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Evaluating Measure of Reformed Rotatability for Second Degree Polynomial Using a Pair of Incomplete Block Designs with Two Unequal Block Sizes

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Authors' contributions

This work was carried out in collaboration between both authors. Author PJ designed the study, performed the statistical analysis, wrote the protocol and wrote the first draft of the manuscript. Author BRV managed the analyses of the study and managed the literature searches. Both authors read and approved the final manuscript.

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Abstract

In this paper, a study on evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes (like symmetrical unequal block arrangements with two unequal block sizes) is suggested which enables us to assess the degree of reformed (modified) rotatability for a given response surface design.

Keywords: Response surface methodology; reformed rotatability; measure.

1 Introduction

Response surface process is a collection of mathematical and statistical techniques appropriate for analysing problems in which several independent variables influence a dependent variable. The regressor variables are often called input or explanatory variables and the regressand variable is often the response variable. An important development of polynomial designs was the introduction of rotatable designs suggested by Box

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and Hunter [1]. Rotatable designs using balanced incomplete block designs (BIBD) was proposed by Das and Narasimham [2]. A design is said to be rotatable, if the variance of the response estimate is a function only of the distance of the point from the design centre. Raghavarao [3] constructed second order rotatable designs (SORD) using incomplete block designs. Das et al. [4] developed reformed second degree polynomial designs. Park et al. [5] introduced measure of rotatability for second degree polynomial designs. Victorbabu et al. [6] studied measure of rotatability for second degree polynomial design using a pair of incomplete block designs with two unequal block sizes (like symmetrical unequal block arrangements (SUBA) with two unequal block sizes). Victorbabu [7] studied reformed rotatable designs of second order using a pair of incomplete block designs with two unequal block sizes.

A lot of work was carried out by Victorbabu and some other authors on reformed rotatability, measure of rotatability and measure of reformed slope rotatability on second degree polynomial designs respectively Victorbabu and Vasundharadevi [8,9], Victorbabu et al. [9,10], Victorbabu [7,10], Victorbabu and Surekha [11,12,13], Victorbabu et al. [6,14], Victorbabu and Jyostna [15,16], Jyostna and Victorbabu [17,18]. Recently, evaluating measure of reformed rotatability is studied by Jyostna and Victorbabu [19,20,21,22] using central composite design (CCD), BIBD, pairwise balanced design and incomplete block designs (like SUBA with two unequal block sizes) respectively. Raghavarao [23] wrote a book "constructions and combinatorial problems in design of experiments". Victorbabu [24,25] suggested a review on rotatability of second order and measure of rotatability for second degree polynomial designs respectively.

In this paper, a new measure of reformed rotatability for second degree polynomial designs using a pair of incomplete block designs with two unequal block sizes (like SUBA with two unequal block sizes) is suggested.

2 Conditions for SORD

Suppose we want to use the second degree polynomial model D= $((x_{iu}))$ to fit the surface,

$$Y_u = b_0 + \sum_{i=1}^{v} b_i x_{iu} + \sum_{i=1}^{v} b_{ii} x_{iu}^2 + \sum\sum_{i<j} b_{ij} x_{iu} x_{ju} + e_u \,, \tag{1}$$

where x_{iu} denotes the level of the i^{th} factor (i =1,2,…,v) in the u^{th} run (u=1,2,…,N) of the experiment, e_u's are uncorrelated random errors with mean zero and variance σ^2 is said to be rotatable design of second order, if the variance of the estimated response of $\hat{Y}_u$ from the fitted surface is only a function of the distance ($d^2 = \sum_{i=1}^{v} x_i^2$) of the point ($x_1, x_2, \ldots, x_v$) from the origin (centre) of the design. Such a spherical variance function for estimation of second degree polynomial is achieved if the design points satisfy the following conditions [1,2]).

