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Sine-cosine Method for a Class of Nonlinear Fourth Order Variant of a generalized Camassa-holm equation

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Abstract

In this paper we study a class of nonlinear fourth order analogue of a generalized Camassa-Holm equation by using sine-cosine method. The compactons, solitary wave, solitary patterns, periodic wave and solitary patterns solutions of a class of nonlinear fourth order analogue of a generalized Camassa-Holm equation are successfully obtained. It is shown that the sine-cosine provides a powerful mathematical tool for solving a great many nonlinear partial differential equations in mathematical physics.

Keywords: Compactons; Solitary wave solutions; Solitary patterns solutions; Periodic solutions; Sine-Cosine method; Camassa-Holm equation

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1 Introduction

Studies of various physical structures of nonlinear dispersive equations had attracted much attention in connection with the important problems that arise in scientific applications. Mathematically, these physical structures have been studied by using various analytical methods, such as inverse scattering method [1], Darboux transformation method [2,3], Hirota bilinear method [4], Lie group method [5], bifurcation method of dynamic systems [6,7,8], sine-cosine method [9,10], tanh function method [11-13], Fan-expansion method [14], homogenous balance method [15] and so on. Practically, there is no unified technique that can be employed to handle all types of nonlinear differential equations.

In 1999, Clarkson and Priestley [16] studied a class of nonlinear fourth order partial differential equations

$$u_{tt} = (au + bu^2)_{xx} + \gamma u u_{xxxx} + \mu u_{xxtt} + \alpha u_x u_{xxx} + \beta u_{xx}^2, \tag{1.1}$$

where $\alpha, \beta, \gamma, \mu, a, b$ are arbitrary constants. This equation was thought of [16] as a fourth order analogue of a generalization of the Camassa-Holm equation. Further, Eq. (1.1) was also considered as a Boussinesq-type equation. In [16], it was shown that (1.1) admits both conventional solitons and compactons.

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Motivated by the rich treasure of the Camassa-Holm-type equation in the literature, we will study nonlinear dispersive variants CH(n,m,m) and CH(-n,-m,-m) of the generalized Camassa-Holm equation of the form, respectively

$$u_{tt} = (au + bu^n + du^m)_{xx} + k(u^m)_{xxtt},\ m, n \in Z^+ \tag{1.2}$$

and

$$u_{tt} = (au + bu^{-n} + du^{-m})_{xx} + k(u^{-m})_{xxtt},\ m, n \in Z^+ \tag{1.3}$$

Specially, when $n = m$, the two equations respectively called nonlinear dispersive variants CH(n,n) and CH(-n,-n) of the generalized Camassa-Holm equation [17]. In [17], A.M. Wazwaz studied the CH(n,n) and CH(-n,-n) equations by using sine-cosine method, it is shown that these class give compactons, conventional solitons, solitary patterns and periodic solutions. It is also found that the qualitative change in the physical structure of solutions depends mainly on the exponent of the wave function $u(x, t)$, positive or negative, and on the coefficient of $(u^n)''$ as well.

The sine-cosine method will be mainly used to back up our analysis. The sine-cosine method was proved to be powerful in handling nonlinear problems, with genuine nonlinear dispersion, where compactons and solitary patterns solutions are generated. This method will be described briefly, where details can be found in [9-13,17-18] and the references therein.

Let $u(x, t) = u(x - ct) = u(\xi)$, where c is the wave speed. Then Eq.(1.2) and Eq.(1.3) become to

$$c^2u'' = (au + bu^n + du^m)'' + kc^2(u^m)^{(4)} \tag{1.4}$$

and

$$c^2u'' = (au + bu^{-n} + du^{-m})'' + kc^2(u^{-m})^{(4)}, \tag{1.5}$$

where "$\prime$" is the derivative with respect to ξ. Integrating Eq. (1.4) and Eq. (1.5) twice, using the constants of integration to be zero we have the following ordinary differential equation

$$(a - c^2)u + bu^n + du^m + kc^2(u^m)'' = 0 \tag{1.6}$$

and

$$(a - c^2)u + bu^{-n} + du^{-m} + kc^2(u^{-m})'' = 0. \tag{1.7}$$

In what follows, we highlight the main steps of the sine-cosine algorithm and the extended algorithm.

2 Analysis of the Sine-cosine Method

The sine-cosine method has been applied for a wide variety of nonlinear problems. The main features of the method will be reviewed briefly.

