



## Field Experiments on Hydraulic Jump in a Sloped Triangular Channel

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### Authors' contributions

This work was carried out in collaboration between both authors. Both authors read and approved the final manuscript.

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### ABSTRACT

Triangular ditch irrigation may be used to raise the downstream water surface. In this paper hydraulic jump in a sloped triangular channel of 90° central angle has been studied and analyzed using field experiments. The paper aims to determine the effect of the triangular channel's bed slope on the hydraulic jump characteristics such as sequent depth ratio, relative jump height, relative energy loss, and the relative jump length. For this motive, wide range of bed slopes, nine positive slopes were tested. The obtained results were represented in dimensionless curves and empirical models using Buckingham's  $\pi$ -Theorem. The developed curves and empirical computational models were used to estimate the different hydraulic jump characteristics for different initial Froude number and triangular channel bed slope. Also, the obtained results were compared with the previous results of horizontal triangular channel layout.

*Keywords:* Hydraulic jump; triangular channel; positive slope; open channels; irrigation ditches.

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## LIST OF SYMBOLS

- $E_1$  : Energy upstream the jump (m)  
 $E_2$  : Energy downstream the jump (m)  
 $E$  : Relative energy loss =  $(E_1 - E_2)/E_1$  (-)  
 $F_1$  : Initial Froude number (-)  
 $g$  : Gravity acceleration ( $ms^{-2}$ )  
 $H$  : Relative jump height =  $(y_2 - y_1)/y_1$  (-)  
 $L$  : Relative jump length =  $L_j/y_1$  (-)  
 $L_j$  : Jump length (m)  
 $S$  : Bed slope (-)  
 $y_1$  : Initial depth (m)  
 $y_2$  : Sequent depth (m)  
 $Y$  : Sequent depth ratio =  $y_2/y_1$  (-)

## 1. INTRODUCTION

Hydraulic jump in a triangular open channel has not studied with a great interest that granted his counterpart of the rectangular channel. However, although rare studies were devoted to suggest that, the hydraulic jump in a triangular channel is much more advantageous vis-a-vis the energy dissipation and requires a lower sequent depth to the same initial value of Froude number Hager and Wanoschek [1]. However, the cross section geometry depends on the stability of the formed jump the earliest known studies are those of Argyropoulos [2], Rajaratnam [3], and Silvester [4]. The first and second led used a symmetrical channel 47° and 60° opening angles respectively. The results showed, as a general rule, that the momentum equation remains sufficient to evaluate the sequent depth ratio,  $Y$  of the jump, and the obtained experimental  $Y$  values are slightly lower than the theoretical  $Y$  values. This difference was about 5% which is also observed by Hager and Wanoschek [1] for 90° triangular channel opening angle. Larry [5] concluded that the main design and analysis principles of open channel flow are valid for ditches. Achour and Debabeche [6] showed that the formed hydraulic jump in triangular ditch may be used to raise the downstream water surface. Das [7] used solutions of quadratic and cubic equations for determination of alternate depths and sequent depths in trapezoidal, rectangular and triangular channels. Vatankhah and Kouchakzadeh [8] presented solutions of specific energy and specific force equations in trapezoidal, rectangular and triangular open channels using iterative fixed-point method. In 2010, Vatankhah [9] solved analytically the specific energy and specific force equations for the three channel cross sections. Vatankhah and Omid [10] showed steps to reach an acceptable physical analytic solution for sequent depth ratios

in horizontal triangular channels. Rashwan [11] developed theoretically equations for calculating hydraulic jump characteristics in horizontal bed triangular channel for given initial conditions but using these equations depends on estimation value for relative head loss. Achour and Debabeche [12] investigated theoretically the hydraulic jump in a sloped triangular channel with a central angle of 90°. A theoretical relationship for sequent depth ratio,  $Y$  as a function of initial Froude number  $F_1$  and bed slope was proposed. The proposed relationship was obtained by application of the momentum equation applied between the upstream and downstream sections of the jump. Only four bed slopes were used in their study. Mahmoud [13] studied experimentally the effect of rectangular channel bed slope on the hydraulic jump characteristics.