1. $\sum x_{iu}=0$, $\sum x_{iu}x_{ju}=0$, $\sum x_{iu}x_{ju}^2=0$, $\sum x_{iu}x_{ju}x_{ku}=0$, $\sum x_{iu}^3=0$, $\sum x_{iu}x_{ju}^3=0$, $\sum x_{iu}x_{ju}x_{ku}^2=0$, $\sum x_{iu}x_{ju}x_{ku}x_{lu}=0$; for $i \neq j \neq k \neq l$; (2)

2. (i) $\sum x_{iu}^2=$ constant = $N\lambda_2$;

   (ii) $\sum x_{iu}^4=$ constant = $cN\lambda_4$; for all i (3)

3. $\sum x_{iu}^2 x_{ju}^2 =$ constant $= N\lambda_4$; for $i \neq j$ (4)

4. $\sum x_{iu}^4 = c\sum x_{iu}^2 x_{ju}^2$; (5)

5. $\dfrac{\lambda_4}{\lambda_2^2} > \dfrac{v}{(c+v-1)}$ (6)

where, v denotes number of factors c, λ_2 and λ_4 are constants and the summation is over the design points. If the above mentioned conditions are satisfied, the variances and covariances of the estimated parameters become,

$$V(\hat{b}_0) = \frac{\lambda_4(c+v-1)\sigma^2}{N\left[\lambda_4(c+v-1)-v\lambda_2^2\right]},$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\lambda_2},$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4},$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}\left[\frac{\lambda_4(c+v-2)-(v-1)\lambda_2^2}{\lambda_4(c+v-1)-v\lambda_2^2}\right],$$

$$Cov(\hat{b}_0,\hat{b}_{ii}) = \frac{-\lambda_2\sigma^2}{N[\lambda_4(c+v-1)-v\lambda_2^2]},$$

$$Cov(\hat{b}_{ii},\hat{b}_{jj}) = \frac{(\lambda_2^2-\lambda_4)\sigma^2}{(c-1)N\lambda_4[\lambda_4(c+v-1)-v\lambda_2^2]} \tag{7}$$

and other covariances are zero.

3 Conditions for Reformed SORD

Let $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$, $k_1=\sup(k_{11},k_{12})$, $b_{11}+b_{12}=b_1$ with $r_1 \underset{>}{\overset{<}{-}} c\lambda_1$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$, $k_2=\sup(k_{21},k_{22})$, $b_{21}+b_{22}=b_2$ with $r_2 \underset{<}{\overset{>}{-}} c\lambda_2$ be a pair of incomplete block designs with two unequal block sizes (like SUBA with two unequal block sizes) respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ denotes a Resolution V fractional factorial of 2^{k_1} and 2^{k_2} factorials with levels ± 1, such that no interaction with less than five factors is confounded. The design points achieved from the transpose of incidence matrix of design D_1 by "multiplication" (cf. Raghavarao [23]) are denoted by $\left[1\text{-}(v, b_1, r_1, k_{11}, k_{12}, b_{11}, b_{12}, \lambda_1)\right]2^{t(k_1)}$. Let $\left[1\text{-}(v, b_1, r_1, k_{11}, k_{12}, b_{11}, b_{12}, \lambda_1)\right]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generated from D_1 by "multiplication". Let $\left[a\text{-}(v, b_2, r_2, k_{21}, k_{22}, b_{21}, b_{22}, \lambda_2)\right]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points generated from D_2 by "multiplication". Let n_0 be the number of central points. The usual method of construction of rotatable designs using a pair of incomplete block designs with two unequal block sizes is to take combinations with unknown constants, associate a 2^v factorial combinations or a suitable fraction of it with factors each at ± 1 levels to make the level codes equidistant. All such combinations form a design. Generally, rotatable designs of second order need at least five levels (suitably coded) at $0, \pm 1, \pm a$ for all factors ($(0, 0, \ldots 0)$)- chosen centre of the design, unknown level 'a' be chosen suitably to satisfy the conditions of the rotatability) generation of design points this way ensures satisfaction of all the conditions even though the design points contain unknown levels.