We first use the wave variable $\xi = x - ct$ to carry a PDE in two independent variables

$$P(u, u_t, u_x, u_{tx}, u_{xx}, u_{xxx}, ...) = 0 \tag{2.1}$$

into an ODE

$$Q(u, u', u'', u''', ...) = 0. \tag{2.2}$$

Eq. (2.2) is then integrated as long as all terms contain derivatives where integration constants are considered zeros.

The sine-cosine method admits the use of the solution in the form

$$u(x, y, t) = \begin{cases} \lambda \cos^{\beta}(\mu\xi), & \mid \mu\xi \mid < \frac{\pi}{2}, \\ 0, & otherwise, \end{cases} \tag{2.3}$$

or in the form

$$u(x,y,t) = \begin{cases} \lambda \sin^{\beta}(\mu\xi), & |\mu\xi| < \pi, \\ 0, & otherwise, \end{cases} \tag{2.4}$$

where λ, μ, and β are parameters that will be determined. For (2.3),we use

$$\begin{aligned} u(\xi) &= \lambda \cos^{\beta}(\mu\xi), \\ u^n(\xi) &= \lambda^n \cos^{n\beta}(\mu\xi), u^{-n}(\xi) = \lambda^{-n} \cos^{-n\beta}(\mu\xi) \\ (u^n)'' &= -n^2\mu^2\beta^2\lambda^n \cos^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\cos^{n\beta-2}(\mu\xi), \end{aligned} \tag{2.5}$$

and for (2.4) we use

$$\begin{aligned} u(\xi) &= \lambda \sin^{\beta}(\mu\xi), \\ u^n(\xi) &= \lambda^n \sin^{n\beta}(\mu\xi), u^{-n}(\xi) = \lambda^{-n} \sin^{-n\beta}(\mu\xi) \\ (u^n)'' &= -n^2\mu^2\beta^2\lambda^n \sin^{n\beta}(\mu\xi) + n\mu^2\lambda^n\beta(n\beta-1)\sin^{n\beta-2}(\mu\xi). \end{aligned} \tag{2.6}$$

We substitute (2.3) or (2.4) into the reduced ordinary differential equation obtained above in (2.2), balance the terms of the cosine functions when (2.3) is used, or balance the terms of the sine functions when (2.4) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations to obtain all possible valuse of the parameters λ, μ and β.

3 Using the Sine-cosine Method

3.1 For positive exponents

Substituting (2.5) into (1.6) yields

$$\begin{aligned} &(a-c^2)\lambda\cos^{\beta}(\mu\xi) + b\lambda^n\cos^{n\beta}(\mu\xi) + d\lambda^m\cos^{m\beta}(\mu\xi) \\ &+c^2km\mu^2\beta\lambda^m((m\beta-1)\cos^{m\beta-2}(\mu\xi) - m\beta\cos^{m\beta}(\mu\xi)) = 0. \end{aligned} \tag{3.1}$$

Eq. (3.1) is satisfied only if the following system of algebraic equations holds:

$$\begin{aligned} &m\beta \neq 1,\ a-c^2=0,\ n\beta = m\beta - 2, \\ &b\lambda^n = -c^2km\mu^2\lambda^m\beta(m\beta-1),\ d\lambda^m = c^2km^2\mu^2\beta^2\lambda^m. \end{aligned} \tag{3.2}$$

Solving the system (3.2) give

$$\beta \neq \frac{1}{m},\ a = c^2,\ \beta = \frac{2}{m-n},\ \mu = \pm\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}},\ \lambda = \left[\frac{d(m+n)}{-2bm}\right]^{\frac{1}{n-m}}. \tag{3.3}$$

The results (3.3) can be easily obtained if we also use the sine method (2.6).

For $m > n, l \in Z^+$,$h \in Z^+$,combining (3.3) with (2.5) and (2.6), we have the following compactons solutions:

$$u_1 = \begin{cases} \pm\left[\frac{-2bm}{d(m+n)}\cos^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \frac{\pi}{2},\ m-n=2l,\ a>0,\ dk>0,\ bd<0, \\ 0, \qquad otherwise, \end{cases} \tag{3.4}$$

$$u_2 = \begin{cases} \left[\frac{-2bm}{d(m+n)}\cos^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \frac{\pi}{2},\ \ m-n=2h-1,\ a>0,\ dk>0, \\ 0, \qquad otherwise, \end{cases} \tag{3.5}$$

$$u_3 = \begin{cases} \pm\left[\frac{-2bm}{d(m+n)}\sin^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \pi,\ m-n=2l,\ a>0,\ dk>0,\ bd<0, \\ 0, \qquad\qquad otherwise \end{cases} \tag{3.6}$$

and

$$u_4 = \begin{cases} \left[\frac{-2bm}{d(m+n)}\sin^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \pi,\ \ m-n=2h-1,\ a>0,\ dk>0, \\ 0, \qquad\qquad otherwise. \end{cases} \tag{3.7}$$