Few numbers of researchers studied the hydraulic jump through triangular channels and limited number of them studied it through sloped bed. The present paper carried out field experiments on hydraulic jump in a sloped bed triangular channel with a central angle of 90° using wide range of bed slopes (nine slopes). Empirical computational models were developed to estimate any hydraulic jump characteristic for given initial Froude number and bed slope. In addition, the experimental analysis will be proposed to find a better formulation of the obtained theoretical relationship.

## 2. EXPERIMENTAL PROCEDURE

The present paper carried out field experiments on a triangular channel with central angle of 90° with dimensions of 2.50 x 0.40 x 0.40 m, Fig. 1. Series of runs at different values of discharge were conducted and hydraulic jump was formed by operating the tail gate and sluice gate. Four shapes of the used sluice gates were used to have different gate opening. Nine bed slopes,  $S$  with values of 0 (horizontal bed), 0.01, 0.02, 0.03, 0.035, 0.04, 0.05, 0.07, and 0.11 were used. The experimental procedure is as follows:

1. Start the pump and turn the flow control valve open.
2. Allow the flow to establish the water level in the reservoir behind the gate should be steady.
3. Place the first sluice gate carefully to create a hydraulic jump which is fixed at about the first third section of the flume.
4. Measure sequent depths,  $y_1$  and  $y_2$  before and after the jump respectively using a point gage and length of hydraulic jump  $L_j$ .

5. Record the discharge  $Q$  (l/sec) which, was supplied from artesian well using calibrated sharp crested rectangular weir.
6. Repeat steps 2 through 5 for different values of  $Q$ , bed slopes and sluice gate shapes. The value of  $Q$  can be changed by adjusting the flow control valve.

for bed slope,  $S$  of 0.02 increasing in initial Froude number  $F_1$  by 39.5% results in increasing in the sequent depth ratio by 30%. On the other hand, for Froude number  $F_1$  value of 6.5, the sequent depth ratio,  $Y$  increases by 25% as the bed slope increases by 400%. It is evident from the figure that the least square root,  $R^2$  of the fitted lines has values of 0.97 to 0.99.

### 3. RESULTS AND DISCUSSION

Variations of different hydraulic jump characteristics such as the sequent depth ratio  $Y$ , relative jump height  $H$ ,  $((y_2 - y_1) / y_1)$ , relative jump length  $L$ ,  $(L / y_1)$ , and relative energy loss  $E$   $((\text{initial energy}, E_1 - \text{sequent energy } E_2) / E_1)$  with initial Froude number,  $F_1$  and channel bed slope,  $S$  are given below.

#### 3.1 Variation of Sequent Depth Ratio, $Y$ with Initial Froude Number, $F_1$ and Bed Slope, $S$

Fig. 2 shows that, the sequent depth ratio,  $Y$  increases with increase in the initial Froude number,  $F_1$  and the bed slope,  $S$ . For example,

#### 3.2 Variation of Relative Jump Height, $H$ with Initial Froude Number, $F_1$ and Bed Slope, $S$

It is observed from the obtained data that the relative jump height,  $H$  increases as the initial Froude number,  $F_1$  and bed slope,  $S$  increase. It was found that, increasing 71% in the initial Froude number,  $F_1$  results in 67% increasing in the relative jump height,  $H$  at bed slope,  $S$  of 0.03. Also, increasing 300% in the bed slope,  $S$  results in 13.3% increasing in the relative jump height,  $H$  at Froude number of 6.5. It is evident from the figure that approximately over than 96% of the results are lying within  $\pm 10\%$  of the fitted lines, as shown in Fig. 3.