Alternatively, by putting some restrictions indicating some relation among $\sum x_{iu}^2, \sum x_{iu}^4$ and $\sum x_{iu}^2 x_{ju}^2$ some equations involving the unknowns are obtained and their solution gives the unknown levels. In SORD the restriction used is $\sum x_{iu}^4 = 3\sum x_{iu}^2 x_{ju}^2$, i.e., $c=3$. Other restriction is also possible. We shall investigate the restriction $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ ie., $\lambda_2^2 = \lambda_4$ to get another series of symmetrical second degree polynomial designs, which provide more precise estimates of response at specific points of interest than what is available from the corresponding existing designs [4]. Simplification of (7) using the reformed (modified) condition $\lambda_2^2 = \lambda_4$ the variances and covariances of the estimated parameters are,

$$V(\hat{b}_0) = \frac{(c+v-1)\sigma^2}{N(c-1)}$$

$$V(\hat{b}_i) = \frac{\sigma^2}{N\sqrt{\lambda_4}}$$

$$V(\hat{b}_{ij}) = \frac{\sigma^2}{N\lambda_4}$$

$$V(\hat{b}_{ii}) = \frac{\sigma^2}{(c-1)N\lambda_4}$$

$$\operatorname{cov}(\hat{b}_0, \hat{b}_{ii}) = \frac{-\sigma^2}{N\sqrt{\lambda_4}(c-1)} \tag{8}$$

and other covariances are zero. These modifications of the variances and covariances affect the variance of the estimated response at specific points considerably. Using these variances and covariances, variance of estimated response at any point can be obtained. Let $\hat{Y}_u$ denote the estimated response at the point $(x_{1u}, x_{2u}, \ldots x_{vu})$. Then,

$$V(\hat{y}_u) = V(\hat{b}_0)+d^2[V(\hat{b}_i) + 2\text{cov}(\hat{b}_0, \hat{b}_{ii})] + d^4V(\hat{b}_{ii}) + (\sum x_{iu}^2 x_{ju}^2)[(c\text{-}3)\sigma^2 / (c\text{-}1)N\lambda_4]$$

The study of reformed polynomial designs is the same as for SORD except that instead of taking $c=3$ the restriction $(\sum x_{iu}^2)^2=N\sum x_{iu}^2 x_{ju}^2$ is to be used and this condition will provide different values of the unknowns involved.

4 Conditions for Evaluating Measure of Rotatability for Second Degree Polynomial

Following Box and Hunter [1], Das and Narasimham [2], Park et al. [5], conditions (2) to (6) and (7) provide the necessary and sufficient conditions for evaluating measure of rotatability for any general second degree polynomial. Further we have,

$V(b_i)$ are equal for i,

$V(b_{ii})$ are equal for i,

$V(b_{ij})$ are equal for i, j, where $i \neq j$,

$$\text{Cov}(b_i,b_{ii})=\text{Cov}(b_i,b_{ij})=\text{Cov}(b_{ii},b_{ij})=\text{Cov}(b_{ij},b_{il})=0 \text{ for all } i \neq j, j \neq l, l \neq i. \tag{9}$$

Park et al. [5] suggested that if the conditions in (2) to (6) together along with (7) and (9) are satisfied, then the following measure $(P_v(D))$ can be used to assess the degree of rotatability for any general second degree polynomial [5].

$$P_v(D) = \frac{1}{1+R_v(D)}, \tag{10}$$

here,

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \frac{6v\left[V(\hat{b}_{ij}) + 2\,\text{cov}(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})\right]^2 (v\text{-}1)}{(v+2)^2(v+4)(v+6)(v+8)g^8} \tag{11}$$

and g is the scaling factor.

On simplification, numerator of (11), $[V(\hat{b}_{ij}) + 2\,\text{cov}(\hat{b}_{ii}, \hat{b}_{jj}) - 2V(\hat{b}_{ii})]$ using (7) becomes $(c\text{-}3)\sigma^2/(c\text{-}1)N\lambda_4$. Thus $R_v(D)$ becomes

$$R_v(D) = \left[\frac{N}{\sigma^2}\right]^2 \left(\frac{6v[(c\text{-}3)\sigma^2]^2(v\text{-}1)}{[(c\text{-}1)N\lambda_4]^2(v+2)^2(v+4)(v+6)(v+8)g^8}\right) \tag{12}$$

Note: For SORD, we take $c=3$. Substituting the value of $'c'$ and on simplification of (12) we get $R_v(D)$ is zero. Hence from (10), we get $P_v(D)$ is one if and only if a design is rotatable and less than one then it is nearly rotatable design.

5 Reformed Rotatability for Second Degree polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes [7]

The method of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes (SUBA with two unequal block sizes) is given in the following result [7].