However, for $dk < 0$, we obtain the following solitary patterns solutions:

$$u_5 = \pm\left[\frac{-2bm}{d(m+n)}\cosh^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, m-n=2l,\ a>0,\ dk<0,\ bd<0, \tag{3.8}$$

$$u_6 = \left[\frac{-2bm}{d(m+n)}\cosh^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n=2h-1,\ a>0,\ dk<0, \tag{3.9}$$

$$u_7 = \pm\left[\frac{2bm}{d(m+n)}\sinh^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, m-n=2l,\ a>0,\ dk<0,\ bd>0 \tag{3.10}$$

and

$$u_8 = \left[\frac{2bm}{d(m+n)}\sinh^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n=2h-1,\ a>0,\ dk<0. \tag{3.11}$$

For $m < n$, $l \in Z^+$, $h \in Z^+$, combining (3.3) with (2.5) and (2.6), the following periodic wave solutions:

$$u_9 = \pm\left[\frac{d(m+n)}{-2bm}\sec^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m=2l,\ a>0,\ dk>0,\ bd<0, \tag{3.12}$$

$$u_{10} = \left[\frac{d(m+n)}{-2bm}\sec^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m=2h-1,\ a>0,\ dk>0, \tag{3.13}$$

$$u_{11} = \pm\left[\frac{d(m+n)}{-2bm}\csc^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m=2l,\ a>0,\ dk>0,\ bd<0 \tag{3.14}$$

and

$$u_{12} = \left[\frac{d(m+n)}{-2bm}\csc^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}},\ n-m=2h-1,\ a>0,\ dk>0. \tag{3.15}$$

However, for $dk < 0$, we obtain the following solitary wave and solitary patterns solutions:

$$u_{13} = \pm\left[\frac{d(m+n)}{-2bm}sech^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m=2l,\ a>0,\ dk<0,\ bd<0, \tag{3.16}$$

$$u_{14} = \left[\frac{d(m+n)}{-2bm} sech^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m = 2h-1,\ a>0,\ dk<0, \quad (3.17)$$

$$u_{15} = \pm\left[\frac{d(m+n)}{2bm} csch^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m = 2l,\ a>0,\ dk<0,\ bd>0 \quad (3.18)$$

and

$$u_{16} = \left[\frac{d(m+n)}{2bm} csch^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}},\ n-m = 2h-1,\ a>0,\ dk<0. \quad (3.19)$$

3.2 For negative exponents

We consider Eq. (1.3) equation. Substituting (2.5) into (1.7) yields

$$(a-c^2)\lambda\cos^{\beta}(\mu\xi) + b\lambda^{-n}\cos^{-n\beta}(\mu\xi) + d\lambda^{-m}\cos^{-m\beta}(\mu\xi)$$
$$+c^2km\mu^2\beta\lambda^{-m}((m\beta+1)\cos^{-m\beta-2}(\mu\xi) - m\beta\cos^{-m\beta}(\mu\xi)) = 0. \quad (3.20)$$

Eq. (3.20) is satisfied only if the following system of algebraic equations holds:

$$m\beta \neq -1,\ a-c^2 = 0,\ n\beta = m\beta+2,$$
$$b\lambda^{-n} = -c^2km\mu^2\lambda^{-m}\beta(m\beta+1),\ d\lambda^{-m} = c^2km^2\mu^2\beta^2\lambda^{-m}. \quad (3.21)$$

Solving the system (3.21) give

$$\beta \neq -\frac{1}{m},\ a = c^2,\ \beta = \frac{2}{n-m},\ \mu = \pm\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}},\ \lambda = \left[\frac{d(m+n)}{-2bm}\right]^{\frac{1}{m-n}}. \quad (3.22)$$

The results (3.22) can be easily obtained if we also use the sine method (2.6).

