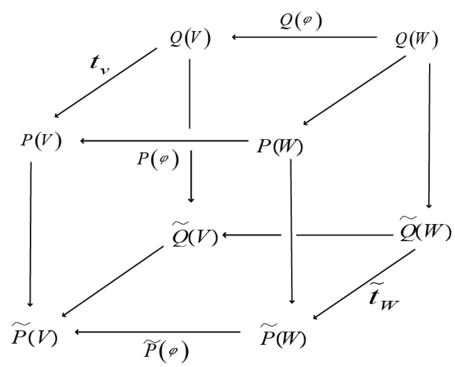
where π_Q and π_P are the canonical epimorphisms (surjectivemorphisms) to the quotient objects. The above epimorphisms π_Q and π_P in Eq.13 are the projections from individual thoughts to the associated image-interpreted classes. The definition of $\tilde{t}_V$ in Eq.12 has the following interpretation. For the communication morphism t_V, the created image $\overline{x}$ by the thought x of P (over V) is communicated by the induced morphism $\tilde{t}_V$ to the image $\overline{t_V}(x)$ associated with the thought $t_V(x)$ of Q (over V). Since P and Q are both Type (I), their thoughts x and $t_V(x)$ of P and Q, respectively, are interpreted (or understood) as the images $\overline{x}$ and $\overline{t_V}(x)$ by P and Q over V, respectively.

In our paper [1], we have introduced a formulation for "understanding (recognizing)." For Type (I) P (and Q), such a formulation must be a morphism

$$\tilde{P}(W) \xrightarrow{\tilde{P}(\varphi)} \tilde{P}(V) \tag{14}$$

where φ is a morphism from V to W, and $P(\varphi)$ is the induced morphism (acting as the restriction) from $P(W)$ to $P(V)$ satisfying the commutativity of the following diagrams:

$$\begin{array}{ccc} P(W) & \xrightarrow{P(\varphi)} & P(V) \\ \downarrow \pi_{P(W)} & & \downarrow \pi_{P(V)} \\ \tilde{P}(W) & \xrightarrow{\tilde{P}(\varphi)} & \tilde{P}(V) \end{array} \tag{15}$$



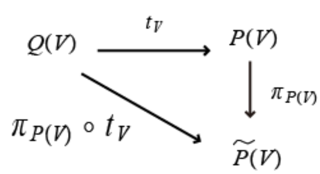
(16)

(ii) Communication from Type (*L*) to Type (*I*)

For this case our goal is to establish a morphism:

$$Q(V) \longrightarrow \tilde{P}(V). \tag{17}$$

Using the same notation as in Case (i), consider the following diagram.



(18)

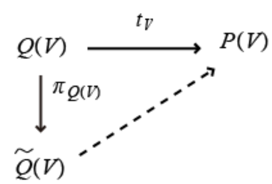
Hence, the morphism in (17) is obtained by the composition $\pi_{P(V)} \circ t_V$ of the commutative diagram (18). Namely, element-wise, for a thought $x \in Q(V)$ we obtain

$$(\pi_{P(V)} \circ t_V)(x) = \pi_{P(V)}(t_V(x)) = \bar{t}_V(x). \tag{19}$$

The meaning of Eq.(18) is the following. Individual P receives the information $t_V(x)$ as a thought of $P(V)$ (the state of P determined by V) from Q over V. Then P interprets the received thought $t_V(x)$ as the "image" $\overline{t}_V(x)$ in $\tilde{P}(V)$. Since Q is of Type (L), the communication method of Q is element-wise (i.e., individual thoughts are communicated with P via t_V). However, since P is Type (I), reinterprets the thought $t_V(x)$ as the image $\overline{t_V}(x)$ induced by that thought which is the meaning of the composition of $t_V$with the projection eprimorphism$\pi_{P(V)}$.

(iii) Communication from Type (*I*) to Type (*L*)

Instead of diagram (9) in Case (ii), we need to consider the diagram:



(20)

That is, there does not induce any canonical morphism from $\tilde{Q}(V)$ to $P(V)$. This means that information flows (communication) from Type (I) to Type (L) are not smooth (i.e., difficult). That is, for language Type (L) people, it is not easy (requires an effort) to understand image Type (L) people since there does not canonically exist a morphism from $\tilde{Q}(V)$ to $P(V)$.

The formulation of this case provides the following conjecture. That is, the communication from Type (I) to Type (L) is not smooth. Since Q is of Type (I), Q prefers to interpret the thoughts as the images (classes). However, P prefers the element level to the class level for communication. This conflict is the consequence of the fact that we can not compose $t_V$with $\pi_{Q(V)}$ in Eq.(19).

(iv) Communication between Type (*L*)

This is simply $Q(V) \longrightarrow P(V)$ as (11). Namely, this type of communication is the most straightforward combination among the four types listed earlier.

4. CONCLUSION

We have used categorical motions to express two kinds Type (I) and Type (L) of thinking patterns of brain activity.

We have formulated Type (I), i.e., the image-thinking type in terms of the notion of a quotient object where an image associated with a thought is an equivalence class in the quotient object. Intuitively speaking, by collecting individual thoughts sharing a common concept, the equivalence class is formed. Then the image interpretation of thoughts is the following assignment: for an individual thought x to the class $\overline{x}$ to which x belongs. Furthermore, we have analyzed information flows (communication) in terms of (induced) morphines for all four ordered combinations between the Type (I) and Type (L).

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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