If $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ are two incomplete block designs with two unequal block sizes, Let $k_1=\sup(k_{11},k_{12})$, $b_{11}+b_{12}=b_1$ with $r_1 \underset{>}{\overset{<}{-}} c\lambda_1$ of D_1 and $k_2=\sup(k_{21},k_{22})$, $b_{21}+b_{22}=b_2$ with $r_2 \underset{<}{\overset{>}{-}} c\lambda_2$ of D_2 designs respectively. Let $2^{t(k_1)}$ and $2^{t(k_2)}$ Resolution V fractional replicates of 2^{k_1} and 2^{k_2} factorials with levels ± 1, such that no interaction with less than five factors is confounded. Let $[1\text{-}(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]$ denote the design points generated from the transpose of incidence matrix of a pair of incomplete block designs with two unequal block sizes, $[1\text{-}(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]2^{t(k_1)}$ are the $b_1 2^{t(k_1)}$ design points generated from design D_1 by "multiplication" (cf. Raghavarao [23]). Repeat these design points y_1 times. Let $[a\text{-}(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)]2^{t(k_2)}$ are the $b_2 2^{t(k_2)}$ design points generated from design D_2 by multiplication. The design D_2 repeated by y_2 times and n_0 be the number of central points. Then the method of construction of reformed rotatability for second degree polynomial is established as follows.

The design points,

$y_1[1\text{-}(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]2^{t(k_1)} \cup y_2[a\text{-}(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)]2^{t(k_2)} \cup (n_0)$ will give a v-dimensional reformed rotatability for second degree polynomial in

$$N=\frac{(y_1r_1 2^{t(k_1)}+y_2r_2 2^{t(k_2)}a^2)^2}{y_1\lambda_1 2^{t(k_1)}+y_2\lambda_2 2^{t(k_2)}a^4}$$ design points if,

$$a^4=\frac{y_1(3\lambda_1\text{-}r_1)}{y_2(r_2\text{-}3\lambda_2)}2^{t(k_1)\,\text{-}\,t(k_2)},$$

$$n_0=\frac{(y_1r_1 2^{t(k_1)}+y_2r_2 2^{t(k_2)}a^2)^2}{y_1\lambda_1 2^{t(k_1)}+y_2\lambda_2 2^{t(k_2)}a^4}-y_1b_1 2^{t(k_1)}-y_2b_2 2^{t(k_2)}$$ and n_0 turns out to be an integer.

6 Measure of Rotatability for Second Degree Polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes

The result of measure of rotatability for second degree polynomial using a pair of incomplete block designs (like SUBA with two unequal block sizes) is suggested here [6].

Let $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$ and $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ are two incomplete block designs with two unequal block sizes. Then the design points, $y_1[1\text{-}(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]2^{t(k_1)} \cup y_2[a\text{-}(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)]2^{t(k_2)} \cup (n_0)$ will give a v-dimensional measure of rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes in $N = y_1 b_1 2^{t(k_1)} + y_2 b_2 2^{t(k_2)} + n_0$ design points without any additional set of points with 'a' prefixed and $c = \dfrac{y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^4}{y_1 \lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4}$ as follows (we take $y_1 = 1, y_2 = 1$).

We can obtain the measure of rotatability values for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. Here

$$R_v(D) = \left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2 (v+2)^2 (v+4)(v+6)(v+8) g^8}$$

where,

$$g = \begin{cases} \dfrac{1}{a}, \text{ if } a \le \sqrt{\dfrac{1}{r_2}\left[\dfrac{y_1(b_1\text{-}r_1)2^{t(k_1)\text{-}t(k_2)}}{y_2} + b_2\right]} \\ \dfrac{1}{\sqrt{\dfrac{1}{r_2}\left[\dfrac{y_1(b_1\text{-}r_1)2^{t(k_1)\text{-}t(k_2)}}{y_2} + b_2\right]}}, \text{ otherwise} \end{cases}$$

$$P_v(D) = \frac{1}{1+R_v(D)}$$

If $P_v(D)$ is 1 if and only if the design is rotatable, and it is smaller than one for a nearly rotatable designs.

7 Evaluating Measure of Reformed Rotatability for Second Degree Polynomial using a Pair of Incomplete Block Designs with Two Unequal Block Sizes

The proposed method of evaluating measure of reformed rotatability for second degree polynomial designs using a pair of incomplete block designs with two unequal block sizes (like SUBA with two unequal block sizes) is suggested as follows.