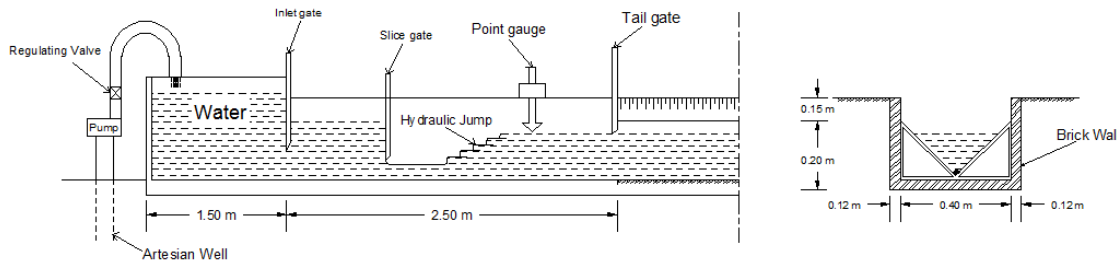


Fig. 1. Schematic layout of experimental setup

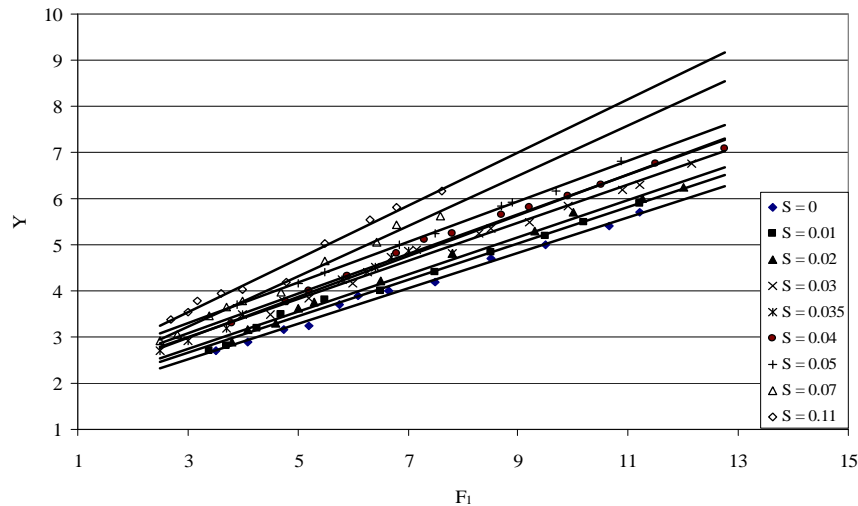


Fig. 2. Variation of  $Y$  with  $F_1$  at different  $S$

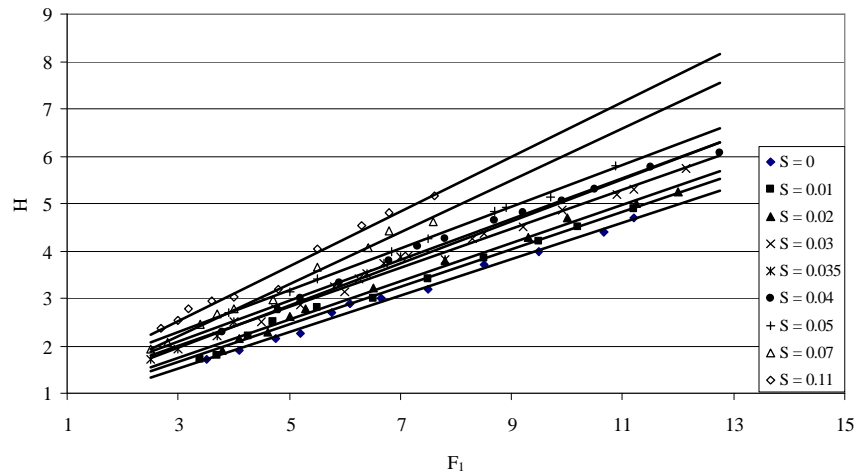


Fig. 3. Variation of H with  $F_1$  at different S

### 3.3 Variation of Relative Energy Loss, E with Initial Froude Number, $F_1$ and Bed Slope, S

From the Fig. 4, it is observed that the relative energy loss, E increases as the initial Froude number,  $F_1$  increases and decreases as the bed slope, S increases. Analysis of the collected results shows as example that at bed slope of 0.03, the relative energy loss, E increases by 53% as the initial Froude number,  $F_1$  increases by 88.9%. Also, the relative energy loss, E decreases by 14% as a result of 400% increasing in bed slope, S. It shows also, that approximately over 98% of experimental data are lying within  $\pm 10\%$  of the fitted lines.