For $n > m$, $l \in Z^+$, $h \in Z^+$, combining (3.22) with (2.5) and (2.6), the following compactons solutions:

$$u_{17} = \begin{cases} \pm\left[\frac{-2bm}{d(m+n)}\cos^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \frac{\pi}{2}, n-m = 2l,\ a>0,\ dk>0,\ bd<0, \\ 0, \qquad otherwise, \end{cases} \quad (3.23)$$

$$u_{18} = \begin{cases} \left[\frac{-2bm}{d(m+n)}\cos^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \frac{\pi}{2},\ n-m = 2h-1,\ a>0,\ dk>0, \\ 0, \qquad otherwise, \end{cases} \quad (3.24)$$

$$u_{19} = \begin{cases} \pm\left[\frac{-2bm}{d(m+n)}\sin^2\left|\frac{m-n}{2m}\right|\sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \pi, n-m = 2l,\ a>0,\ dk>0,\ bd<0, \\ 0, \qquad otherwise \end{cases} \quad (3.25)$$

and

$$u_{20} = \begin{cases} \left[\frac{-2bm}{d(m+n)} \sin^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, \\ \left|\frac{m-n}{2m}\sqrt{\frac{d}{ak}}(x-ct)\right| < \pi,\ n-m = 2h-1,\ a>0,\ dk>0, \\ 0. \qquad\qquad otherwise. \end{cases} \tag{3.26}$$

However, for $dk < 0$, we obtain the following solitary patterns solutions:

$$u_{21} = \pm\left[\frac{-2bm}{d(m+n)} \cosh^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}}, n-m = 2l,\ a>0,\ dk<0,\ bd<0, \tag{3.27}$$

$$u_{22} = \left[\frac{-2bm}{d(m+n)} \cosh^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}},\ n-m = 2h-1,\ a>0,\ dk<0, \tag{3.28}$$

$$u_{23} = \pm\left[\frac{2bm}{d(m+n)} \sinh^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}},\ n-m = 2l,\ a>0,\ dk<0,\ bd>0 \tag{3.29}$$

and

$$u_{24} = \left[\frac{2bm}{d(m+n)} \sinh^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{n-m}},\ n-m = 2h-1,\ a>0,\ dk<0. \tag{3.30}$$

For $n < m$, $l \in Z^+$, $h \in Z^+$, combining (3.22) with (2.5) and (2.6), the following periodic wave solutions:

$$u_{25} = \pm\left[\frac{d(m+n)}{-2bm} \sec^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2l,\ a>0,\ dk>0,\ bd<0, \tag{3.31}$$

$$u_{26} = \left[\frac{d(m+n)}{-2bm} \sec^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2h-1,\ a>0,\ dk>0, \tag{3.32}$$

$$u_{27} = \pm\left[\frac{d(m+n)}{-2bm} \csc^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2l,\ a>0,\ dk>0,\ bd<0 \tag{3.33}$$

and

$$u_{28} = \left[\frac{d(m+n)}{-2bm} \csc^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2h-1,\ a>0,\ dk>0. \tag{3.34}$$

However, for $dk < 0$, we obtain the following solitary wave and solitary patterns solutions:

$$u_{29} = \pm\left[\frac{d(m+n)}{-2bm} sech^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2l,\ a>0,\ dk<0,\ bd<0, \tag{3.35}$$

$$u_{30} = \left[\frac{d(m+n)}{-2bm} sech^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2h-1,\ a>0,\ dk<0, \tag{3.36}$$

$$u_{31} = \pm\left[\frac{d(m+n)}{2bm} csch^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}},\ m-n = 2l,\ a>0,\ dk<0,\ bd>0 \tag{3.37}$$

and

$$u_{32} = \left[\frac{d(m+n)}{2bm} csch^2 \left|\frac{m-n}{2m}\right| \sqrt{\frac{-d}{ak}}(x-ct)\right]^{\frac{1}{m-n}}, \; m-n = 2h-1, \; a>0, \; dk<0. \qquad (3.38)$$

4 Discussion

By using the sine-cosine method to study a class of nonlinear fourth order variant of a generalized Camassa-Holm equation the results are similar to the reference [9,17] conclusion. The study of compactons may give insight into many scientific processes such as the super deformed nuclei, preformation of cluster in hydrodynamic models, the fission of liquid drops (nuclear physics), inertial fusion and others as discussed in. The basic goal of this work has been the study of a class of nonlinear fourth order analogue of a generalized Camassa-Holm equation. The solitary wave and compactons solutions for a class of nonlinear fourth order variant of a generalized Camassa-Holm equation is obtained analytically by using the sine-cosine method. The obtained results in this work clearly demonstrate the effect of the purely nonlinear dispersion and the qualitative change made in the genuinely nonlinear phenomenon. This approach may be applied to seek traveling wave solutions for other types of nonlinear dispersion partial differential equations which satisfy certain restrictions.

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Competing Interests

The authors declare that no competing interests exist.

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