Let $D_1=(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)$, $D_2=(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)$ are two incomplete block designs with two unequal block sizes. Then design points $y_1[1\text{-}(v,b_1,r_1,k_{11},k_{12},b_{11},b_{12},\lambda_1)]2^{t(k_1)} \cup y_2[a\text{-}(v,b_2,r_2,k_{21},k_{22},b_{21},b_{22},\lambda_2)]2^{t(k_2)} \cup (n_0)$ will give a measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. From 2 of (i), (ii) of equation (3) and 3 of equation (4) we have,

$$\sum x_{iu}^2 = y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^2 = N\lambda_2 \tag{13}$$

$$\sum x_{iu}^4 = y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^4 = cN\lambda_4 \tag{14}$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1 \lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4 = N\lambda_4 \tag{15}$$

From (14) and (15), we get

$$a^4 = \frac{y_1(3\lambda_1 - r_1)}{y_2(r_2 - 3\lambda_2)} 2^{t(k_1)\,-\,t(k_2)}$$

The reformed condition $(\sum x_{iu}^2)^2 = N\sum x_{iu}^2 x_{ju}^2$ leads to N which is given by

$$N = \frac{(y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^2)^2}{y_1\lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4}.$$

Alternatively N may be obtained directly as $N = y_1 b_1 2^{t(k_1)} + y_2 b_2 2^{t(k_2)} + n_0$ design points without any additional set of points, where n_0 is given by $n_0 = \frac{(y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^2)^2}{y_1\lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4} - y_1 b_1 2^{t(k_1)} - y_2 b_2 2^{t(k_2)}$ n_0 turns out to be an integer with 'a' prefixed and $c = \frac{y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^4}{y_1\lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4}$.

From equations (13) and (15) and on simplification we get $\lambda_2 = \frac{y_1 r_1 2^{t(k_1)} + y_2 r_2 2^{t(k_2)} a^2}{N}$ and $\lambda_4 = \frac{y_1 \lambda_1 2^{t(k_1)} + y_2 \lambda_2 2^{t(k_2)} a^4}{N}$.

To obtain evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes, we have

$$P_v(D)=\frac{1}{1+R_v(D)}$$

$$R_v(D)=\left[\frac{(c\text{-}3)}{(c\text{-}1)}\right]^2 \frac{6v(v\text{-}1)}{\lambda_4^2(v+2)^2(v+4)(v+6)(v+8)g^8}$$

Here g is a scaling factor,

$$g=\begin{cases}\dfrac{1}{a}, \text{ if } a\le\sqrt{\dfrac{1}{r_2}\left[\dfrac{y_1(b_1\text{-}r_1)2^{t(k_1)\text{-}t(k_2)}}{y_2}+b_2\right]} \\ \dfrac{1}{\sqrt{\dfrac{1}{r_2}\left[\dfrac{y_1(b_1\text{-}r_1)2^{t(k_1)\text{-}t(k_2)}}{y_2}+b_2\right]}}, \text{ otherwise}\end{cases}$$

The following table gives the values of an evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes. It can be verified that $P_v(D)$ is 1 if and only if the design is reformed rotatable, and it is smaller than one for nearly reformed rotatable designs.

Example: We illustrate the evaluating measure of reformed rotatability for second degree polynomial for $v=9$ factors with the help of a pair of incomplete block designs with two unequal block sizes with parameters $D_1=(v=9,b_1=15,r_1=7,k_{11}=3,k_{12}=5,b_{11}=6,b_{12}=9,\lambda_1=3)$ and $D_2=(v=9,b_2=18,r_2=5,k_{21}=2,k_{22}=3,b_{21}=9,b_{22}=9,\lambda_2=1)$. The design points, $y_1[1\text{-}(v=9,b_1=15,r_1=7,k_{11}=3,k_{12}=5,b_{11}=6,b_{12}=9,\lambda_1=3)]2^4\cup y_2[a\text{-}(v=9,b_2=18,r_2=5,k_{21}=2,k_{22}=3,b_{21}=9,b_{22}=9,\lambda_2=1)]2^3\cup(n_0)$ will give a measure of reformed rotatability for second degree polynomial in N=722 design points. From (13), (14) and (15), we have