### 3.4 Variation of Relative Jump Length, L with Initial Froude Number, $F_1$ and Bed Slope, S

Through the analysis of the obtained results in this section, it was found that the relative jump length, L has a directly relationship with the initial Froude number,  $F_1$  and inversely relationship with the bed slope S. For instance, for bed slope, S of 0.03 the relative jump length ratio, L has 57% increasing corresponding to 71% increasing in the initial Froude number,  $F_1$ . Approximately over than 97% of experimental data are lying within  $\pm 10\%$  of the fitted lines, see Fig. 5. The reason of deviation of few data points from the fitted lines may be due to inaccuracy in measurement of  $y_1$ , discharge and  $L_j$ .

### 3.5 Empirical Computational Models

Using Buckingham's  $\pi$ -theorem and regression analysis of the obtained experimental data, empirical computational models were developed. On the basis of linear fitting between the sequent depth ratio, Y, relative jump height, H, relative energy loss, E, and relative jump length, L and dimensionless sequent depth factor  $I_D$ , jump height factor  $I_H$ , energy loss factor  $I_E$ , and jump length factor  $I_L$  respectively, the empirical models were developed, Equations (5,6,7, and 8). The values of Y, H, E, and L can be estimated using the fitted lines in Figs. (6,7,8, and 9) or directly from Equations (5,6,7, and 8). Also, Figs. (6,7,8, and 9) show comparison of Y, H, E, and L of these equations with the theoretical equations developed by Debabeche [12] and Rashwan [11]. It was observed that approximately all the data of Debabeche [12] are close to the present model but the equation developed by Rashwan [11] gives values beyond the present model this may be due to the fact that his model requires an estimation value of the dimensionless head loss. It proves that the present model and Debabeche model are equally better than the model of Rashwan.