$$\sum x_{iu}^2 = y_1 112+y_2 40a^2 = N\lambda_2 \tag{16}$$

$$\sum x_{iu}^4 = y_1 112+y_2 40a^4 = cN\lambda_4 \tag{17}$$

$$\sum x_{iu}^2 x_{ju}^2 = y_1 48+y_2 8a^4 = N\lambda_4 \tag{18}$$

From equations (17) and (18) with rotatability value $c=3$, $y_1=2$ and $y_2=1$, we get $a^4=4\Rightarrow a^2=2\Rightarrow a=1.414213$. From equations (16) and (18) using the reformed condition with $(\lambda_2^2=\lambda_4)$ along with $a^2=2$, $y_1=2$ and $y_2=1$, we get $N=722$, $n_0=98$. For reformed SORD we get $P_v(D)=1$ by taking $a=1.414213$ and scaling factor $g=0.7071$. Then the design is reformed SORD using a pair of incomplete block designs with two unequal block sizes.

Instead of taking $a=1.414213$ if we take $a=2.2$ for the above pair of incomplete block designs with two unequal block sizes $D_1=(v=9,b_1=15,r_1=7,k_{11}=3,k_{12}=5,b_{11}=6,b_{12}=9,\lambda_1=3)$ and $D_2=(v=9,b_2=18,r_2=5,k_{21}=2,k_{22}=3,b_{21}=9,b_{22}=9,\lambda_2=1)$ from equations (17) and (18), we get $c=4.096698$. The scaling factor $g=0.4545$, $R_v(D)=2.3585$ and $P_v(D)=0.2978$. Here $P_v(D)$ becomes smaller it deviates from reformed rotatability.

Table gives the values of evaluating measure of reformed rotatability $P_v(D)$ for second degree polynomial using a pair of incomplete block designs with two unequal block sizes, for different values of 'a'. It can be verified that $P_v(D)$ is one, if and only if a design 'D' is reformed rotatable. $P_v(D)$ becomes smaller as 'D' deviates from a reformed rotatable design.

Table 1. Evaluating measure of reformed rotatability for second degree polynomial using a pair of incomplete block designs with two unequal block sizes

$(9,15,7,3,5,6,9,3)(9,18,5,2,3,9,9,1)$, N=722, a=1.414213, $n_0=98, y_1=2, y_2=1$				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.5385	1	3.084×10^{-3}	0.9969
1.3	2.8460	0.7692	1.945×10^{-3}	0.9981
*1.414213	3	0.7071	0	1
1.6	3.2753	0.625	0.0215	0.9789
1.9	3.7216	0.5263	0.4092	0.7096
2.2	4.0967	0.4545	2.3585	0.2978
2.5	4.3733	0.4	8.6659	0.1035
2.8	4.5644	0.3571	24.9381	0.0386
3.1	4.6933	0.3226	61.4359	0.016

$(10,15,8,4,6,5,10,4)(10,25,8,4,3,5,20,2)$, N=1024, a=1.414213, $n_0=144, y_1=1, y_2=1$				
a	c	g	$R_v(D)$	$P_v(D)$
1.0	2.4	1	2.733×10^{-3}	0.9973
1.3	2.8332	0.7692	1.0055×10^{-3}	0.9989
*1.414213	3	0.7071	0	1
1.6	3.2419	0.625	0.0074	0.9926
1.9	3.5303	0.5263	0.111	0.9001
2.2	3.7083	0.4545	0.5585	0.6416
2.5	3.8142	0.4529	0.7036	0.5870
2.8	3.8778	0.4529	0.7820	0.5612
3.1	3.9169	0.4529	0.8306	0.5463

**indicates exact reformed (modified) slope rotatability value using a pair of SUBA with two unequal block sizes [7]*

8 Conclusion

The evaluating measure of reformed rotatability for second degree polynomial designs using a pair of incomplete block designs with two unequal block sizes, for different values of 'a'. It can be verified that $P_v(D)$ is one if and only if the design is reformed rotatable design and it is less than one for a nearly reformed rotatable design.

Competing Interests

Authors have declared that no competing interests exist.

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