$$I_D = 0.4482 F_1 + 8.7798 S + 1.4194 \quad (1)$$

$$I_H = 0.4482 F_1 + 8.7798 S + 0.4194 \quad (2)$$

$$I_E = 0.1202 F_1 - 2.0668 S - 0.1176 \quad (3)$$

$$I_L = 5.9695 F_1 - 45.854 S + 6.1432 \quad (4)$$

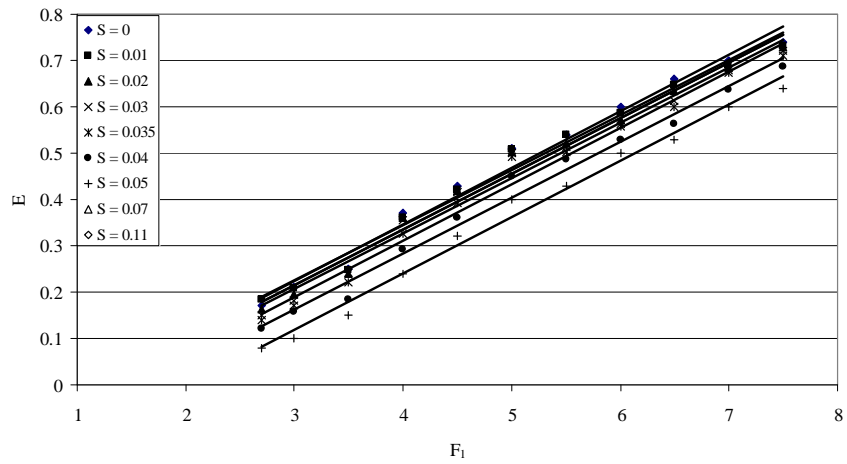


Fig. 4. Variation of E with  $F_1$  at different S

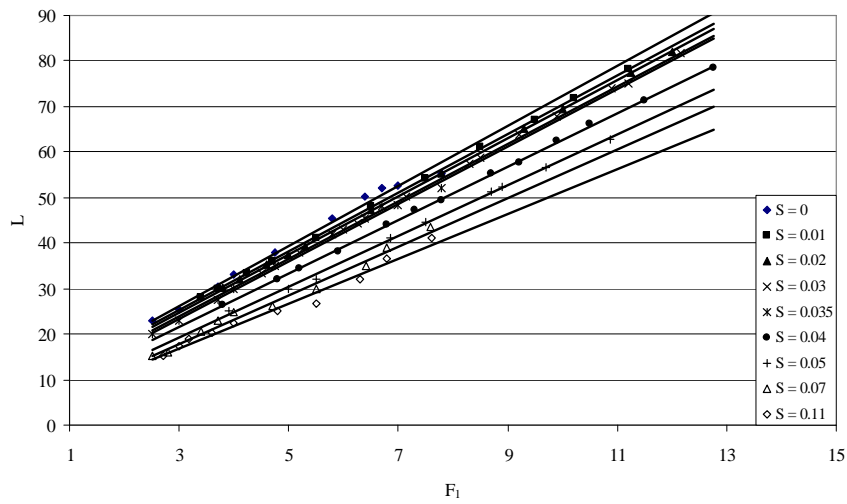


Fig. 5. Variation of L with  $F_1$  at different S

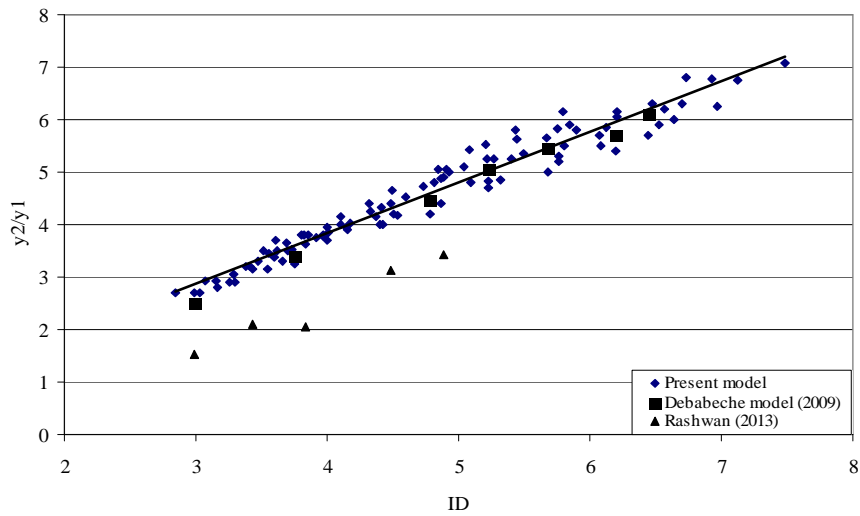


Fig. 6. Comparison of model equation 5 with Debabeche and Rashwan models

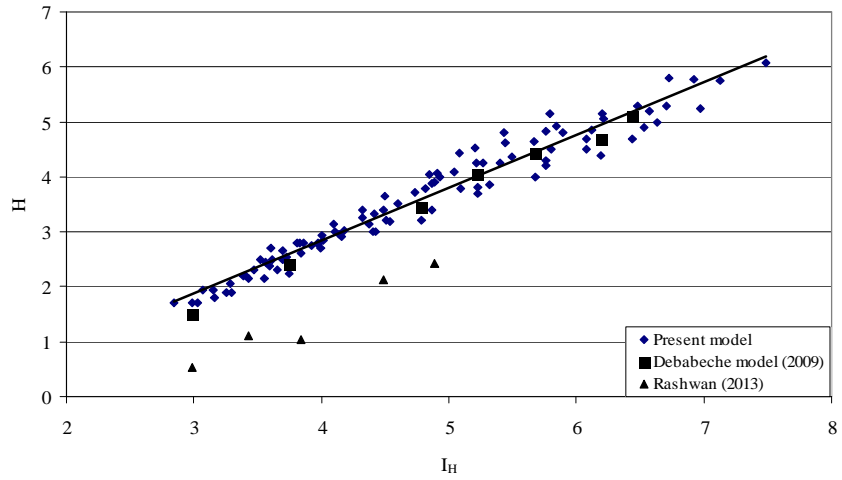


Fig. 7. Comparison of model equation 6 with Debabeche and Rashwan models

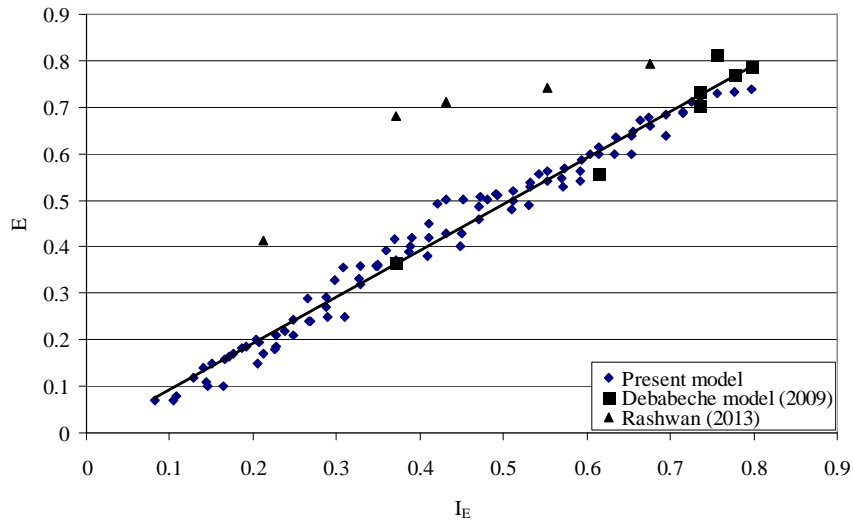


Fig. 8. Comparison of model equation 7 with Debabeche and Rashwan models

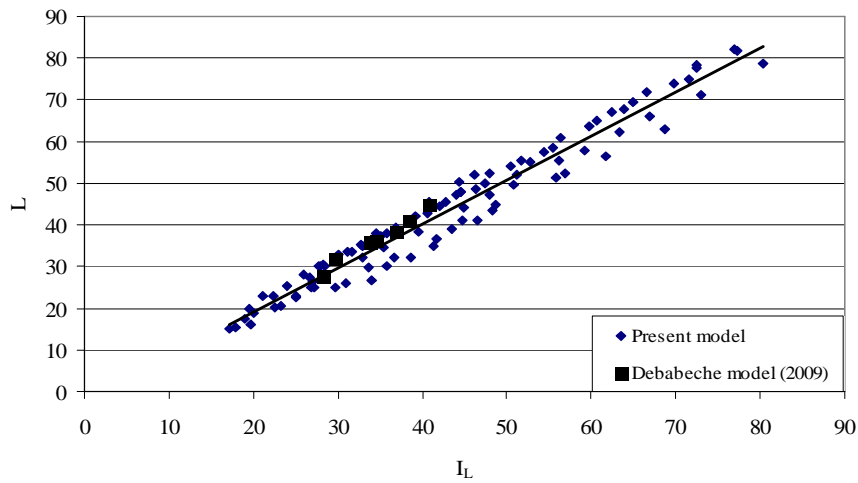


Fig. 9. Comparison of model equation 8 with Debabeche models

$$Y = 0.4315 F_1 + 8.4532 S + 1.3504 \quad (5)$$

$$H = 0.4315 F_1 + 8.4532 S + 0.3504 \quad (6)$$

$$E = 0.1193 F_1 - 2.0511 S - 0.1222 \quad (7)$$

$$L = 6.302 F_1 - 48.408 S + 4.464 \quad (8)$$

#### 4. CONCLUSION

Through the analysis of the obtained field experimental results of hydraulic jump in triangular channel with different bed slopes, the following conclusions were obtained;

1. For a given bed slope, S, the hydraulic jump characteristics, the sequent depth ratio, Y, relative jump height, H, relative energy loss, E, and relative jump length, L increase with the increase of the initial Froude number,  $F_1$ .
2. At a known initial Froude number,  $F_1$ , the sequent depth ratio, Y, and relative jump height, H increase with the increase of the bed slope, S but relative energy loss, E, and relative jump length, L decreases with the increase of the bed slope, S.
3. These results are important for using the developed empirical computational models for estimating the values of hydraulic jump characteristics, which are the sequent depth ratio, Y, relative jump height, H, relative energy loss, E, and relative jump length, L for given initial Froude number,  $F_1$  and the bed slope, S.

#### COMPETING INTERESTS

Authors have declared that no competing interests exist.

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